Identification and simulation of manufacturing uncertainties

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Identification et simulation des incertitudes de fabrication

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Résumé

L'étude présente les méthodes pour identifier et simuler les défauts de fabrication tridimensionnels. Les méthodologies ont été élaborées sur la base des travaux antérieurs, tels que la méthode de simulation MMP (Model of Manufactured Part) présentée par F. Villeneuve et F. Vignat, associée à la méthode de la double mesure présentée par S. Tichadou.

Dans cette thèse, la première méthode proposée, basée sur la méthode des petits déplacements (TPD) est présentée et permet l'identification des défauts de fabrication. Cette méthode permet de distinguer les défauts d'usinage et les défauts de positionnement d'un lot de pièces au cours d'un processus de fabrication. Les résultats obtenus dans cette méthode représentent les dispersions géométriques des pièces usinées. En outre, une méthode d'analyse modale de défauts a été réalisée pour analyser les défauts de forme d'une pièce mesurée sur une MMT avec un nombre restreint de points de mesure (10 points sur chaque surface usinée). Les résultats montrent que les modes des défauts de forme sont obtenus correctement (bombé, ondulation, vrillage, etc.)

En raison de l'importance du rôle du défaut de positionnement dans la qualité d'un produit en cours de fabrication, ensuite deux indicateurs simples ont été proposés pour évaluer la qualité globale d'un montage de fixation de pièces.

Par ailleurs, un modèle permettant de simuler les défauts de positionnement d'une pièce fixée sur un mandrin à trois mors a été développé. Le modèle final de simulation est une combinaison de trois méthodes: plan d'expérience, simulation par éléments finis, et simulation de Monte Carlo. Pour la méthode des plans d'expérience, trois facteurs, qui sont supposés être les plus importants dans les défauts de positionnement, sont utilisés dans le modèle. Les résultats obtenus à partir des simulations sont exprimés sous forme de distributions et de paramètres statistiques caractéristiques. Ceux-ci sont ensuite utilisés pour effectuer les simulations en appliquant la méthode de Monte Carlo.

Enfin, un modèle global est proposé, pour simuler la gamme de fabrication d'une pièce fraisée. Ce modèle permet de vérifier la gamme choisie avec des tolérances fonctionnelles de la pièce imposée. De plus, cette méthode permet de vérifier une gamme de fabrication en garantissant les tolérances fonctionnelles imposées ou une utilisation inverse qui permet de déterminer les tolérances garantissant un nombre de pièces usinées hors des zones de tolérance.

Abstract

The research presents methodologies to identify and simulate manufacturing defects in three-dimension. The methodologies have been developed based on the previous works, such as the MMP (Model of Manufactured Part) simulation method presented by F. Villeneuve and F. Vignat, and the double measurement method is presented by S. Tichadou.

In this thesis, the first proposed method based on the Small Displacement Torsor (SDT) concept is presented for identification of manufacturing defects. This method allows distinguishing the machining defects and positioning defects of a batch of parts during a process plan. The results obtained in this method represent geometric dimension errors of machined parts. In addition, we applied the parameterization method, which is usually used to analyze form defects of a part measured on a CMM with hundreds of measurement points, to complete the analysis of the form defects with a restricted number of measurement points (10 points on each machined surface). Even though this number appears to be low, the modes of the form defects are almost obtained (comber, undulation, twist, etc).

Because of the important role of the positioning defect in the quality of a product during manufacturing, we then propose two simple indicators for evaluating the global quality of a fixture.

Furthermore, we developed a model for simulating positioning defects of a workpiece fixed on a three-jaw chuck. The model is a combination of three methods: design of experiments, finite element simulation, and Monte Carlo simulation. Three factors, which are assumed to be the most important in positioning defects, are used in this model. Based on the simulated results, the influences of these factors are estimated. The results obtained from simulations can be expressed by form of distributions or statistical parameters. These allow using simulation of tolerance analysis based on Monte Carlo simulation.

Finally, a model is developed based on MMP for tolerance analysis. This model allows us to verify a given process plan with functional tolerances of the machined part by determination of a number of machined parts out of tolerance zones or to determine functional tolerances of a batch of machined parts based on a given process plan (without functional tolerances) and a number of rejected parts per million.

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INTRODUCTION

1. DEFAUTS DE FABRICATION

Les défauts de fabrication, par définition, sont des imperfections qui apparaissent inévitablement sur les produits comme résultat de l'imperfection des procédés de fabrication. Un défaut de fabrication sur un produit est déterminé en comparant ce produit au produit conçu avec ses spécifications. Une pièce fabriquée est rejetée si ses défauts de fabrication sont en dehors des spécifications et vice-versa.

Dans cette thèse, on s'intéresse aux écarts de mise en position et d'usinage d'une pièce. Dans ces travaux, les défauts de fabrication sont divisés en deux catégories principales : les défauts de mise en position et les défauts d'usinage.

1.1. Identification des defaults d'usinage

Une méthode de mesure et un traitement de ces mesures sont développés à partir d' études antérieures [TICHADOU 2005, TICHADOU *et al.* 2007] et permettent de distinguer les erreurs d'usinage et les erreurs de mise en position.

1.2. Évaluation du défaut de mise en position

La pièce doit occuper une position unique sur son montage. Les déviations sont dues entre autres aux efforts de serrage et de coupe, et aux frottements au contact pièce-montage. Nous proposons des indicateurs simples pour évaluer la qualité globale d'un montage.

1.3. Simulation du défaut de positionnement d'une pièce sur son montage

Les résultats expérimentaux permettent de proposer des modèles pour étudier les influences de différents facteurs sur la position d'une pièce sur son montage puis d'utiliser ces modèles en simulation.

2. ANALYSE DES TOLERANCES

C'est une étape essentielle en fabrication pour évaluer la qualité des produits en termes de tolérances fonctionnelles.

L'analyse des tolérances est le terme général pour les activités relatives à l'étude de l'accumulation des variations des pièces usinées et assemblées. D'une part, les tolérances fonctionnelles sont spécifiées sur les dessins de définition, et d'autre part les contraintes de fabrication sont imposées par la précision des machines outils, des outils de coupe et des machines à mesurer, etc.

La plupart des outils récents de simulation d'analyse de tolérances et de chaînes de cotes sont plutôt unidirectionnels. Ces simulations ne tiennent pas compte des petits écarts angulaires. Aussi des modèles tridimensionnels ont été développés pour résoudre les problèmes d'analyse de tolérances.

3. ORGANIZATION DE LA THESE

Ce document est organisé en six parties :

- Une introduction pour présenter le sujet, les objectifs, et la méthodologie de cette étude.
- Le Chapitre 1 donne une revue bibliographique relative à l'identification et à la simulation des défauts de fabrication. Les problèmes devant être abordés dans ce domaine de recherche sont discutés dans cette revue bibliographique. Un énoncé du problème est finalement proposé.
- Dans le chapitre 2, on développe une méthode basée sur le concept de torseur de petits déplacements pour identifier les défauts d'usinage et de mise en position d'un lot de pièces tout au long de la gamme d'usinage.
- Le chapitre 3 introduit des indicateurs simples qui permettent d'évaluer la qualité globale d'un montage ainsi que les défauts de mise en position de la pièce sur son montage.
- Le chapitre 4 introduit un modèle mathématique pour l'analyse de tolérances basé sur le modèle MMP qui permet de vérifier la gamme en termes de tolérances fonctionnelles ou bien de déterminer les tolérances minimales d'un lot de pièces usinées en se basant sur une gamme donnée et un taux de rebut donné (ppm).
- Le dernier chapitre « conclusion et perspectives » récapitule les résultats obtenus dans cette thèse et propose des travaux futurs.

CHAPITRE 1

REVUE BIBLIOGRAPHIQUE ET POSITION DU PROBLEME

Ce chapitre donne une revue des travaux en rapport avec cette thèse. D'abord, la littérature est relative aux incertitudes de fabrication, puis les travaux dans le domaine de l'analyse des tolérances sont examinés. Les problèmes qui doivent être traités dans ce domaine de recherche sont discutés. Le problème est posé dans la conclusion de ce chapitre.

1. DEFAUTS DE FABRICATION

Les pièces ne peuvent pas être fabriquées aux dimensions idéales de conception parce que les imprécisions sont inhérentes aux procédés de fabrication. Ces imperfections sont reconnues comme des erreurs qui peuvent provenir de différentes sources. Dans [ZHANG 1997], K. Whybrew et G.A. Britton ont résumé 27 sources d'erreurs dans un processus de fabrication pour les 8 éléments de fabrication suivants : machine outil, outil de coupe, montage d'usinage, pièce usinée, fluide de coupe, opérateur, conditions extérieures, et variables du procédé.

1.1. Facteurs affectant la qualité du produit en fabrication

1.1.1 Erreurs de machine-outil

La précision d'une machine-outil est un des facteurs qui affecte la précision des cotes de la pièce usinée. Différentes sources d'erreurs peuvent induire des erreurs de machines-outils. Plusieurs travaux de recherche se sont intéressés à deux sortes de sources d'erreurs de machines-outils. Les erreurs dues aux imprécisions géométriques, et les erreurs induites par les effets thermiques.

La plupart des modèles utilisés pour l'identification, l'évaluation ou la compensation des erreurs géométriques des machines sont basés sur les matrices de transformation homogènes (HTM). Il s'agit d'une matrice 4x4 utilisée pour représenter n'importe qu'elle position et orientation d'un repère dans l'espace tridimensionnel. [EMAN *et al.* 1987] ont développé un modèle d'erreurs basé sur l'imprécision géométrique et les relations mutuelles des éléments structuraux de la machine aussi bien que les erreurs résultant des mouvements relatifs entre ces éléments. [KIM *et al.* 1991] présentent un modèle volumique d'erreurs géométriques généralisé par les matrices homogènes de transformation (HTM). [RAHMAN *et al.* 2000] utilisent un modèle d'erreurs volumiques tridimensionnel pour compenser les erreurs de la machine–outil. Ce modèle est obtenu par combinaison de différentes mesures comme la mise en position, la rectitude, l'équerrage, les erreurs angulaires etc. [OKAFOR *et al.* 2000] ont développé un modèle cinématique d'erreurs compatible avec la géométrie et les erreurs thermiques sur un centre d'usinage vertical. Dans ce modèle les erreurs de déplacements des trois axes machine sont modélisés en utilisant les matrices HTM.

Le modèle est alors utilisé pour calculer et prédire le vecteur d'erreur résultant à l'interface outil-pièce pour la compensation des erreurs. [RAMESH *et al.* 2000a] tentent une revue des travaux de recherche pour analyser les sources variées des erreurs géométriques et méthodes pour éliminer ou compenser les défauts sur les machines outils.

D'autre part, [KAKINO *et al.* 1993] ont utilisé un système « double-billes » (Fig. 1-3) pour mesurer les erreurs de positionnement sur une machine-outil multiaxes. Concernant les erreurs cinématiques dues aux imprécisions géométriques, [UDDIN *et al.* 2009] présentent un simulateur des erreurs géométriques en usinage 5 axes basé sur les effets des erreurs cinématiques sur les interférences 3D entre outil et pièce.

Plusieurs modèles cinématiques pour machines 5 axes ont été construits pour étudier les effets des erreurs cinématiques sur la précision des mouvements [LIN *et al.* 2003, SOONS *et al.* 1992, SRIVASTAVA *et al.* 1994, SUH *et al.* 1998]. On trouve aussi de nombreux travaux sur l'identification des erreurs cinématiques basés sur les mesures des erreurs de mouvements de la machine. Des systèmes de mesure basés sur la méthode de la tige à deux billes ont été souvent utilisés pour identifier les erreurs cinématiques de machines 5 axes [ABBASZAHEH-MIR *et al.* 2002, KAKINO *et al.* 1994, MAYER *et al.* 1999, SAKAMOTO *et al.* 1994, TSUTSUMI *et al.* 2003, 2004].

Les erreurs thermiques ont été considérées comme une autre source majeure d'imprécisions. [RAMESH *et al.* 2000b] .

En considérant l'effet significatif des erreurs thermiques sur la broche de la machine, [SRINIVASA *et al.* 1996, YANG *et al.* 2004] présentent une méthode pour mesurer les dérives dues aux effets thermiques sur la broche par un système de calibration laser. On mesure les coordonnées du centre de broche et la direction de l'axe de broche dans le système de coordonnées machine et ceci pour différents états thermiques. [YANG *et al.* 2003] proposent un modèle dynamique et une stratégie pour compenser les erreurs dues aux effets thermiques basé sur le comportement dynamique du champ de température et des déformations de la broche.

1.1.2. Erreurs d'outil de coupe

Plusieurs approches ont été développées concernant les conditions de coupe (vitesse, profondeur de passe ...). [BEAUCHAMP *et al.* 1996] ont proposé six variables indépendantes (vitesse de coupe et d'avance, profondeur de passe, rayon d'outil, longueur d'outil ou type de barre d'alésage) pour évaluer leurs effets sur la rugosité des surfaces usinées pour des opérations de tournage ou de fraisage. [BENGA *et al.* 2003] ont étudié les effets de la vitesse de coupe et d'avance sur la rugosité et la durée de vie de l'outil à partir d'expérimentations par usinage d'acier à roulements 100Cr6 (62-64HRC) en utilisant des outils traités au nitrure de bore poly-cristallin ou outils céramique. Ils montrent que la vitesse de coupe est le facteur le plus influent, affectant la finition de la surface. [GADELMAWLA *et al.* 2008] ont présenté une méthode qui exprime les relations entre les conditions de coupe et la texture des surfaces usinées dans des opérations de fraisage. [TEKINER *et al.* 2004] introduisent une méthode basée sur les bruits de coupe enregistrés pendant l'usinage d'un acier inoxydable AISI 304 avec des paramètres idéaux de coupe, pour développer un système d'alarme.

D'autre part, la flexion d'outils est un autre facteur qui intervient pendant l'usinage. Des modèles ont été utilisés pour prédire les déflexions d'outils [DÉPINCÉ *et al.* 2006a, 2006b, KLINE *et al.* 1982, RAO *et al.* 2006].

1.1.3. Erreurs de montage et de pièce

Un montage est un système mécanique pour positionner et maintenir une pièce pendant un usinage, une mesure ou une opération d'assemblage.

Une pièce fixée sur son montage doit occuper une position unique et rester en équilibre. Les variations à partir de la localisation désirée sont dues essentiellement aux efforts de blocage, déformations, frottements, efforts d'usinage.

Assurer un positionnement précis de la pièce est un rôle important d'un montage d'usinage. En particulier on cherchera à réaliser une mise en position isostatique. [Y. RONG 2001]

Plusieurs études ont été menées sur l'analyse des montages. [WANG *et al.* 2006] présentent une méthode pour optimiser une mise en position en se basant sur le critère de répétabilité et de précision de localisation. L'optimisation du serrage consiste à minimiser la force de blocage tout en satisfaisant la stabilité requise. Cette condition de stabilité a été étudiée dans [WANG *et al.* 1999, WU *et al.* 1998]. Une méthode proposée par [DEMETER 1994] tient compte des forces de friction et des frottements au contact.

Des études de fiabilité de localisation sont présentées par [XIONG *et al.* 2004] et [CHU *et al.* 1999]. Ils vérifient la précision de localisation à partir de points de mesure. [LI *et al.* 1999] améliorent la précision de localisation en tenant compte de la déformation élastique des contacts. [WANG 2000] utilise le déterminant d'une matrice de localisation qui caractérise la variance totale de mise en position de façon à réduire les erreurs de mise en position. D'autres études portent sur l'influence des forces de serrage [RAGHU *et al.* 2004], [CHEN *et al.* 1996] et [SCHIMMELS *et al.* 1994].

Dans ces études de déformation, la méthode des éléments finis permet de considérer les différents facteurs comme les forces de serrage, la position des éléments de serrage, les forces de coupe [MAYER *et al.* 1999, SOONS *et al.* 1992, SRIVASTAVA *et al.* 1994, SUH *et al.* 1998]. Le rôle de ces forces sur la position de la pièce a été largement étudié [YEH *et al.* 2000]. Pour éviter les déformations de pièces et erreurs de localisation, différents montages sont optimisés [ABBASZAHEH-MIR *et al.* 2002, DEMETER *et al.* 2001, DENG *et al.* 2006, JENG *et al.* 1995, MANNAN *et al.* 1997, RAGHU *et al.* 2004, WEIFANG CHEN].

Les défauts d'une surface dans la région de contact affectent également la précision de mise en position. Des modèles sont proposés pour tenir compte de ces défauts [SRIVASTAVA *et al.* 1994] et [SANGNUI *et al.* 2001].

Le frottement au contact pièce-montage a été étudié dans de nombreuses publications pour estimer son influence sur la précision de la mise en position. [XIE *et al.* 2000] présentent une étude expérimentale sur ce sujet et [DEIABA *et al.* 2004] examinent les conditions tribologiques du contact pièce-montage (matériaux, rugosité, charge normale).

La qualité du montage est évaluée en considérant la précision de mise en position. [LIN *et al.* 2003] présentent un modèle en 3D utilisant une matrice jacobienne. [SONG *et al.* 2005] définissent un critère pour évaluer la mise en position par une matrice construite à partir de 6 points de contact. [BO LI 1999] présentent un modèle permettant de réduire les erreurs de localisation. [BRISSAUD *et al.* 1998] présentent un indicateur de qualité de localisation. [PARIS *et al.* 2005] définissent trois types d'indices pour caractériser les performances d'un montage : respect de la qualité du positionnement, respect de la stabilité pendant l'usinage, respect de l'accessibilité d'usinage.

1.2. Identification des défauts de fabrication

Les défauts de fabrication considérés ici comprennent deux principales catégories : les écarts de mise en position et les écarts d'usinage. Pour analyser ces écarts, plusieurs études ont été conduites par [BOURDET *et al.* 1995b, DURET 1988, LEHTIHET *et al.* 1990]. Néanmoins certaines difficultés persistent. Un exemple expérimental présenté par [TICHADOU 2005] illustre ces difficultés. Une autre expérimentation d'un lot de 55 pièces tournées a été conduite par KamaliNejad [TICHADOU *et al.* 2007]. L'objectif de cette étude est aussi de quantifier les défauts de mise en position et d'usinage en se basant sur la méthode de la double mesure. Les pièces sont fixées et usinées sur un tour CNC. Chaque pièce est alors mesurée à l'intérieur de la machine-outil à la fin de l'usinage sans désassembler la pièce de son montage. La seconde mesure est effectuée sur une machine à mesurer MMT pour chaque pièce.

1.3. Discussions

Il y a plusieurs limitations aux études suivantes :

- Les défauts d'usinage sont obtenus par la mesure de points sur les surfaces usinées, ce qui est insuffisant pour représenter les défauts des surfaces.
- Les études précédentes ne montrent pas que les résultats obtenus avec différentes machines sont comparables ne serait-ce qu'à cause de la précision des systèmes de mesure utilisés.

Pour résoudre ces problèmes, nous proposons plusieurs solutions pour identifier, analyser et estimer les défauts de fabrication [BUI *et al.* 2010, SERGENT *et al.* 2010] .

- Utiliser la méthode de double mesurage des pièces usinées, (CNC et MMT)
- Utiliser la même méthode pour associer une surface à partir d'un nuage de points, par exemple les moindres carrés.
- Vérifier la qualité des mesures et traitements numériques
- Déterminer les défauts de lots de pièces usinées sous forme de lois de distributions et paramètres statistiques utilisables en simulation numérique.

2. ANALYSE DES TOLERANCES

L'analyse des tolérances est le terme général pour l'étude de l'accumulation des écarts de géométrie dans les pièces et assemblages. Ceci comprend :

- la méthode pour déterminer la signification des spécifications individuelles des tolérances
- la méthode pour déterminer la variation cumulée possible entre deux ou plusieurs éléments géométriques. Ce qui est souvent désigné sous le terme d'empilage de tolérances.

[FISCHER 2004] explique que les cotes et leur tolérance sont additionnées. Elles « s'empilent » pour déterminer la variation totale possible.

On distingue deux types majeurs d'analyse de tolérances : empilage au pire des cas et empilage statistique. [EVANS 1974, 1975] explique la différence entre deux politiques distinctes de tolérancement, statistiques et le pire des cas, par la façon dont les activités tolérancement sont généralement effectuées. Dans ce travail, le problème général est formulé par l'équation Y=f(Xi) ou Y est la résultante de l'empilage et Xi sont les écarts individuels (composantes). [NIGAM *et al.* 1995] présentent une revue des approches statistiques d'analyse de tolérances.

2.1. Différents modèles d'analyses de tolérances

[FORTINI 1967] présente le modèle le plus commun pour l'empilage où la tolérance résultante d'assemblage T est la somme des tolérances Ti des composantes. Le second modèle statistique classique conduit à une relation quadratique : le carré de la tolérance T est la somme des carrés des tolérances Ti.

Grâce à ces deux approches, [GREENWOOD *et al.* 1988, 1990] effectue l'analyse de tolérances. Une méthode similaire est appliquée à des assemblages mécaniques plus complexes basée sur des méthodes de linéarisation [CHASE *et al.* 1995, GAO *et al.* 1998, GLANCY *et al.* 1999, WITTWER *et al.* 2004].

Analyse statistique des tolérances

On suppose donnée les fonctions densité de probabilité des variables Xi de la fonction Y=f(Xi) pour prédire la variabilité sur la résultante Y. Une procédure standard consiste à déterminer les 4 premiers moments de la densité de probabilité de Y [COX 1979]. Plusieurs publications utilisent cette méthode [CHASE *et al.* 1991, NIGAM *et al.* 1995].

La méthode de simulation de Monte Carlo peut être utilisée pour des fonctions non linéaires et des distributions non normales [CVETKO *et al.* 1998, SKOWRONSKI *et al.* 1997, TURNER *et al.* 1987]. Plusieurs travaux de recherche sont centrés sur l'analyse de sensibilité en utilisant la géométrie variationnelle [DONG 1997, DONG *et al.* 1995, 1997, SHAH *et al.* 1995].

Des modèles de description variationnelle de solides ont été développés pour éviter les limites des modèles où seules les dimensions varient [BOYER *et al.* 1991], [AKELLA *et al.* 2000, LI *et al.* 2001, WHITNEY *et al.* 1999a, 1999b].

Différentes approches ont été présentées pour analyser la propagation des tolérances géométriques en 3D, en se basant sur différentes formulations :

- une formulation cinématique [RIVEST *et al.* 1994] [LAFOND *et al.* 1999, LAPERRIÈRE *et al.* 2000, LAPERRIÈRE *et al.* 2001, LAPERRIÈRE *et al.* 1998].
- Les torseurs de petits déplacements (TPD) ou une représentation matricielle [WHITNEY *et al.* 1993], [DESROCHERS *et al.* 1997]
- le tolérancement vectoriel [WIRTZ 1991, WIRTZ *et al.* 1993], [MARTINSEN 1993].

Avec le modèle basé sur les torseurs de petits déplacements, les conditions d'assemblage des pièces se traduisent par des relations entre composantes de torseurs écarts et jeux

[BOURDET *et al.* 1996] [BALLOT *et al.* 1998]. Partant de cette approche, [GIORDANO *et al.* 1992] présentent le concept de domaines jeux et écarts pour tenir compte des zones de tolérances et conditions de non interpénétration des contacts [GIORDANO *et al.* 2003] [SAMPER *et al.* 2006]. Un autre modèle basé sur le concept de torseur de petits déplacements est présenté par [TEISSANDIER *et al.* 1998, 1999a].

2.2. Analyse d'empilage, propagation des erreurs

La propagation des erreurs dans une fabrication à plusieurs procédés de fabrication a été étudiée en se basant sur le modèle de l'espace d'état [DING *et al.* 2002, DJURDJANOVIC *et al.* 2001, HUANG *et al.* 2003a, HUANG *et al.* 2003b, ZHOU *et al.* 2003], c'est une forme différente du modèle cinématique standard. Il s'agit d'estimer l'état à partir de valeurs observées. [HUANG *et al.* 2003b] utilisent un modèle d'espace d'état pour décrire la propagation des erreurs sur les pièces dans un procédé de fabrication multi-phases. [ZHOU *et al.* 2003] utilisent un vecteur de mouvement différentiel (DMV) comme vecteur d'état.

3. CONCLUSIONS

Dans le contexte de production, le défaut de fabrication est le résultat de différentes sources d'erreurs. K. Whybrew et G.A. Britton dans [ZHANG 1997] ont résumé 27 sources d'erreurs dans un processus de fabrication pour les 8 éléments de fabrication suivants : machine outil, outil de coupe, fixation, pièce fabriquée, liquide de coupe, opérateur, conditions d'environnement et variable du procédé. De nombreuses études ont été conduites sur les quatre premiers éléments. Bien que les chercheurs aient reconnu le rôle important de ces sources d'erreur sur la qualité des pièces fabriquées, peu de recherches ont été réalisées sur l'effet des défauts de fabrication sur la qualité finale des pièces. Pour cette raison, nous considérons que les défauts de fabrication sont divisés en deux catégories : les défauts de positionnement et les défauts d'usinage qui peuvent être dus à différentes sources d'erreurs. Avec les limitations de quelques recherches sur la quantification des défauts de positionnement et d'usinage, nous proposons plusieurs solutions pour développer la méthode qui peut être utilisée pour identifier, analyser et estimer les défauts de fabrication [BUI *et al.* 2010, SERGENT *et al.* 2010] :

- utilisation de la méthode de la double mesure pour mesurer les pièces fabriquées,
- association des surfaces à partir des mesures de points,
- comparaison des mesures par les deux moyens,
- identification des défauts des paramètres statistiques d'un lot de pièces fabriquées.

De plus, une étape essentielle en fabrication consiste à évaluer la qualité des produits en termes de tolérances fonctionnelles. Durant la fabrication, un produit peut passer par un ou plusieurs processus, plusieurs machines-outils, montages, outils de coupes suivant le séquençage des opérations appelé gamme de fabrication. En conséquence, l'évaluation d'une gamme en termes de tolérances fonctionnelles est une étape directe pour maîtriser la qualité des produits, ce qui est connu comme l'analyse des tolérances. En d'autres termes, contrôler la qualité des produits consiste à vérifier pour une gamme donnée, si les tolérances fonctionnelles issues de la conception sont vérifiées pour des écarts de fabrications connus. Pour cela, nous présentons un modèle mathématique pour l'analyse de tolérances basé sur le «Modèle des pièces fabriquées » (MMP). Dans ce modèle, les résultats expérimentaux ou simulés, sont utilisés comme variables d'entrée pour la simulation de défauts de fabrication (défauts d'usinage et de montage) durant les phases d'une gamme de fabrication. Ceci permet de vérifier la gamme en termes de tolérances fonctionnelles, ou de déterminer les tolérances minimales d'un lot de pièces, en se basant sur une gamme donnée et un taux de non-conformité donné (en ppm).

De plus, des résultats peuvent être obtenus par des mesures expérimentales. Néanmoins, l'expérimentation est couteuse et prends beaucoup de temps. D'autre part, elle nécessite un équipement approprié. Par exemple, la machine outil doit être équipée par un moyen de mesure de façon à mesurer la pièce sur son montage à l'intérieur de la machine (sans démontage). Le développement de modèles et de méthodes précis pour simuler les défauts de montage et l'évaluation des facteurs qui affectent ces défauts peuvent permettre de limiter ces défauts. Ainsi, la détermination des influences des différents facteurs sur la mise en position de la pièce sur son montage est présentée. On propose d'étudier trois facteurs : les erreurs géométriques de localisation, les forces de serrage, et le coefficient de frottement au contact pièce-montage.

Dans ce but, on effectue un certain nombre de simulations par éléments finis et un plan d'expérience complet à deux niveaux de façon à déterminer les modèles mathématiques.

Les modèles sont alors utilisés dans une simulation de Monte-Carlo pour évaluer les défauts de mise en position du montage.

CHAPTER 2

IDENTIFICATION DES DEFAUTS DE FABRICATION

1. INTRODUCTION

Une étape essentielle pour parvenir à évaluer la qualité des produits consiste à utiliser une méthode pour identifier les défauts de fabrication. Ce chapitre présente une méthode qui permet de distinguer les défauts d'usinage des défauts de mise en position. Pour cela, on fait référence à quelques études antérieures relatives à la quantification des défauts de fabrication. Les limitations de ces études sont discutées et plusieurs propositions sont faites pour résoudre les problèmes soulevés.

Ces propositions s'appuient sur une application expérimentale. Les résultats de ces expériences sont exprimés de différentes façons, comme à l'aide des composantes de TPD, les défauts de forme et d'orientation, ou les distributions et les paramètres statistiques. Les résultats peuvent être utilisés comme variables d'entrée de simulations pour prédire les défauts de fabrication.

2. LIMITATIONS DES TRAVAUX ANTERIEURS ET PROPOSITIONS

Nous avons évoqué dans le chapitre précédent les limitations liées à la quantification des défauts de fabrication dans les études de [BOURDET *et al.* 1995b, DURET 1988, LEHTIHET *et al.* 1990]. Concernant la quantification de ces défauts, les limitations peuvent se résumer au fait que nous ne pouvons pas utiliser seulement les mesures sur la machine outil. Cela ne produit pas assez de données pour analyser les défauts de mise en position.

Pour résoudre ce problème, une mesure supplémentaire a été proposée par [TICHADOU 2005]. Une autre mesure sera effectuée après la dernière étape d'usinage de la pièce. Les mesures sont donc effectuées à l'intérieur de la machine outil sans démontage de la pièce puis sur une machine MMT après démontage de la pièce hors de son montage d'usinage.

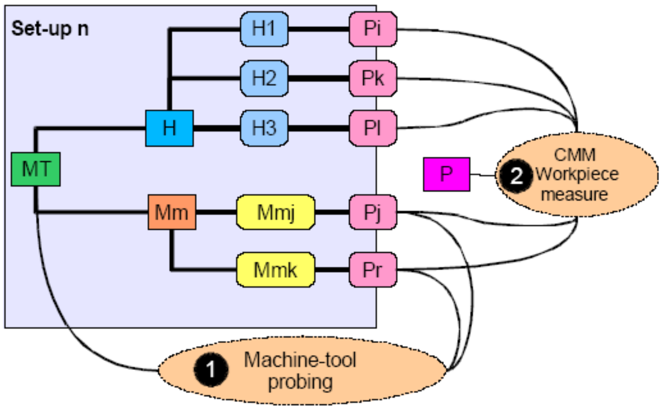


Fig. 2-1 Principe de la méthode de double mesure [TICHADOU *et al.* 2007]

Nous avons proposé de développer cette méthode pour analyser les défauts de mise en position et d'usinage [BUI *et al.* 2010, SERGENT *et al.* 2010].

3. IDENTIFICATION DES DEFAUTS D'USINAGE

3.1. Méthode

Les composantes de torseurs de petits déplacements (TPD) sont déterminées à partir des surfaces associées obtenues par les points de mesure sur la machine outil ou sur une machine à mesurer.

Pour reconstruire une surface mesurée à partir d'un nuage de points mesurés, plusieurs approches sont possibles, par exemple par des surfaces B-Spline [PATRIKALAKIS *et al.* 2002], par surfaces associées par le critère du maximum de vraisemblance [ARANDA *et al.* 2010], ou par les moindres carrés [FORBES 1989].

On construit un plan associé par la méthode des moindres carrés, ce plan est spécifié par un point sur le plan, et par un vecteur normal au plan. Un cylindre associé est spécifié par un point de l'axe, un vecteur orienté sur l'axe et le rayon.

3.2. Description des pièces brutes et de la pièce définie en conception

Un lot de 50 pièces en aluminium (2017 A) de diamètre 30 mm et de longueur 50 mm est utilisé dans cette étude. Les pièces brutes sont obtenues par sciages à partir de barres. La pièce à réaliser comporte deux plans usinés (1 et 2) parallèles distants de 10 mm.

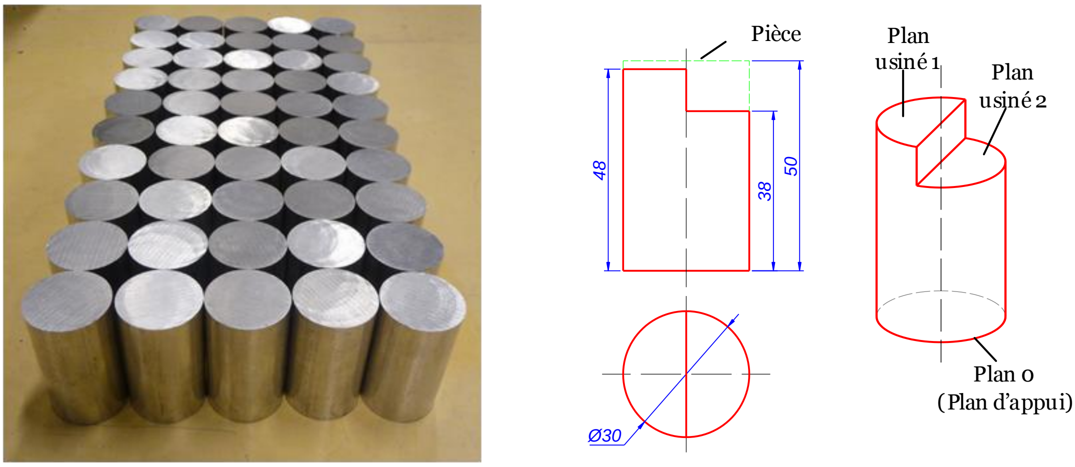


Fig. 2-2 Pièces brutes et pièce usinée

3.3. Processus de fabrication et de mesure

La pièce est fixée et localisée sur la table d'un centre d'usinage 5 axes CNC (DMG Dekel maho-DMU50) par un mandrin à 3 mors doux. En théorie, les trois mors viennent en contact avec la pièce par une surface cylindrique et plane. Les mors doux ont été usinés directement sur la machine. Les surfaces de mise en position du montage sont mesurées par un palpeur de la machine pour construire un système de coordonnées utilisé comme repère d'origine machine.

Le plan OXY est le plan associé au plan de mise en position. Le point O est l'intersection de ce plan avec l'axe du cylindre du mandrin 3 mors.

Les plans 1 et 2 sont alors usinés par fraisage en bout (fraise ϕ20) avec deux parcours d'outils différents, ce qui permet d'évaluer les influences des différents parcours d'outils sur les surfaces.

Chaque pièce est ensuite mesurée par un palpeur monté à la place de l'outil. Comme la pièce n'est pas démontée, on mesure ainsi l'influence des défauts d'usinage.

Chaque point mesuré est défini par les 3 coordonnées cartésiennes. Une étude de capabilité des mesures (utilisation de calibres classe 0), permet de vérifier que les bruits de mesure sont négligeables et permettent d'estimer la dispersion de mesure [SERGENT *et al.* 2008]. L'écart type de mesure de longueur est de l'ordre de 0.27 microns, faible par rapport à l'écart type des mesures de défauts de fabrication obtenus dans cette étude (de l'ordre de 2 microns). Pour réduire la variation due aux effets thermiques [RAMESH *et al.* 2000b], une phase d'échauffement est effectuée au préalable de l'usinage et des mesures.

3.4. Exploitation des résultats

A partir de ces mesures, les défauts sont analysés de différentes façons:

3.4.1. Défauts exprimés sous forme de torseurs de petits déplacements

Pour chaque plan usiné, on détermine un torseur de petits déplacements avec trois composantes non nulles : r_X et r_Y pour les rotations autour des axes X et Y et t_Z le déplacement suivant l'axe Z. Le repère est celui construit sur la machine. Les écarts d'usinage de chaque plan sont définis par un écart de rotation et un écart de translation. Un écart de rotation est calculé à partir des composantes des normales aux plans. Les défauts de translation sont calculés par la coordonnée de la projection du centre des faces sur l'axe Z.

On constate que les défauts de translation des deux plans croissent avec le temps (de la 1ère à la 50ème pièce). Les accroissements de ces deux dérives sont similaires. Cela signifie que les défauts de translation des deux faces usinées sont dépendants.

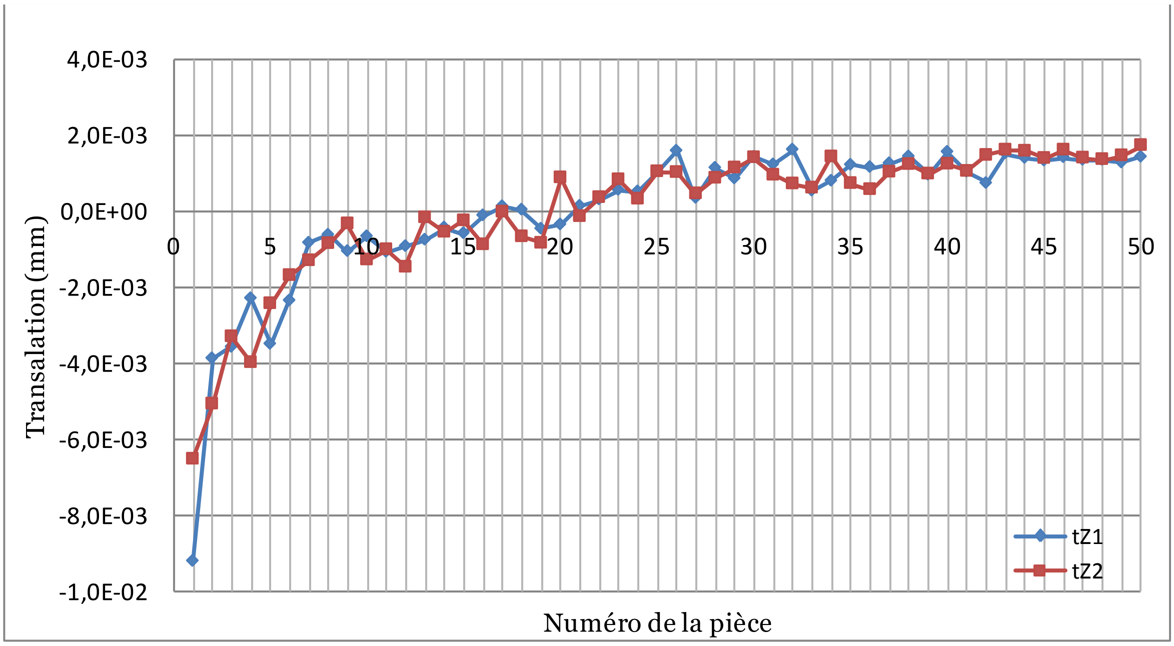


Fig. 2-3 Déviation des surfaces usinées en translation

Les défauts d'usinage en rotation et translation sont résumés Tab. 2-1.

Components	CNC (MCS)	
	Machined planes 1	Machined planes 2
$s_{t_Z}^2(mm^2)$	3.711E-6	3.123E-6

Tab. 2-1 Défauts de translation

3.4.2. Défauts d'usinage exprimés sous forme de défaut de planéité, de perpendicularité et de parallélisme

La planéité est calculée de la façon suivante : soit A un plan associé au nuage de points mesurés. Soit Am et AM deux plans parallèles au plan A et de coordonnées minimale et maximale suivant la normale à A. La planéité sera définie par la distance t entre ces deux plans.

	Plan usiné 1	Plan usiné 2
Moyen (mm)	9.517×10^{-3}	1.943×10^{-3}
Ecart type	0.458×10^{-3}	0.462×10^{-3}

Tab. 2-2 Planéité

Le défaut de parallélisme est défini en choisissant le plan 1 en référence (surface associée), le défaut de parallélisme du plan 2 est la distance entre les deux plans parallèles au plan de référence et tels que les points de 2 soient compris entre ces deux plans.

Pour les 50 pièces mesurées, on obtient les résultats suivants : (Tab 2-3)

	Plan usiné 2 par rapport avec plan usiné 1
Moyen (mm)	16.013×10^{-3}
Ecart type	1.268×10^{-3}

Tab. 2-3 Parallélisme

Le défaut de perpendicularité d'un plan usiné par rapport au cylindre est défini ici par la distance entre deux plans perpendiculaires à l'axe de référence et passant par deux points de coordonnées minimale et maximale suivant l'axe du cylindre.

	Machined planes 1	Machined planes 2
Average (mm)	17.30×10^{-3}	14.03×10^{-3}
Standard deviation (mm)	4.935×10^{-3}	5.637×10^{-3}

Tab. 2-4 Perpendicularité

Les résultats expérimentaux sont ensuite analysés du point de vue statistique. Pour déterminer les types de distributions statistiques associées aux défauts, on effectue les étapes suivantes [WEB-PAGE 2011].

- représentation graphique des mesures à l'aide d'histogrammes,

- on détermine ensuite le type de loi de distribution le plus approché par le test du ki2 pour chaque distribution possible. Le plus grand ki2 correspond à la loi la plus vraisemblable. Le calcul peut être effectué par des logiciels adaptés (Mathematica).

Les résultats montrent que les défauts d'usinage ne suivent pas toujours des lois normales ni uniformes comme cela est souvent proposé en simulation.

Évaluation des défauts de forme des surfaces usinées

Une surface moyenne est construite à partir des 50 pièces pour chacun des deux plans. Une forme conique est mise en évidence. Le choix particulier du parcours d'outil permet d'expliquer ce défaut de forme systématique. Pour la surface 1 l'épaisseur du copeau augmente de 0 à un maximum pendant l'opération. L'inverse se produit avec la surface 2.

Par ailleurs, un modèle de déflexion d'outil et de pièce est proposé pour expliquer ces défauts de forme. La déflexion d'outil dépend de l'effort de coupe supposé proportionnel à la section du copeau. Le défaut de forme des deux plans peut alors être expliqué :

- Pour le plan 1, la déflexion de l'outil au centre de la pièce est plus faible qu'au bord de cette surface. Comme la montre la figure ci-après, la déflexion de l'outil provoque une variation de la hauteur de coupe. D'où la forme bombée de la surface.
- Pour le plan 2, la déflexion de l'outil de coupe au bord droit de la surface usinée est plus grande qu'au bord gauche (suivant l'axe Y) aussi la coupe sur le bord gauche est plus faible que sur le bord droit d'où l'inclinaison autour de l'axe Y.

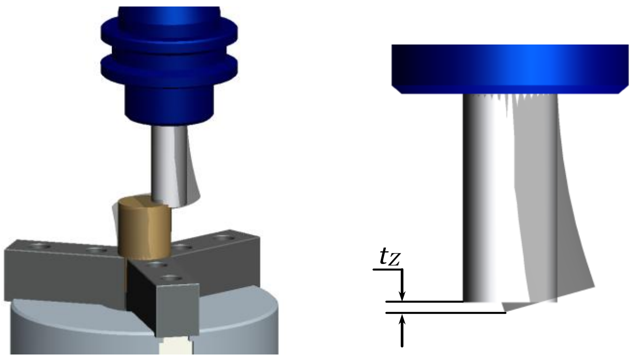


Fig. 2-4 Déflexions de l'outil de fraisage et de la pièce

Explication des dérives des défauts d'usinage

Les résultats montrent que les dérives des deux surfaces usinées sont similaires. Les erreurs peuvent être subdivisées en deux types : les erreurs aléatoires et les erreurs systématiques. Les dérives sont considérées comme des erreurs systématiques dues probablement à des effets thermiques. [RAMESH *et al.* 2000b] étudient la variation de température comme facteur critique source principale d'imprécision.

L'analyse des résultats montre que les erreurs systématiques jouent un rôle significatif dans les défauts d'usinage. Si ces erreurs peuvent être corrigées, alors les défauts de translation seront fortement réduits. Néanmoins, en pratique, les erreurs systématiques sont difficiles à corriger.

3.5. Évaluation des défauts d'usinage par la méthode de paramétrage de forme dite méthode modale

La méthode basée sur le concept de TPD ne tient pas compte des défauts de forme. Nous proposons d'utiliser la méthode d'analyse modale de forme pour compléter l'analyse des défauts des surfaces usinées.

Cette méthode permet d'analyser les défauts de forme d'une pièce à partir d'un grand nombre de points de mesure [FAVRELIERE *et al.* 2009, FORMOSA *et al.* 2007]. Elle est basée sur la décomposition modale discrète (DMD) [ADRAGNA *et al.* 2007]. Les surfaces mesurées sont décrites comme un ensemble discret de fonctions. La méthode est conduite par les étapes suivantes :

- La surface est discrétisée en éléments finis.
- Les déplacements sont décrits par un vecteur $(\vec{V})$.
- L'analyse modale permet d'obtenir une base modale de déplacement (Q_i) utilisée alors pour décomposer le vecteur $\vec{V}$ sur cette base, d'où les coefficients λ_i qui représentent les écarts de forme sur la base modale de forme.

$$Q^{*}.V = ((Q^{T}.Q)^{-1}.Q^{T}).V = \lambda \qquad (0\text{-}1)$$

La matrice Q est constituée des vecteurs de base Qi.

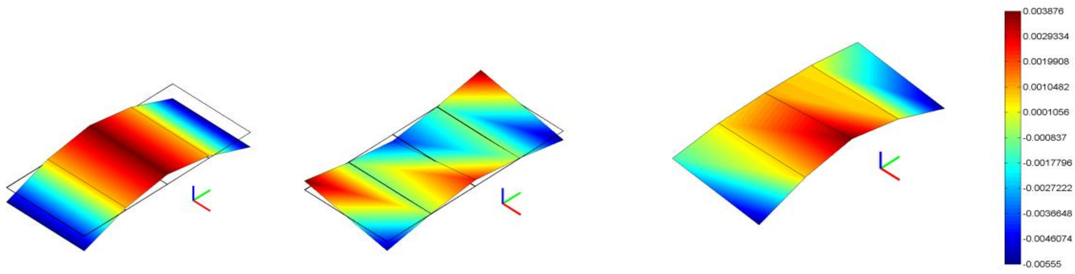
$$V = \sum_{i=1}^{m} \lambda_i . Q_i + \varepsilon \qquad (0\text{-}2)$$

$\vec{\varepsilon}$ est le résidu qui dépend du nombre de modes retenus dans la sommation.

On utilise la base duale Q^{*} qui est orthonormale.

Les modes Qi sont déterminés par résolution d'un problème classique de mécanique de vibration de structure résolu généralement par la méthode des éléments finis.

Cette méthode est appliquée pour quantifier les défauts de forme des plans usinés. Chaque point mesuré a un degré de liberté : translation suivant z. Le maillage utilise le même nombre de nœuds que de points mesurés. On retient les 8 premiers modes. Ils permettent de reconstituer la surface. Les modes 3 et 6 sont les plus significatifs.



a) Mode 3 b) Mode 6 c) Moyen de défauts de forme

Fig. 2-5 Défauts de forme du plan usiné 1

3.6. Conclusion sur l'identification des défauts d'usinage

Nous proposons dans cette section deux méthodes pour identifier les défauts d'usinage. La première méthode basée sur le concept de TPD permet de déterminer les composantes des torseurs d'écarts des surfaces usinées. Les résultats obtenus par cette méthode sont exprimés par des paramètres statistiques : moyenne, variance et type de distribution. Ils peuvent alors être utilisés pour simuler une gamme et détecter les défauts de fabrication qui sont dus au parcours d'outil, à l'outil, à la déformation de la pièce, etc. La seconde méthode est utilisée pour compléter les résultats de la première méthode. Elle détermine les défauts de forme et leur type (courbure, ondulation, torsion, etc.). Comme il a été mentionné plus haut, les défauts de fabrication sont divisés en deux catégories : défauts d'usinage et défauts de mise en position. Les défauts d'usinage ont été obtenus par essais expérimentaux. Dans le chapitre suivant, nous identifions les défauts de mise en position en se basant sur les résultats de ce chapitre et des compléments de mesures effectuées sur une machine MMT.

4. IDENTIFICATION DES DEFAUTS DE MISE EN POSITION

4.1. Méthode

Les 50 pièces sont fixées sur la fraiseuse CNC par un mandrin à trois mors doux. L'objectif de cette section est de déterminer les défauts de mise en position de la pièce sur son montage. Pour cela, une première mesure des deux plans usinés et du cylindre de la pièce est effectué sur la CNC, puis une seconde mesure sur la MMT (MarVision MS222). Les deux mesures sur les deux machines différentes sont illustrées par la figure ci-après:

4.2. Évaluation de la capabilité comparée des résultats de mesure obtenus par deux moyens de mesure

Le plan OM XMYM lié à la machine CNC est défini par la mesure sur le plan de localisation de l'assemblage. L'axe Op Zp du repère pièce est l'axe associé au cylindre de la pièce.

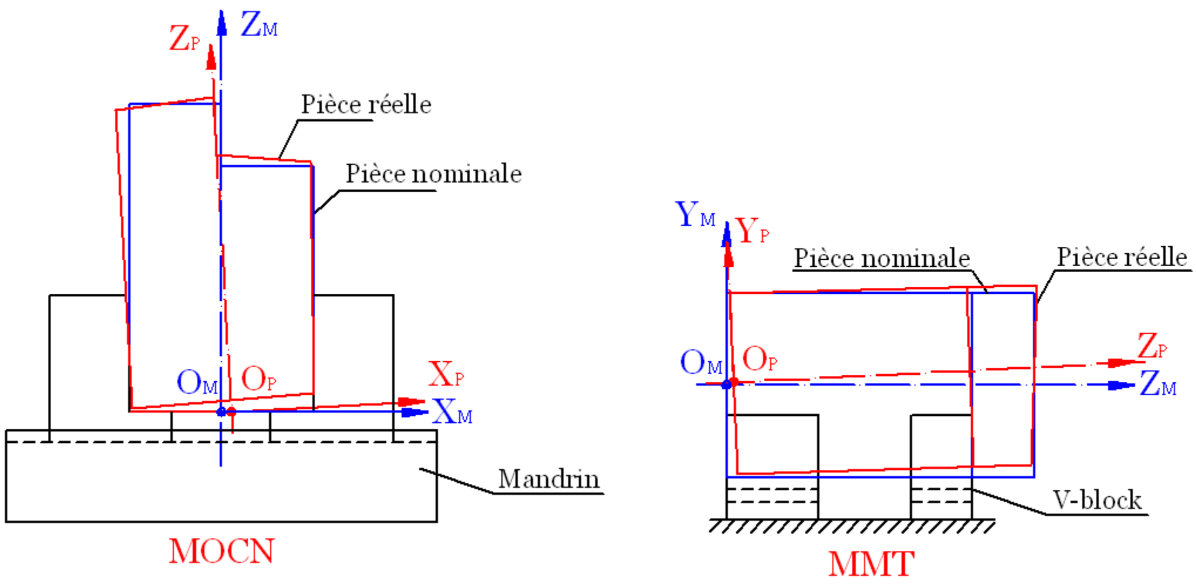


Fig. 2-6 Les repère sur les machines MOCN et MMT

L'angle entre l'axe du cylindre et le vecteur normal à chaque plan usiné peut alors être déterminé. Pour chaque composante, un test d'égalité des variances permet de vérifier si les variances sont significativement différentes pour les deux méthodes de mesure.

4.3. Détermination des composantes de translation des défauts de mise en position

On détermine ici la distance entre les deux plans usinés. Comme ils ne sont pas parfaitement parallèles, on doit définir cette distance. Deux méthodes sont proposées :

- par projection des centres des points sur l'axe OZ de la pièce,
- par la distance entre les points intersection des plans et de l'axe OZ de la pièce.

4.4. Détermination des composantes de rotation des défauts de mise en position

L'écart de rotation de la pièce sur son montage est défini comme un angle entre les axes de la pièce et l'axe du montage.

4.5. Caractérisation des défauts de mise en position

Les défauts de mise en position des pièces sur le montage sont caractérisés par les variances des 2 rotations autour des axes X et Y et la translation suivant Z.

Deux méthodes de mesure d'écart de mise en position

$s_{r_X}^2$ (rad²)	$s_{r_Y}^2$ (rad²)	$s_{t_Z}^2$ (mm²)
3.141E-07	2.235E-07	4.161E-04

Tab. 2-5 Défauts de mise en position

Les trois composantes peuvent aussi être exprimées sous forme de lois de distribution (fonction densité de probabilité).

4.6. Évaluation de la position du plan de localisation de la pièce sur son montage

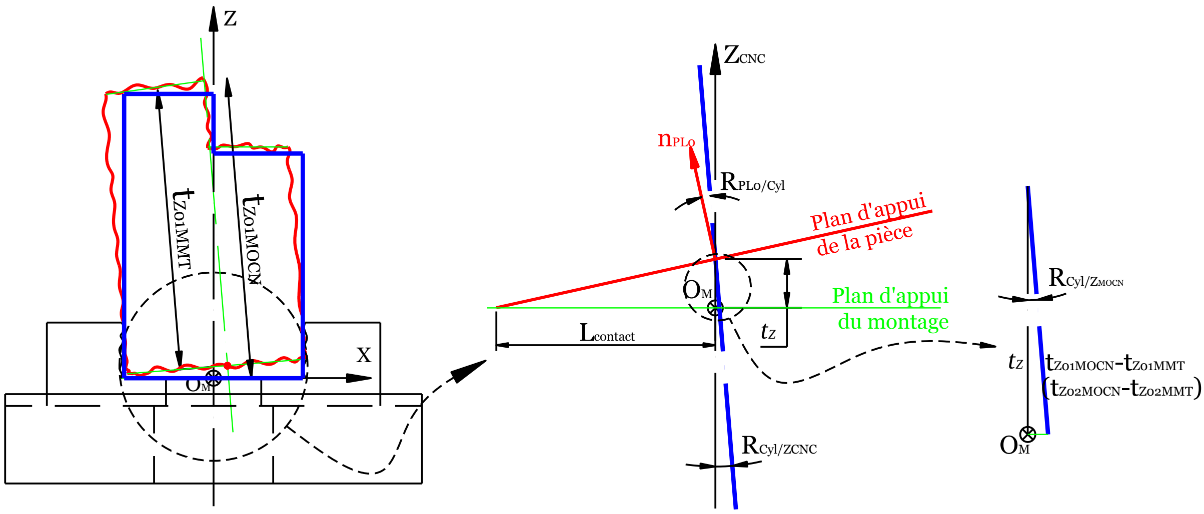


Fig. 2-7 Différence des distances obtenues par deux machines

$\delta_1 = t_{Z01CNC} - t_{Z01CMM}$ est la différence des distances obtenues par chaque mesure. L'écart de mise en position sur l'axe Z de la machine est :

$$t_Z = \delta_1 . \cos(R_{Cyl/Z_{CNC}}) \qquad (0\text{-}3)$$

Il s'agit ici d'évaluer la position des plans de contact sur le montage. Ceci permet de déterminer s'il y a contact entre le plan de la pièce et le plan du montage. Pour cela, on suppose que le plan de localisation de la pièce n'a pas de défaut de forme. A partir des mesures sur la CNC et sur la machine à mesurer MMT, une distance entre le plan de mise en position de la pièce et le plan du montage est déterminée.

Les résultats de cette analyse montrent que 61% des pièces étaient en contact avec le plan d'appui du montage, tandis que 39% sont sans contact.

4.7. Conclusion sur l'identification des écarts de mise en position

Nous avons expliqué la méthode pour identifier les défauts de positionnement à partir de deux moyens de mesure différents. Les résultats sont exprimés sous forme de paramètres statistiques : moyenne, variance et type de distribution. Les défauts de mise en position peuvent alors être utilisés pour simuler une gamme d'usinage.

5. EVALUATION DES DEFAUTS D'USINAGE ET DE POSITIONNEMENT

Le tableau suivant (2-6) synthétise les défauts d'usinage et de mise en position obtenus à partir de l'application expérimentale. La variance de chaque composante est calculée pour évaluer les dispersions de ces défauts. Ici l'ensemble des valeurs est un ensemble d'écarts de rotation ou translations des surfaces usinées. D'autre part, les défauts d'usinage et de mise en position peuvent aussi être caractérisés par une loi de distribution.

Composantes	**Défauts d'usinage**		**Défauts de mise en position**
	Plan usiné 1	Plan usiné 2	
$s_{rX}^2 (rad^2)$	7.907E-10	6.356E-10	3.141E-07
$s_{rY}^2 (rad^2)$	41.714E-10	163.72E-10	2.235E-07
$s_{tZ}^2 (mm^2)$	3.711E-6	3.123E-6	4.161E-04

Tab. 2-6 Les défauts d'usinage et de mise en position

Les résultats du tableau 2-6 montrent que les défauts de mise en position sont plus significatifs que ceux d'usinage. En conséquence, la méthode proposée peut être utilisée non seulement pour identifier les défauts de mise en position et d'usinage, mais aussi pour détecter s'il y a des différences entre les surfaces usinées ou entre une surface usinée et les surfaces de mise en position.

6. RESUME

Dans ce chapitre, a été présentée la méthode basée sur le concept de torseur de petits déplacements qui peut être utilisée pour identifier les défauts d'usinage et de mise en position. La méthode d'analyse modale est également présentée pour compléter l'analyse des défauts de forme [SERGENT *et al.* 2010].

D'autre part dans ce chapitre, nous avons pu :

- Analyser les résultats de mesure obtenus à partir de deux moyens de mesure différents.
- Évaluer la qualité de ces deux moyens de mesure,
- Définir les composantes d'écarts de position et d'orientation de chaque surface usinée,
- Expliquer les défauts de forme de plans usinés par les erreurs de déflexion de l'outil et de la pièce,
- Calculer les défauts de forme et d'orientation des plans usinés pour compléter l'interprétation des défauts,

- Vérifier s'il y a contact ou pas entre une face plane de la pièce et un plan de mise en position appartenant au montage.

CHAPTER 3

EVALUATION DE LA QUALITE D'UN MONTAGE PAR DES INDICATEURS

1. INTRODUCTION

Dans le précédent chapitre, nous avons réalisé des mesures sur différentes machines (machine-outil, machine à mesurer) et analysé ces mesures de façon à déterminer les défauts de fabrication et de mise en position. Les résultats expérimentaux montrent que les défauts de mise en position jouent un rôle important dans le processus de fabrication. Néanmoins, ces résultats montrent seulement les défauts de positionnement de la pièce sur son montage, mais la qualité du montage n'est cependant pas estimée. Dans ce chapitre, deux indicateurs simples sont proposés pour évaluer la qualité d'un montage d'usinage [DURET *et al.* 2010].

2. DEFINITION DE LA QUALITE D'UN MONTAGE

Dans cette étude, la qualité de montage est définie par :

- La dispersion de localisation de la pièce obtenue par comparaison du déplacement de la nième pièce et de la première pièce (ou par comparaison avec la position théorique).
- La sensibilité du montage qui dépend des forces de réaction au contact entre la pièce et le montage lorsque des forces extérieures sont appliquées (forces de serrage).

En conséquence, deux indicateurs sont proposés :

- Le premier est utilisé pour évaluer la qualité d'un montage, basé sur l'analyse des déplacements d'une pièce sur son montage. Un montage sera considéré bon si la dispersion des déplacements de la pièce sur son montage est faible.

- Le second indicateur est utilisé pour évaluer la qualité du montage en se basant sur les influences des forces de réaction aux contacts entre la pièce et le montage. Donc un montage sera considéré comme bon si les influences d'une force extérieure (forces de serrage) appliquée à la pièce, ont une faible influence sur les forces de réactions aux contacts de la pièce sur son montage.

3. PROPOSITION POUR DES INDICATEURS

3.1. Hypothèses d'étude

- le système d'assemblage n'est pas surcontraint, (mise en position isostatique)
- les contacts entre la pièce et le montage sont ponctuels. Il y a 6 points de contact dans notre application.
- la pièce et le montage sont des solides rigides sans déformation.

Pour chaque mise en position, les coordonnées des 6 points de contact sont déterminées à partir de mesures sur une machine CMM.

3.2. Premier indicateur propose « *Déterminant* »

Une pièce unique est assemblée puis mesurée et démontée un grand nombre de fois. En supposant que la pièce est un solide rigide, et qu'elle est mise ne position par 6 contacts, on peut écrire une relation linéaire sous forme matricielle entre les déplacements δ_1 à δ_6 des 6 points de contact suivant la normale au contact et les composantes du torseur de petits déplacements dans un repère donné (matrice A).

Chaque matrice A dépend de la géométrie du montage. Le déterminant est une caractéristique du montage. Si on considère les déplacements δ_i donnés, le montage qui donne les plus petits déplacements de la pièce aura un déterminant plus élevé et sera de meilleure qualité.

On a mené une application expérimentale en comparant deux types de montages : mise en position du type Kelvin et montage de Boys.

Le montage de Kelvin est réalisé sur une pièce parallélépipédique de dimensions 100x50x20.

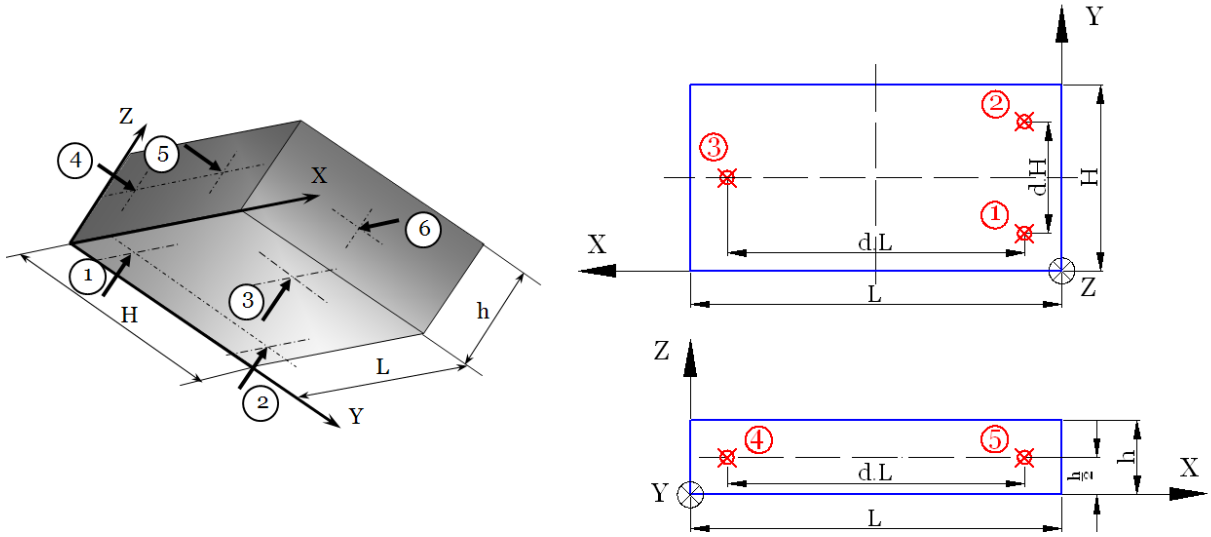


Fig. 3-1 Montage et le coefficient d

En faisant varier le coefficient d, on fait varier le déterminant. Cet indicateur caractérise donc la qualité de l'assemblage.

Le montage de type Boys est obtenu par 3 liaisons d'une sphère dans un V. Chaque sphère est en contact par 2 points dans son V. On fait varier l'angle des V (90°, 120°) et la distance entre un point central et les 3 sphères. On montre que le déterminant croît avec cette distance et décroit avec l'angle des V.

En conclusion, ce calcul de déterminant est un indicateur simple pour caractériser la qualité d'un montage.

3.3. Deuxième indicateur proposé : *le coefficient « K »*

Il s'agit d'évaluer l'influence d'une force extérieure sur les forces aux points de contact entre la pièce et le montage. On écrit les équations d'équilibre statique de la pièce soumise à une force quelconque et aux actions de contact sans frottement aux 6 points de contact. On obtient une équation linéaire de la forme :

$$[\boldsymbol{B}].[\boldsymbol{F}] = [\boldsymbol{P}] \tag{0-4}$$

$[\boldsymbol{B}]$ est une matrice 6x6, F la matrice colonne des 6 efforts de contact et P les 6 composantes du torseur d'efforts extérieurs.

On choisit comme indicateur la norme euclidienne K :

$$\mathrm{K} = \|\boldsymbol{B}\|_E.\|\boldsymbol{B}^{-1}\|_E \tag{0-5}$$

Où b_{ij} est le terme générique de la matrice B.

Dans le cas du montage de Boys, on montre que le coefficient K croît avec la distance entre les points de contact et diminue avec l'angle des V.

On en conclue que les deux indicateurs proposés donnent une méthode pour évaluer la qualité globale d'un montage. Il s'agit d'une estimation préliminaire du montage. Une application expérimentale est détaillée ci-après.

4. APPLICATION EXPERIMENTALE

4.1. Montage expérimental

Des mesures sont effectuées sur un montage expérimental. Les valeurs sont analysées pour évaluer les défauts de positionnement de la pièce sur son montage.

Fig. 3-2 Montage expérimental

La pièce est un tripode fixé sur trois V. On fait varier deux paramètres : l'angle des V (90 et 120°) et la distance des zones de contact au centre. On nomme R1 et R2 deux distances différentes.

4.2. Procédure de mesure

Comme les points de contact pièce-montage ne sont pas directement accessibles, on mesure des points sur les trois cylindres dans la partie haute près de la zone de contact.

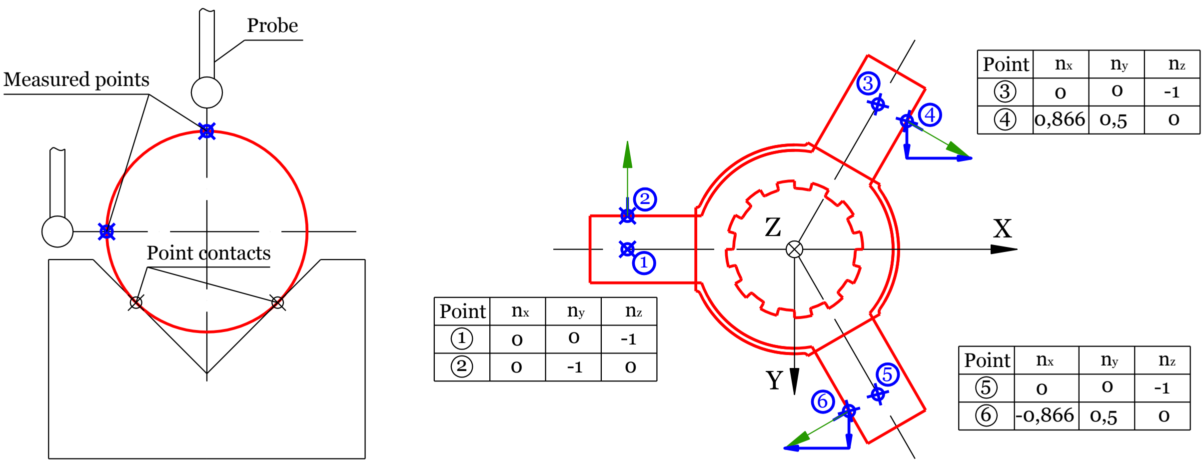


Fig. 3-3 Points mesurés sur la pièce

Les forces de serrage sont maintenues constantes pour chaque mesure grâce à un système à ressort. C'est toujours la même pièce qui est montée, mesurée puis démontée. La mesure est réalisée sur une machine à mesurer Sip-Orion à l'aide d'un capteur à touche de résolution 0.1 µm et de précision 0.8 µm sur L=800mm.

Concernant le paramètre de distance entre le centre du tripode et chacun des 3 V d'appui, les résultats expérimentaux correspondent à ceux donnés par les deux indicateurs. Par contre pour le paramètre angle du V, les résultats ne correspondent pas. Seul le coefficient K semble un bon indicateur pour évaluer la qualité du montage.

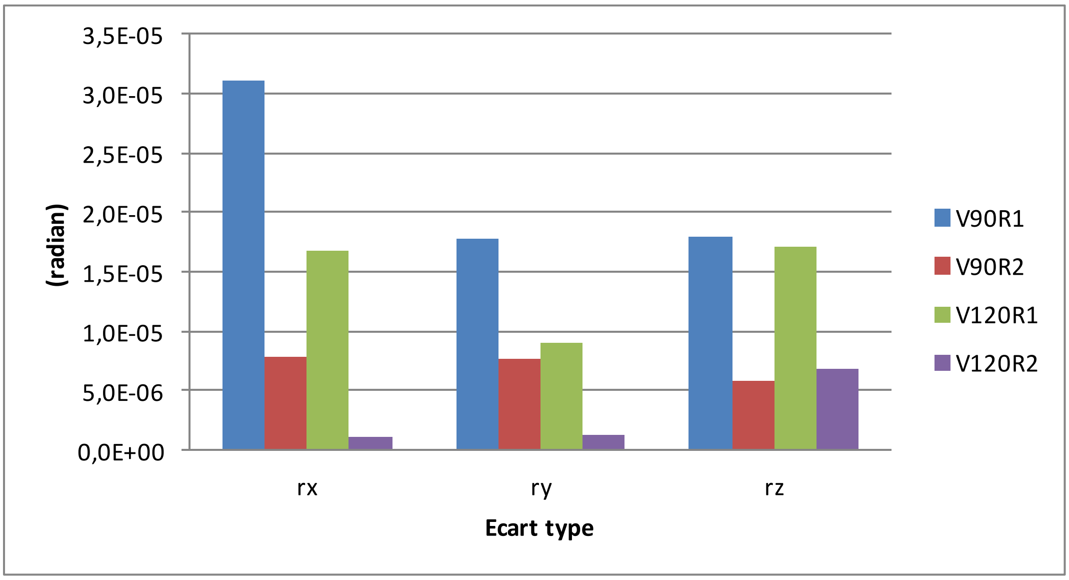


Fig. 3-4 Rotations de la pièce avec différence configurations du montage

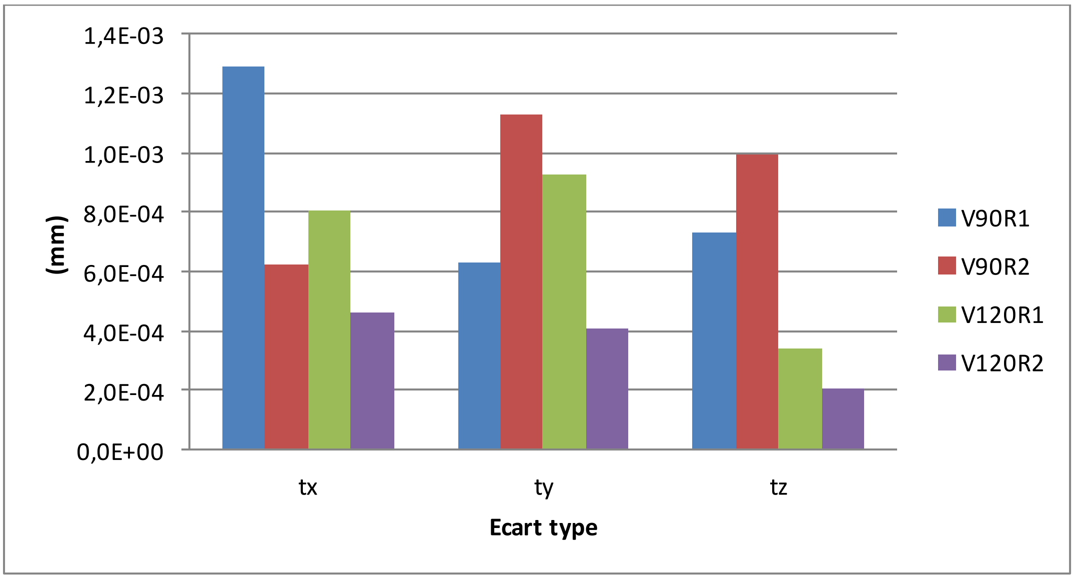


Fig. 3-5 Translations de la pièce avec différence configurations du montage

5. RESUME

Dans ce chapitre, nous avons proposé deux indicateurs pour évaluer la qualité d'un montage [DURET *et al.* 2010].

- Avec le premier indicateur « déterminant », la qualité du montage est évaluée en se basant sur les dispersions des déplacements de la pièce sur son montage. Il est représenté par la valeur absolue du déterminant de la matrice de configuration du montage.
- Le second indicateur « coefficient K », la qualité du montage est évaluée en se basant sur la sensibilité de la force de réaction aux points de contact entre la pièce et le montage. Il est représenté par un coefficient qui est calculé par la norme de la matrice de configuration du montage.

Il est important de noter que les matrices de configuration utilisées dans le calcul de ces deux indicateurs, ne sont pas les mêmes.

Les deux types de montages sont appliqués pour montrer les résultats des analyses avec les deux indicateurs.

Un montage expérimental est alors analysé pour comparer les résultats expérimentaux avec les résultats obtenus par les deux indicateurs. La comparaison montre que les résultats obtenus par le montage expérimental peuvent être expliqués par la combinaison des deux indicateurs. Donc un montage est considéré bon si les déplacements de la pièce sur son montage sont faibles et si l'influence des forces extérieures sur les réactions aux points de contact est faible.

De plus, les indicateurs simples qui sont proposés peuvent être intégrés dans un logiciel de façon à choisir rapidement une configuration de montage à partir de différents paramètres géométriques.

CHAPTER 4

SIMULATION DE L'ANALYSE DE TOLERANCES EN FABRICATION

1. INTRODUCTION

L'évaluation d'une gamme de fabrication en termes de tolérances fonctionnelles est une étape de contrôle de la qualité des produits. Il s'agit d'analyser si les exigences fonctionnelles de la pièce sont satisfaites pour une gamme donnée et des écarts de fabrication donnés. Ce chapitre présente un modèle d'analyse basé sur le modèle MMP (modèle des pièces fabriquées). Les résultats expérimentaux issus des chapitres 2 et 3 permettent de définir les variables d'entrée de la simulation. Pour obtenir des résultats expérimentaux, il est nécessaire de disposer d'un équipement approprié et d'y consacrer du temps ce qui est couteux. Pour limiter ces coûts, on peut utiliser des défauts simulés comme variables d'entrée. La simulation permet de vérifier une gamme en termes de tolérances fonctionnelles, ou de déterminer les valeurs minimales des tolérances pour une gamme donnée, un lot de pièces et un taux de non-conformité donnés.

Avant de décrire la méthode d'analyse, on rappelle le modèle MMP et quelques définitions.

2. PLUSIEURS DEFINITIONS

2.1. Erreurs de l'operation d'usinage

On distingue dans [LOOSE *et al.* 2007a] trois types d'erreurs :

- Erreur de référence : les surfaces de mise en position ont des défauts dus aux usinages précédents.
- Erreur de montage : le montage lui-même est mal disposé par rapport à l'outil ou bien une erreur est due à des contacts imparfaits entre la pièce et le montage,

- Erreur d'usinage.

2.2. Variation propagée

Ces erreurs se propagent et génèrent une erreur pour la nouvelle surface usinée. Un modèle d'espace d'état est proposé par [HUANG *et al.* 2000] pour décrire les écarts dimensionnels pour une fabrication avec reprises. [ZHONG *et al.* 2002] proposent un modèle pour étudier la propagation de ces erreurs y compris les déformations de la pièce. La plupart de ces modèles s'appliquent pour des systèmes de montage orthogonaux et ne sont pas assez généraux.

2.3. Modèle de la pièce fabriquée (MMP)

F. Villeneuve et F. Vignat proposent un modèle général de propagation des erreurs où les écarts générés à une étape de fabrication sont pris en compte pour déterminer les erreurs à l'étape suivante. Les écarts sont décrits par des composantes de torseurs de petits déplacements [VIGNAT 2005, VIGNAT *et al.* 2007, VIGNAT *et al.* 2009, VILLENEUVE *et al.* 2004, 2007]

Un écart de mise en position est défini par la différence entre la position réelle de la pièce et sa position nominale sur le montage. Cet écart est du aux erreurs géométriques de la pièce et du montage.

On exprime cet écart par un torseur de petits déplacements sous la forme :

$$\mathcal{T}_{Sj,P} = -\mathcal{T}_{P,Pi} + \mathcal{T}_{Sj,Hk} + \mathcal{T}_{Hk,Pi} \tag{0-6}$$

- $\mathcal{T}_{Sj,P}$ est l'erreur du positionnement j.
- $\mathcal{T}_{P,Pi}$ est l'erreur de référence (erreur obtenue par les opérations précédentes).
- $\mathcal{T}_{Sj,Hk}$ est l'erreur du montage j.
- $\mathcal{T}_{Hk,Pi}$ est l'erreur de liaison au contact entre la pièce et le montage.

L'écart d'usinage est exprimé comme un écart entre la surface usinée et la surface nominale créée par l'outil en position nominale (théorique). Cet écart pendant la phase j est exprimé par un torseur : $\mathcal{T}_{Sj,Pi}$. D'autre part, le torseur qui définit l'écart de la surface usinée par rapport à sa position nominale est noté $\mathcal{T}_{P,Pi}$.

Cet écart est la somme de l'écart d'usinage et de l'écart de mise en position.

$$\mathcal{T}_{P,Pi} = -\mathcal{T}_{Sj,P} + \mathcal{T}_{Sj,Pi} \tag{0-7}$$

Ces définitions concernent une pièce et son montage. Pour un lot de pièces, les valeurs statistiques sont représentatives des défauts de fabrication.

2.4. Domain écart et zone de tolérance

Dans la norme ISO 1101 une tolérance géométrique spécifie la zone dans laquelle la surface doit se situer. Il y a plusieurs approches pour comparer une pièce fabriquée avec la géométrie nominale. On peut citer l'approche par calibre virtuel [MAILHE *et al.* 2008], l'approche « GapGP » [KAMALI NEJAD 2009]. Dans cette étude, nous utilisons les domaines écarts et les zones de tolérances.

Les défauts des surfaces usinées créent un domaine appelé domaine écart. Ce domaine est alors comparé à la zone de tolérance pour vérifier les pièces fabriquées.

Le défaut de parallélisme d'une face plane par rapport à une autre face plane est défini par la dimension de la zone de tolérance qui contient le plan associé tolérancé. On procède de la même façon pour une perpendicularité d'une face plane ou une localisation (voir figure ci-après).

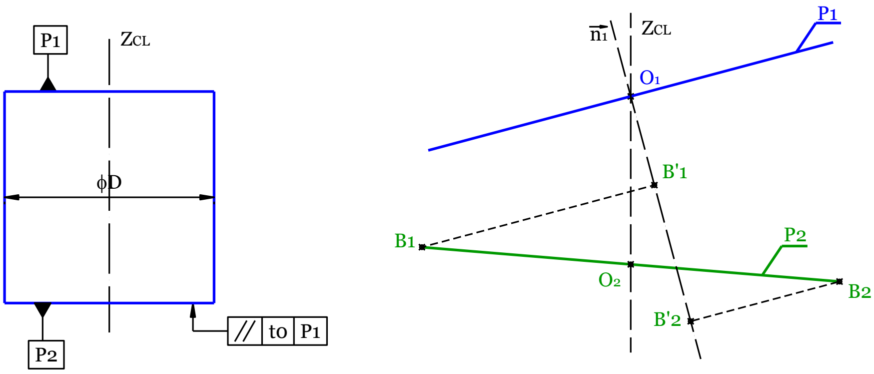


Fig. 4-1 Défaut de parallélisme

L'appartenance aux zones de tolérances se traduit par des inégalités portant sur les composantes des torseurs d'écarts, inégalités qui correspondent aux domaines écarts.

3. MODELES DE L'ERREUR DE FABRICATION

La simulation de l'analyse de tolérances est effectuée sur un exemple de pièce usinée avec une gamme donnée et un taux de non-conformité donné.

3.1. Description de la pièce usinée et de la gamme

Les bruts sont des cylindres fixés sur la fraiseuse par un montage par mandrin 3 mors doux pour fraisage en bout avec un outil de diamètre 20 mm. On distingue 4 étapes :

- sciage du brut dans une barre, le plan sectionné est nommé P0.
- fraisage du plan supérieur P1 par parcours d'outil circulaire en une passe,
- reprise en appui sur P1 et usinage de l'autre extrémité plane P2,

- reprise en appui sur P2 pour usinage en 5 passes suivant un parcours rectiligne de part et d'autre du tenon. On note P3 et P4 les plans usinés dans cette étape. Ces plans sont affectés de tolérances de parallélisme par rapport à P2.

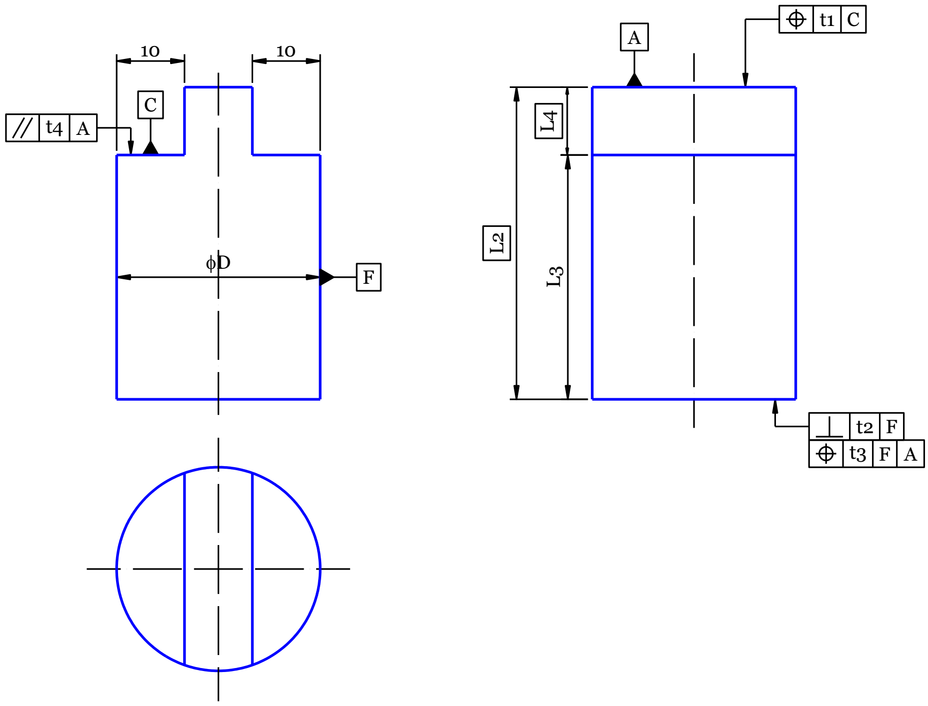


Fig. 4-2 Pièce

3.2. Modèle de la pièce fabriquée

Le modèle MMP permet d'exprimer les torseurs d'écarts de chacun des 4 plans usinés en fonction des composantes d'écarts de mise en position (r_{XS2} , r_{YS2} ...) et d'usinage (r_{X1}, r_{Y1} ...), l'indice correspond au numéro de l'étape (voir tableau ci-après). L'expression $4R\sin 2\theta/2$ $/(3\theta-\sin\theta)$ est la distance du centre d'un arc de cercle de rayon R et d'angle θ au centre de gravité de la face délimitée par cet arc et sa corde.

3.3. Modèle mathématique basé sur le modèle MMP

On rappelle que seuls les écarts de rotation et de translation sont pris en compte dans ce modèle. Il permet toutefois de déterminer les écarts fonctionnels de chacune des surfaces usinées en fonction des écarts de mise en position et d'usinage cumulés au long de l'exécution de la gamme. Pour vérifier par exemple la tolérance de localisation du plan P1 par rapport à P2, on calcule le torseur.

On en déduira les déplacements des points au bord de la zone de tolérance pour vérifier s'ils appartiennent ou non à cette zone.

Plans usinés	TPD
P1	$\mathcal{T}_{P,P1} = \begin{Bmatrix} -r_{X_{S2}} + r_{X_1} & -L_1 r_{X_{S2}} \\ -r_{Y_{S2}} + r_{Y_1} & L_1 r_{Y_{S2}} \\ 0 & -t_{Z_{S2}} + t_{Z_1} \end{Bmatrix}_{O_1X_1Y_1Z_1}$
P2	$\mathcal{T}_{P,P2} = \begin{Bmatrix} -r_{X_{S3}} - r_{X_2} & L_2 r_{X_{S3}} - L_2 r_{Y_{S3}} + L_2 r_{Y_2} \\ -r_{Y_{S3}} + r_{Y_2} & L_2 r_{Y_{S3}} - L_2 r_{X_{S3}} + L_2 r_{X_2} \\ 0 & t_{Z_{S3}} - t_{Z_2} \end{Bmatrix}_{O_1X_1Y_1Z_1}$
P3	$\mathcal{T}_{P,P3} = \begin{Bmatrix} -r_{X_{S4}} + r_{X_3} & -L_2 r_{Y_{S4}} + L_4 r_{Y_3} \\ -r_{Y_{S4}} + r_{Y_3} & L_2 r_{X_{S4}} - L_4 r_{X_3} \\ 0 & -t_{Z_{S4}} - \dfrac{4R\sin^3\frac{\theta}{2}}{3(\theta - \sin\theta)} r_{Y_3} + t_{Z_3} \end{Bmatrix}_{O_1X_1Y_1Z_1}$
P4	$\mathcal{T}_{P,P4} = \begin{Bmatrix} -r_{X_{S4}} + r_{X_4} & -L_2 r_{Y_{S4}} + L_4 r_{Y_4} \\ -r_{Y_{S4}} + r_{Y_4} & L_2 r_{X_{S4}} - L_4 r_{X_4} \\ 0 & -t_{Z_{S4}} + \dfrac{4R\sin^3\frac{\theta}{2}}{3(\theta - \sin\theta)} r_{Y_4} + t_{Z_4} \end{Bmatrix}_{O_1X_1Y_1Z_1}$

Tab. 4-1 TPD des surfaces usinées

4. ANALYSE DE TOLERANCE

L'analyse des tolérances est effectuée par la simulation de Monte Carlo.

4.1. Simulation de Monte-Carlo

L'algorithme utilisé peut être résumé ainsi :

- Créer un modèle paramétrique $y = f(x_i)$
- Générer un ensemble de variables aléatoires d'entrée x_{ij}
- Évaluer la valeur de y d'après le modèle et enregistrer le résultat y_j
- Répéter l'étape précédente pour j=1 à n échantillons.

Analyser les résultats en terme d'histogramme et valeurs statistiques pour y.

4.2. Variables d'entrée obtenues par expérimentation

Les défauts des plans usinés sont exprimés à partir des mesures par des torseurs de petits déplacements. Les résultats expérimentaux obtenus au chapitre 2 sont appliqués pour l'analyse des tolérances des pièces présentées au chapitre 3. On reprendra donc les valeurs des défauts d'usinage et de mise en position qui ont été identifiées. Les lois choisies avec leurs paramètres correspondent à celles qui ont été identifiées.

4.3. Variables d'entrée issues de la simulation

Dans cette partie, on étudie l'influence des différents facteurs sur la mise en position de la pièce sur son montage : les erreurs du plan de mise en position, les forces de serrage, les frottements entre pièce et montage. Un modèle de solides déformable est réalisé pour la pièce en aluminium et les trois mors en acier. Les calculs sont réalisés par le logiciel Abaqus par éléments finis.

A partir de ce modèle, des plans d'expériences statistiques à deux niveaux permettent de déterminer les facteurs les plus influents sur les écarts de mise en position.

Ces modèles sont alors utilisés dans la simulation de Monte Carlo pour évaluer les écarts de mise en position de la pièce.

5. PRESENTATION DES RESULTATS

On peut ainsi en déduire la valeur minimale des tolérances pour respecter un taux de non-conformité imposé. Par exemple si la localisation entre deux plans donne un taux de rebus de 1190 ppm pour une tolérance de 0.25mm (exemple ci-dessus), on vérifiera qu'avec une tolérance d 0.305, le taux de rebus deviens quasiment nul pour le nombre d'échantillons choisi.

6. RESUME

Dans ce chapitre, nous avons introduit la simulation basée sur le modèle MMP pour l'analyse des tolérances en fabrication. Cette simulation est effectuée par la méthode de Monte Carlo avec pour variables d'entrée celles obtenues par expérimentation ou par simulations.

Ces simulations ont permis de :

- Vérifier une gamme en termes de tolérances fonctionnelles pour un lot de pièces usinées,
- Déterminer les tolérances optimales de pièces usinées en suivant une gamme et des exigences données et pour un taux de non-conformité donné.

Nous présentons un modèle qui combine l'expérimentation, le calcul par élément fini et la simulation de Monte Carlo pour déterminer les défauts de mise en position des pièces fixées sur leur montage. Dans ce modèle les différents facteurs qui affectent la mise en position de la pièce sont considérés et leurs effets sont évalués. Lorsque les données d'entrée de la simulation correspondent à des résultats réels, les sorties sont plus proches de la production réelle.

CONCLUSION & PERSPECTIVES

L'objectif de ce travail de thèse est de contribuer à l'analyse, la mesure et la simulation des défauts de fabrication tridimensionnels. Pour cela, nous avons présenté les problèmes et les limitations de l'identification et de la simulation des défauts de fabrication pendant un procédé d'usinage. Nous avons développé plusieurs méthodes basées sur le concept de torseur des petits déplacements. De plus, nous avons appliqué une méthode pour l'analyse des défauts de forme de façon à compléter les analyses des défauts d'usinage.

La première méthode proposée est basée sur le concept de torseur de petits déplacements pour identifier les défauts de fabrication. Cette méthode permet de distinguer le défaut d'usinage et les défauts de montage pour un lot de pièces pendant une phase. Plusieurs propositions sont proposées pour la mesure en s'inspirant des travaux de [TICHADOU 2005]. On peut citer:

- l'analyse des résultats de mesure à partir de plusieurs moyens de mesure.
- l'évaluation des capabilités comparables des résultats de mesure obtenus par deux moyens de mesures.
- la détermination de la relation de translation entre deux plans usinés qui ne peuvent être parfaitement parallèles à cause des imperfections de fabrication.
- le calcul des défauts de forme et d'orientation des plans usinés en compléments des défauts de localisation,
- la vérification de l'existence ou non du contact entre une face plane de la pièce et le plan de mise en position du montage.

Les résultats de l'analyse des défauts d'usinage et de mise en position peuvent être utilisés dans différentes simulations, par exemple les paramètres statistiques sont utilisés dans une simulation de Monte-Carlo et les composantes des torseurs dans des simulations dans le modèle MMP.

De plus, nous appliquons la méthode d'analyse modale, qui est habituellement utilisée pour analyser les défauts de forme d'une pièce mesurée sur une machine MMT avec des centaines de points de mesure. On complète ainsi l'analyse par la mesure des défauts de forme avec un nombre restreint de points de mesure (10 points sur chaque surface usinée). Bien que ce nombre semble faible, les modes de défauts de forme sont assez bien identifiés (courbure, ondulation, torsion etc.).

A cause du rôle important du défaut de mise en position dans la qualité du produit durant la fabrication, nous proposons deux indicateurs simples pour évaluer la qualité globale du montage.

- Avec le premier indicateur dit « déterminant », la qualité du montage est évaluée en se basant sur les dispersions des déplacements de la pièce sur le montage. Il s'agit de la valeur absolue du déterminant de la matrice de configuration.
- Avec le second indicateur, dit « coefficient K », la qualité du montage est évaluée en se basant sur la sensibilité à la force de réaction aux points de contact entre la pièce et le montage. Ce coefficient est calculé à partir de la norme de la matrice de configuration. Le montage est considéré bon lorsque les influences d'une force extérieure sur la pièce sont faibles sur les forces de contact.

Les deux indicateurs sont alors appliqués au système pièce-montage mesuré sur la MMT. La combinaison des deux indicateurs peut expliquer les résultats de l'expérimentation. Ces résultats montrent qu'un montage est considéré bon si les déplacements de la pièce sur le montage sont faibles et si les influences d'une force extérieure sur les réactions aux points de contact sont faibles.

De plus, nous développons un modèle pour la simulation de défauts de positionnement d'une pièce fixée sur un mandrin trois mors. Pour cette modélisation on combine trois méthodes : l'expérimentation, la simulation par éléments finis et la simulation de Monte-Carlo. Trois facteurs, qui sont supposés être les plus importants dans les défauts de positionnement sont utilisés dans ce modèle. En se basant sur les résultats de simulation, les influences de ces facteurs sont estimées. Les résultats obtenus par simulation peuvent être exprimés sous forme de distributions ou de paramètres statistiques.

Enfin, mais non des moindres, un modèle est développé en se basant sur le modèle MMP pour l'analyse de tolérances. Ce modèle est réalisé par les étapes suivantes :

- Étape 1 : les défauts d'usinage et de mise en position sont exprimés pour une gamme donnée.

- Étape 2 : le modèle des pièces fabriquées basé sur les torseurs de petits déplacements est créé pour déterminer les défauts des pièces fabriquées à la fin des opérations d'usinage.
- Étape 3 : Les domaines écarts sont créés à partir des défauts obtenus à l'étape 2. Ces domaines écarts sont des expressions mathématiques qui seront utilisées dans la simulation de Monte Carlo.

Les simulations de Monte Carlo permettent de vérifier une gamme d'une pièce spécifiée par des tolérances fonctionnelles et de déterminer la proportion de pièces usinées hors tolérances, ou bien de déterminer les tolérances fonctionnelles qui correspondent à un taux de rebus donné. Les paramètres d'entrée de ces simulations peuvent être issus de l'expérimentation ou obtenus en combinant expérimentation et simulation des défauts de mise en position (chap. 4).

Cette étude s'est intéressée à l'identification et à la simulation des défauts de fabrication. Des recherches supplémentaires sont nécessaires pour finaliser ces objectifs et les rendre plus précis. Pour cela on propose :

- de développer un programme pour analyser les défauts de fabrication à partir des mesures,
- d'étudier des moyens de mesure plus rapide et précis,
- d'approfondir la notion d'indicateurs pour les différents types de montage d'usinage,
- de prendre en compte d'autres facteurs comme les jeux dans les montages,
- de réduire les temps de simulation.

Enfin, pour que les variables d'entrée pour la simulation donnent de bons résultats, nous proposons :

- d'analyser les défauts de fabrication au préalable sur un échantillon de quelques pièces,
- de créer une bibliothèque de défauts de fabrication pour différents types de montages et de surfaces usinées.

Contents

NOMENCLATURE

d_{01}, d_{02} and d_{12}	the dimensions between two surfaces 0 and 1, 0 and 2, 1 and 2, respectively
L_0, L_1 and L_2	the distances between a reference and surfaces 0, 1, and 2, respectively
s_{01}^2, s_{02}^2 and s_{12}^2	the variances of dimensions d_{01}, d_{02} and d_{12}, respectively
s_0^2, s_1^2 and s_2^2	the variation of surfaces 0, 1, and 2, respectively
$\mathcal{T} = \{\vec{R} \quad \vec{T}\}_{OXYZ} = \begin{Bmatrix} r_X & t_X \\ r_Y & t_Y \\ r_Z & t_Z \end{Bmatrix}_{OXYZ}$	the Small Displacement Torsor (SDT) including rotation vector $\vec{R}(r_X, r_Y, r_Z)$ and translation vector $\vec{T}(t_X, t_Y, t_Z)$ in coordinate system OXYZ
r_X, r_Y, r_Z	the small rotations around X, Y, and Z axis
t_X, t_Y, t_Z	the small translations around X, Y, and Z axis
$s_{r_X}^2, s_{r_Y}^2, s_{t_Z}^2$	the variances of r_X, r_Y, and t_Z, respectively
$s_{(a-b)}^2$ or $s_{(a+b)}^2$	the variance of sum the two random variables a and b
$Cov(a, b)$	the covariance of a and b
MCS	Machine Coordinate System
PCS	Part Coordinate System
$r_{Xi,MCS}, r_{Yi,MCS}$	the rotations around X and Y axis of plane i in MCS
$\vec{n}_i(n_{Xi}, n_{Yi}, n_{Zi})$	the normal vector of machined plane i

m_{r_X}, m_{r_Y}	the means of rotation defects
t_{Z01CNC}, t_{Z02CNC}	the variations of the distances along Z axis between plane $O_M XY$ and machined surface 1, and 2, respectively. These are calculated from the measurements on the CNC machine.
$O_M XY$	the plane is created by origin O_M (measured point) and X, Y axis of the CNC milling machine. It is considered as a perfect plane in this study.
t_{Z12CNC}	the variation of the distance between machined surface 1 and 2 calculated by measurement on the CNC machine
$r_{Xi,PCS}, r_{Yi,PCS}$	the rotations of machined plane i in PCS
$\vec{n}_{Cyl}(n_{X,Cyl}, n_{Y,Cyl}, n_{Z,Cyl})$	the direction vector of workpiece cylinder axis
$t_{Z01CMM}, t_{Z02CMM}, t_{Z12CMM}$	translation relationships between plane 0 and plane 1, plane 0 and plane 2, plane 1 and plane 2 measured on the CMM, respectively
H, L, h	the dimensions of a prismatic workpiece
$[\boldsymbol{A}]$	the matrix of a fixture configuration for the first indicator “determinant” of the quality of a fixture
$\lvert\det[\boldsymbol{A}]\rvert$	the absolute value of determinant of matrix $[\boldsymbol{A}]$
$[\boldsymbol{\delta}]$	the matrix of the displacements of the contact points along their normal vector
$L_i = \{\vec{n}_i \quad \vec{n}_i \times \overrightarrow{OM}_i\}$	the normal line (L_i) are defined in the Plücker coordinates
$\vec{n}_i \times \overrightarrow{OM}_i$	the cross product/vector product of two vectors
$\vec{n}_i \cdot \overrightarrow{OM}_i$	the dot product of two vectors
$\vec{n}_i$	the direction vector (or unit vector) of (L_i)
$\overrightarrow{OM}_i$	a vector is defined from origin O of coordinate system OXYZ to point M_i

$[\boldsymbol{B}]$	the matrix of a fixture configuration for the second indicator "coefficient K" of the quality of a fixture
$\vec{F}_i$	the reacting forces at point contact i
$\vec{P}$	the clamping force (external force)
$\mathrm{K} = \lVert\boldsymbol{B}\rVert_E . \lVert\boldsymbol{B}^{-1}\rVert_E$	the pseudo condition of the matrix $[\boldsymbol{B}]$, namely coefficient K
$\lVert\boldsymbol{B}\rVert_E, \lVert\boldsymbol{B}^{-1}\rVert_E$	pseudo-Euclidean norms of matrix $[\boldsymbol{B}]$ and $[\boldsymbol{B}]^{-1}$
$\mathcal{T}_{P,Pi}$	SDT of the surface Pi compared with its nominal
$\mathcal{T}_{Sj,Pi}$	SDT of machined surface Pi in setup j
$\mathcal{T}_{Sj,P}$	SDT of workpiece position in setup j
$\mathcal{T}_{Sj,Hk}$	the fixture errors of setup j
$\mathcal{T}_{Hk,Pi}$	the link errors of contacts between the workpiece and the fixture in setup j

LIST OF FIGURES

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INTRODUCTION

1. MANUFACTURING DEFECTS

Manufacturing defects, by definition, are imperfections that inevitably occur in products of a given design as a result of the fallibility of the manufacturing process. A manufacturing defect in a product is determined by comparing that product to the product design and specifications. A machined part is a rejected part if its manufacturing defects are out of the product specifications, and vice versa.

Parts cannot be machined to ideal or design dimensions because inaccuracies are inherent in the manufacturing process. These imperfections are recognized as errors that can come from different sources, such as machine tools, cutting tools, fixtures, workpieces, etc. Each of these errors contains other various error sources. In [ZHANG 1997], K. Whybrew and G. A. Britton have summarized 27 error resources in a machining process for the following 8 items in machining: machine tool, cutting tool, fixture, workpiece, coolant, operator, environment conditions, process variable.

From a manufacturing or measurement point of view, the manufacturing defects are observed on a state of a machined part at the end of manufacturing operations. A manufacturing defect is the amount of positioning deviations and machining deviations. Measurements determine whether the manufacturing defects are within the specified limits. Tolerances set the boundaries within which manufacturing must operate.

In this thesis, the positioning deviation and machining deviation are considered for a workpiece/machined part during manufacturing. If we calculate variances of the positioning or machining deviation of a batch of machined parts, the values obtained represent positioning or machining defects. In our research, the manufacturing defects are divided into two principle categories: positioning defects and machining defects.

1.1. Identification of manufacturing defects

There are different approaches for classifying the manufacturing defects. As mentioned previously, we consider that manufacturing defects include machining defects and positioning defects. Determination of the manufacturing defects is necessary for evaluating the quality of a product as well as finding causes of defects in order to improve their quality. In this study, we are interested in methods for identifying the positioning and machining defects. However, in literature, there is little research on investigation of methods for identifying manufacturing defects.

Consequently, we have developed a measurement method and a measuring data treatment method that are based on the study of Tichadou [TICHADOU 2005, TICHADOU *et al.* 2007]. The methods allow identifying and distinguishing between machining and positioning defects. The results obtained are expressed by different forms, such as: SDT form; flatness, perpendicular, and parallelism defects; distributions and statistical parameters. These results can be further used in simulation, for example Monte Carlo, to evaluate the capacity of a process to produce parts consistent with the functional tolerances.

The results obtained from experiments show that the positioning defect strongly influences the quality of a product. Thus, we proposed simple indicators to evaluate the positioning defect of a workpiece fixed on a fixture.

1.2. Evaluation of positioning defect

Generally, a workpiece is located and held on a fixture during a machining, measuring, or assembly operation. The workpiece must satisfy a unique position, orientation, and static equilibrium. If it deviates from its required location, it is mostly due to clamping force errors, workpiece/fixture deformations, friction at contact surfaces between a workpiece and a fixture, cutting force errors, etc. Deviations of a batch of workpieces fixed on a fixture are also known as positioning defects. Different types of fixture or a fixture that has different geometric parameters can be used to fix a workpiece. Each fixture has different precisions. Hence, evaluating the quality of a fixture is also known as evaluating the positioning defects of workpieces on that fixture.

Although many studies investigated on fixture evaluation by indicators, they are complicated for performing during a process plan. We propose in this thesis simple indicators that make it possible to evaluate the global quality of a fixture. These indicators can be used in fixture design for the preliminary estimation of fixture configurations.

1.3. Simulating positioning defect of a workpiece on a fixture

Results obtained from experiments are very useful for evaluating the quality of product as well as simulating the defects of a new design part. However, we have to machine and measure on appropriate equipments. For instance, a machined tool must be equipped of a measurement mean in order to measure workpieces on the fixture inside this machine. In other words, it is very costly and time-consuming. Therefore, developing accurate models and methodologies for simulating the manufacturing defects are necessary to overstep these drawbacks.

We investigate influences of different factors on positions of a workpiece fixed on a fixture. Three treatment factors are proposed: geometrical errors of a workpiece locating plane, clamping force and coefficient of friction at contact surfaces between the workpiece and the fixture. For this, a number of finite element simulations based on a statistical two-level full factorial design of experiments method are conducted in order to determine mathematical models. The models are then used in Monte Carlo simulation for determination of the positions defect. These results can be used in manufacturing tolerance analysis.

2. TOLERANCE ANALYSIS

This is an essential step in manufacturing to evaluate the quality of products in terms of functional tolerances. During manufacturing, a product can pass through one or several processes, machine tools, fixtures, cutting tools and the sequencing of operations that are called the process plan. Consequently, evaluation of a process plan in terms of functional tolerance is a direct step to control the quality of products, which is also known as tolerance analysis. In other words, controlling the quality of products is verifying whether the design tolerance requirements meet a given process plan with specified manufacturing deviations.

Tolerance analysis is the general term for activities related to the study of accumulated variation in machined parts or assemblies. This allows determining geometric dimensions of a machined part at any operation during a process plan. To do this, on the one hand, requirements of functional tolerances are provided by drawing designs, on the other hand, manufacturing constraints are given by precision of a machine tool/measuring machine, ability of cutting tools, etc.

Some recent simulation tools of the tolerance analysis and dimensional chains are usually unidirectional, e.g. dimensional chains in manufacturing, Δl method is presented by [BOURDET 1973]. These simulation tools do not take into account small angular deviations between different machining set-ups because they are projected on a reference system.

Hence, three-dimensional (3D) models have been developed in order to solve the problems in tolerance analysis.

3D approaches have been performed to solve the small angular deviations between different machining set-ups. These models are based on an interpretation of 3D geometric specifications [CLÉMENT *et al.* 1998, CLÉMENT *et al.* 1997], a matrix approach [DESROCHERS *et al.* 1997], a tensor approach [WHITNEY *et al.* 1994] for simulating the accumulated variation. A transfer procedure that is used for determining the three-dimensional manufacturing specification defined with ISO tolerancing standards is presented in [ANSELMETTI *et al.* 2005].

∆TOL method of 3D tolerance are developed by [BALLOT *et al.* 1996, BOURDET *et al.* 1995a, THIEBAUT 2001]. This method models deviations of surfaces and interactions between different parts in an assembly by the Small Displacement Torsor concept. Thereafter, [VILLENEUVE *et al.* 2001] modeled a process plan as succession of assemblies (machine, workpiece, fixture, and machining operation) for analyzing manufacturing variations and verifying with functional tolerances. Vignat followed the work of Villeneuve by proposing a Model of Manufactured Part (MMP) [VIGNAT 2005]. A development of the solution techniques associated with the MMP is then presented in [KAMALI NEJAD 2009].

Thanks to these studies, we propose in this thesis a mathematical model for tolerance analysis based on Model of Manufactured Part (MMP). In this model, the experimental results or simulated results are used as input variables for simulating manufacturing defects (machining and positioning defects) during design phases of a process plan. This allows to:

- verify the process plan in terms of functional tolerances
- or determine minimum tolerances of a batch of machined parts based on an existing process plan and number of rejected parts per million (ppm).

3. ORGANIZATION OF THE THESIS

This dissertation is organized into six parts:

- **"Introduction"** introduces the background, rationale, objective, and methodologies related to this study;
- **"Chapter 1 - Literature review and problem statement"** gives a review of previous studies related to identification and simulation of manufacturing defects. Problems needed to be addressed in these research domains are discussed during the literature review. The problem statement is finally proposed;

- **“Chapter 2 - Identification of manufacturing defects”** develops a method based on SDT concept for identifying machining defects and positioning defects of a batch of parts during a process plan;
- **“Chapter 3 - Evaluating the quality of a fixture by indicators”** introduces simple indicators that allow evaluating the global quality of a fixture as well as positioning defects of a workpiece fixed on that fixture;
- **“Chapter 4 - Simulation of tolerance analysis in manufacturing”** introduces a mathematical model for tolerance analysis based on MMP that allows us to verify the process plan in terms of functional tolerances or determine minimum tolerances of a batch of machined parts based on an existing process plan and number of rejected parts per million (ppm);
- **“Conclusions and perspectives”** summarizes the results obtained in this thesis and proposes future works.

CHAPTER 1

LITERATURE REVIEW AND PROBLEM STATEMENT

This chapter provides a review of literature related to this thesis. Firstly, the literature related to uncertainties in manufacturing is reviewed, then previous works in tolerance analysis are examined. During the literature review, problems that need to be addressed in these research domains are discussed. The problem statement is given in the concluding parts .

1. MANUFACTURING DEFECTS

Manufacturing defects, by definition, are imperfections that inevitably occur in products as a result of the fallibility of the manufacturing process. A manufacturing defect in a product is determined by comparing that product to the product design and specifications. A machined part could be a rejected part if its manufacturing defects are out of the product specifications, and vice versa.

Parts cannot be machined to ideal or nominal dimensions because inaccuracies are inherent in the manufacturing process. These imperfections are recognized as errors that can come from different sources, such as machine tools, cutting tools, fixtures, workpieces, etc. Each of these errors contains other various error sources. In [ZHANG 1997], K. Whybrew and G. A. Britton have summarized 27 error resources in a machining process for the following 8 items in machining:

1. *Machine tool*
    - Clearance between moving parts of slideways and bearings
    - Geometric error in slideways, bearings, and leadscrews
    - Dynamic stiffness

- Resolution of the measuring system
- Resolution of positioning system
- Thermal stability

2. *Cutting tool*
    - Tool wear
    - Variation of tool size and cutting geometry
    - Rigidity of the tool and support
    - Thermal stability
3. *Fixture*
    - Variation between duplicate fixture
    - Variation in location
    - Wear and contamination of locating surfaces
    - Deflection of locators and fixture
    - Thermal stability
4. *Workpiece*
    - Variation in workpiece size
    - Rigidity of workpiece
    - Thermal stability
    - Stress relaxation
    - Variation in physical and chemical properties
5. *Coolant*
    - Variation of flow
    - Variation of temperature
    - Contamination
    - Degradation
6. *Operator*

    Variations are particularly apt to occur if the finished size is under the direct control of the operator

7. *Environment conditions*

    Changes in temperature affect the machine, fixture and tool geometry, and hysteresis in moving parts (e.g., slideways and bearings)

8. *Process variable*

    Changes in process variables, such as feed and depth of cut, have a direct effect on workpiece size and geometric variation

From a manufacturing or measurement point of view, a manufacturing defect is observed on a state of a machined part at the end of manufacturing operations. A manufacturing defect is the amount of positioning deviation and machining deviation from its specified position. Measurements determine whether the manufacturing defect is within the specified limits. Tolerances set the boundaries within which manufacturing must operate.

Usually, the manufacturing defects are divided into two principle categories: positioning deviation and machining deviation. Each deviation depends on different source errors, such as machine tools, cutting tools, workpieces, etc. Using the diagram of Ishikawa, the effects of the manufacturing defect are shown in Fig. 1-1.

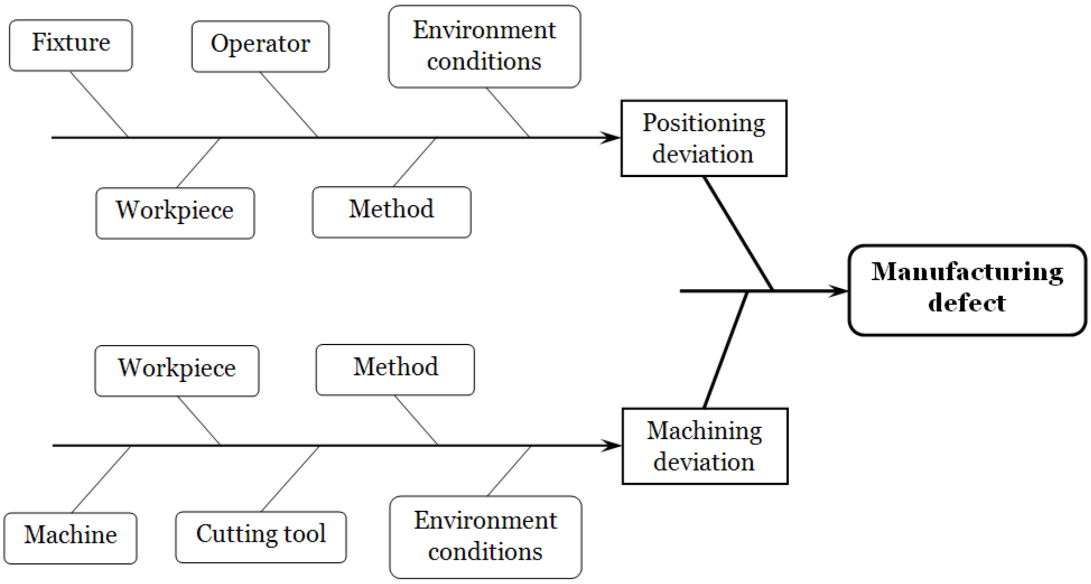


Fig. 1-1 Source errors of manufacturing defect

We are interested in methods for identifying the positioning and machining deviation. However, there is little research investigating methods for identifying manufacturing defects. Firstly, we shortly summarize the studies that help us obtain an overview of methods as well as equipments for evaluating factors that affect the quality of a product in manufacturing. Then, we analyze several studies that have been used for identifying manufacturing defects.

1.1. Factors affecting the quality of product in manufacturing

Analyses and evaluations of the error sources play an important role in reducing or eliminating the errors in manufacturing. Numerous studies have been conducted on the four items: machine, cutting tool, fixture, and workpiece. We will shortly describe the state of the art concerning different sources of the fours items.

1.1.1. Machine tool errors

The accuracy of a machine tool is one of the factors that affect the accuracy of machined part dimensions. Different error resources can induce machine tool errors. Many research works have been interested in two main source errors in machine tools. They are errors due to geometric inaccuracies, and thermally induced errors.

The machine tool errors can be reduced by the structural improvements in design and manufacturing practices. Nevertheless, due to physical limitations, production and design techniques solely cannot improve accuracy. Hence, several models have been developed for identification, characterization and compensation of these error sources.

Most models used for identification, evaluation or compensation of the geometrical machine errors are based on a Homogeneous Transformation Matrix (HTM). This is a 4×4 matrix used to represent any position and orientation of a frame in three-dimensions.

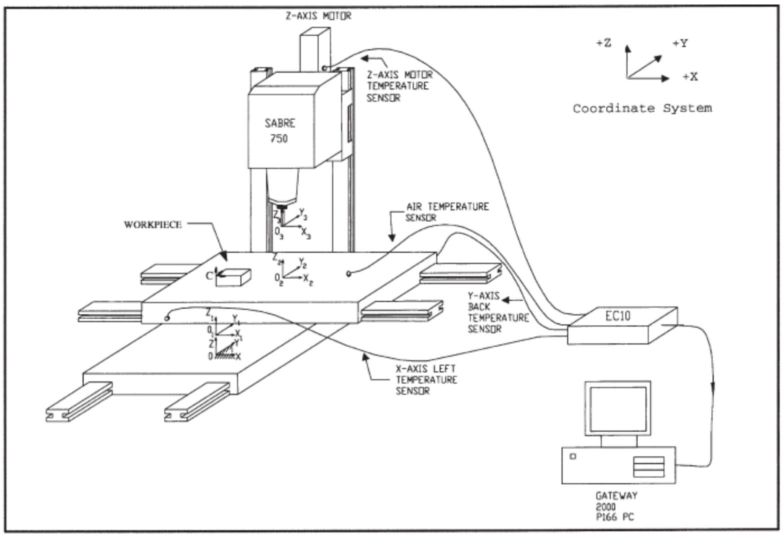


Fig. 1-2 Schematic of the system for measuring error displacements of a three axes machine [OKAFOR *et al.* 2000]

[EMAN *et al.* 1987] developed an error model based on inaccuracies in the geometry and mutual relationships of the machine structural elements as well as errors resulting from the relative motion between these elements. [KIM *et al.* 1991] presented a volumetric error model based on a HTM for generalization geometric errors. [RAHMAN *et al.* 2000] used a three-dimensional volumetric error model for compensation of errors of the considered machined tools. This model is obtained by a combination of different measurements such as positional, straightness, squareness, and angular error etc. [OKAFOR *et al.* 2000] have developed a kinematic error models accounting for geometric and thermal errors in vertical machining center. In this model, error displacements of the three axes machine (Fig. 1-2) are modeled using a HTM. The model is then used for calculating and predicting the resultant error vector at the tool-workpiece interface for error compensation. [RAMESH *et al.* 2000a] tried to review research works for analyzing the various sources of geometric errors and methods for elimination or compensation employed in machine tools.

In addition, [KAKINO *et al.* 1993] have used double ball bar device (Fig. 1-3) for measuring positioning errors of multi axes machine tools in a volumetric sense. For the reason of kinematic errors due to geometric inaccuracies, [UDDIN *et al.* 2009] presented a simulator of machining geometric errors in five axes machining based on the effect of kinematic errors on the three-dimensional interference of a tool and a workpiece. Several kinematic models of various types of five axes machines have been constructed for investigating the effect of kinematic errors on motion accuracies [LIN *et al.* 2003, SOONS *et al.* 1992, SRIVASTAVA *et al.* 1994, SUH *et al.* 1998]. There are also numerous research works available on the identification of kinematic errors based on the measurement of a machine motion errors. Measurement devices based on double ball bar method have been often used for identification of kinematic errors on five axes machines [ABBASZAHEH-MIR *et al.* 2002, KAKINO *et al.* 1994, MAYER *et al.* 1999, SAKAMOTO *et al.* 1994, TSUTSUMI *et al.* 2003, 2004].

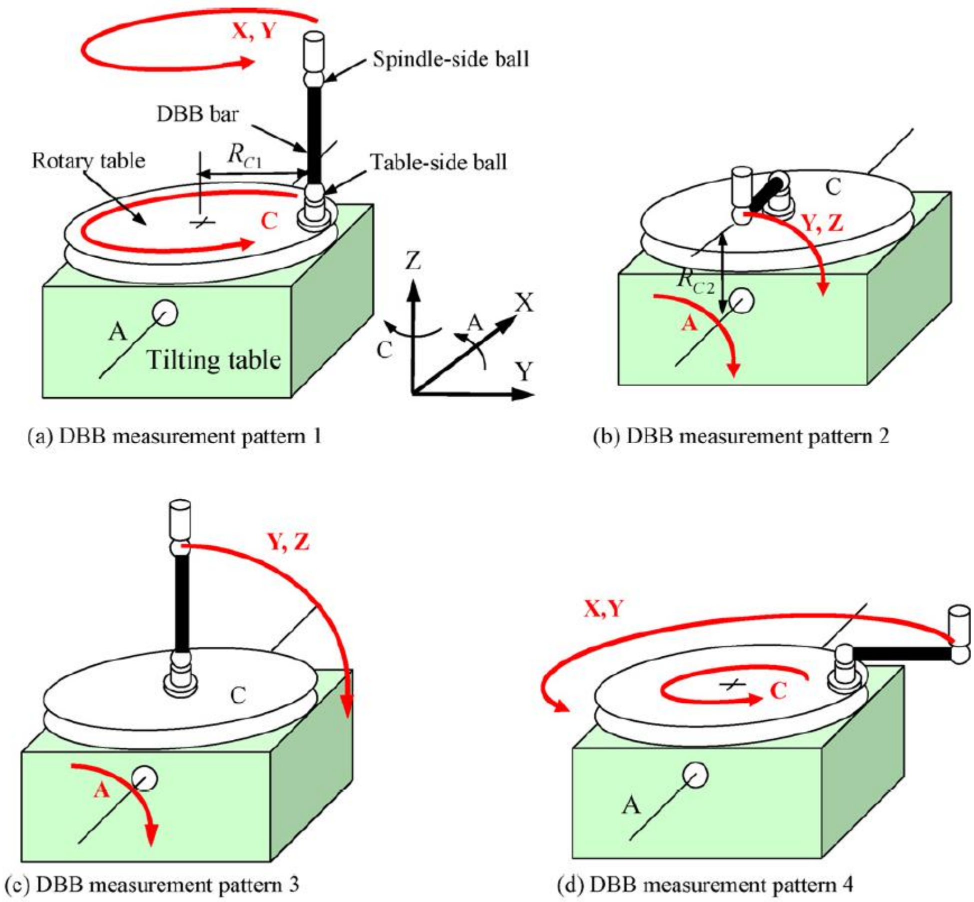


Fig. 1-3 Double ball bar device [UDDIN *et al.* 2009]

Thermal error has been considered as another major source of inaccuracy. [RAMESH *et al.* 2000b] said that *"Continuous usage of a machine tool causes heat generation at the moving elements and this heat causes expansion of the various structural elements of the machine tool. It is this expansion of the structural linkages of the machine that leads to inaccuracy in the positioning of the tool"*. Many studies on estimation and compensation of temperature errors during machining are summarized in this paper. Considering a significant effect of thermal errors on a spindle, [SRINIVASA *et al.* 1996, YANG *et al.* 2004] presented a method for measuring spindle thermal drifts in machine tools by a laser ball bar as the calibration instrument. This method allowed measuring the spindle center coordinates and spindle axis direction with respect to the machine coordinate system for various thermal states of the machine. [YANG *et al.* 2003] proposed a dynamic modeling strategy for machine tool thermal error compensation based on dynamic behaviors of temperature field and thermal deformation of spindle.

We have just summarized the studies about the effects of different machine tool errors and the methods as well as the devices that are used for identifying the errors during machining. In the next section, studies on cutting errors are provided.

1.1.2. Cutting tool errors

In the context of machining, a cutting tool is used to remove material from the workpiece by means of shear deformation. Thus, the factors relative to the cutting tool play an important role in the quality of products, such as cutting conditions (cutting speed, feed rate, and depth of cut…), tool deflection, tool wear, etc. Over the years, there has been a lot of research on effects of the above factors.

Several approaches have been developed to investigate cutting conditions (typically, cutting speed, feed rate, depth of cut). [BEAUCHAMP *et al.* 1996] have proposed six independent variables (cutting speed, feed rate, depth of cut, tool nose radius, tool length and type of boring bar) to evaluate their effects on roughness of machined surfaces in a lathe dry boring operation. They showed that using a short tool length always provides good surface roughness, and slight improvements on surface roughness are achieved by controlling cutting speed, feed rate and tool nose radius. Alternatively, with a long tool length, the cutting variables are important factors on surface roughness. [BENGA *et al.* 2003] have investigated the effect of cutting speed and feed rate on surface roughness and tool life using design of experiments on machining of hardened 100Cr6 bearing steel (62-64HRC) using polycrystalline cubic boron nitride and ceramic tools. They showed that feed rate is the most significant factor affecting surface finish. [GADELMAWLA *et al.* 2008] presented a method that expresses relationships between cutting conditions and image texture feature of machined surfaces in milling operations. [TEKINER *et al.* 2004] introduced a method based on cutting process sounds, which were recorded during machining of AISI 304 stainless steels with ideal cutting parameters, to develop an alarming system.

In addition, tool deflection is another factor that occurs during machining, especially if flexible tools are used in milling operations. Models that are based on the cantilever beam theory have been used to illustrate or predict tool deflection (Fig. 1-4) [DÉPINCÉ *et al.* 2006a, 2006b, KLINE *et al.* 1982, RAO *et al.* 2006].

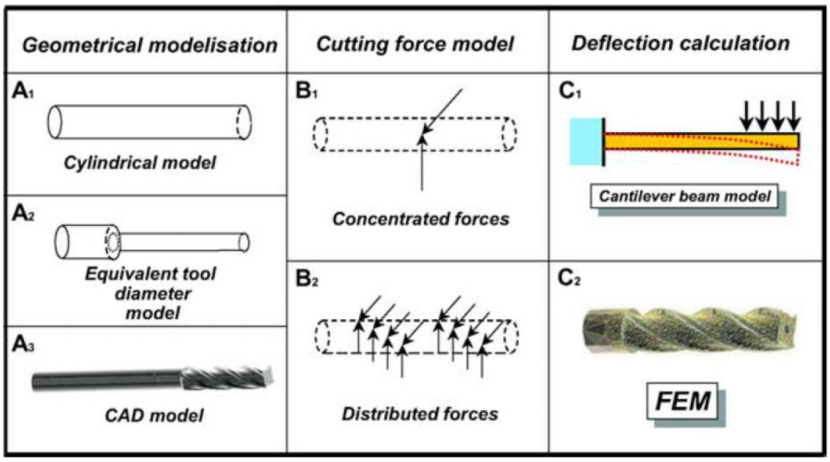


Fig. 1-4 Tool deflection models [DÉPINCÉ *et al.* 2006a]

Although new substrate materials, coatings and tool geometry have been developed [BYRNE *et al.* 2003] to increase cutting speed, tool life etc., tool wear is an unavoidable factor during machining operations. Thus, it affects the accuracy of machined parts. [COOK 1973] provided the background of tool wear problems and tool life. He then presented several typical data to show the general effect of machining variables on wear and life.

Different errors of cutting tools that can occur during machining are provided. Typical cutting tool errors are presented in the previous studies, such as cutting conditions, tool wear, and tool deflection. The results obtained from these studies are used as strategies in order to limit the cutting tool errors during machining. Last but not least, effects of fixture and workpiece errors as well as methods for evaluating the quality of a fixture are reviewed in the next section.

1.1.3. Fixture and workpiece errors

In this section, we first provide the literature on effects of fixture and workpiece errors. We then summarize several studies on evaluation of the quality of a fixture.

1.1.3.1. *Effects of fixture and workpiece errors*

A fixture is a device for positioning and holding a workpiece in the desired location during a machining, measuring, or assembly operation.

A workpiece that is fixed on a fixture must satisfy a unique position, orientation, and static equilibrium. If it deviates from its required location, it is mostly due to the following reasons:

- Clamping force,
- Deformations of workpiece or fixture elements,
- Friction at contact surfaces of workpiece and fixture,
- Effect of cutting force on workpiece during manufacturing operation,

In addition, locator scheme is a vital factor that needs to be considered in fixture layout [Y. RONG 2001]. In particular, a workpiece should not be over constrained.

Many studies on fixturing analysis are available in the literature. [WANG *et al.* 2006] presented the method used to optimize a locator layout based on the criteria of workpiece repeatability and location accuracy. Clamping optimization is used to minimize the clamping force while satisfying the stability requirement was also described. Thus, the stability requirement is an important condition that needs to be considered for a good fixture. It has been intensively investigated in [WANG *et al.* 1999, WU *et al.* 1998]. A method that was proposed by [DEMETER 1994] applies restraint analysis to a fixture, which relies on frictionless or frictional contact surface.

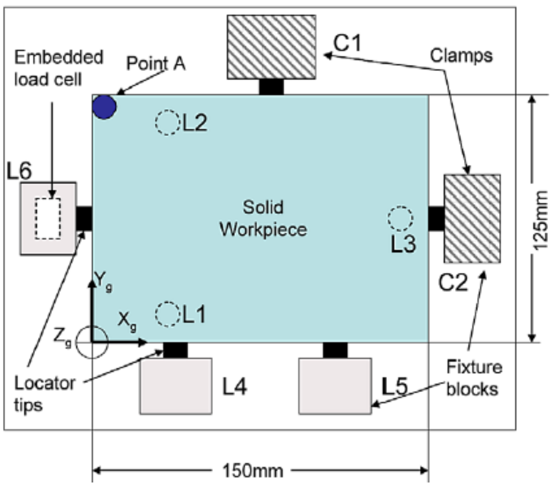


Fig. 1-5 Schematic of 3-2-1 fixture setup for evaluation of the effect of clamping force on workpiece location errors [RAGHU *et al.* 2004]

Localization-related studies, [XIONG *et al.* 2004] presented a probing strategy for the measurement a reliable workpiece localization where a reliability-analysis method [CHU *et al.* 1999] was used to check the localization accuracy with the proposed measurement points.

[LI *et al.* 1999] improved workpiece location accuracy based on the elastic deformation of contact surfaces. [WANG 2000] used the determinant of a locator information matrix, which characterizes the total variance of workpiece positions and orientation, in order to reduce the workpiece positions errors.

Some studies about the influence of clamping force on workpiece location error are available, such as in [RAGHU *et al.* 2004] where an analytical model is presented to predict workpiece location on the 3-2-1 fixture (Fig. 1-5); [CHEN *et al.* 1996] showed the effect of clamping sequences on the stability of fixturing prismatic parts and a model that was used for determining clamping force. [SCHIMMELS *et al.* 1994] identified the satisfied condition for force-assembly with friction.

In the studies on deformation of workpiece-fixture, researches usually use finite element analysis (FEA) to consider the deformation of models under different factors, such as clamping force, positions of locator-clamp pairs or cutting force. Finite element models (FEMs) of the workpiece-fixture systems [MAYER *et al.* 1999, SOONS *et al.* 1992, SRIVASTAVA *et al.* 1994, SUH *et al.* 1998] were used to determine the positions of locator-clamp pairs in order to minimize the elastic deformation at chosen points on the workpiece. The role of forces on workpiece positions has been considered in several papers. Yeh and Liou [YEH *et al.* 2000] analyzed the contact condition between fixtures and components in a modular fixturing system based on an error of clamping force. In order to avoid workpiece deformation and workpiece location errors during machining operation, [ABBASZAHEH-MIR *et al.* 2002, DEMETER *et al.* 2001, DENG *et al.* 2006, JENG *et al.* 1995, MANNAN *et al.* 1997, RAGHU *et al.* 2004, WEIFANG CHEN] used different workpiece-fixture systems to optimize the clamping force as well as positions of clamps.

Surface error at the contact region is also one factor that affects the workpiece position. [SRIVASTAVA *et al.* 1994] presented a model to predict workpiece location and orientation due to locating planes that contain surface errors. This model is just valid for 3-2-1 fixturing method. Additionally, a mathematical model was created by [SANGNUI *et al.* 2001] in order to estimate the impact of surface errors on the positions of a cylindrical workpiece.

In addition, friction coefficient at contact surfaces is a factor that has been considered in various papers to estimate its sensitivity on the accuracy of fixture. [XIE *et al.* 2000] presented experimental fixture-workpiece pairs to evaluate sensitivity of friction coefficient to clamping force, fixture geometry and workpiece surface topography. They showed that the friction coefficient was reduced significantly with presence of slight amounts of cutting oil at the contact surfaces. [DEIABA *et al.* 2004] investigated tribological conditions of the workpiece/fixture elements based on the effect of workpiece material, workpiece surface roughness, fixture element roughness, and normal load.

1.1.3.2. Evaluating the quality of a fixture

In fixture evaluation, many researchers have considered locating performance. [LIN *et al.* 2003] presented a model of the fixture-workpiece in 3D using the Jacobian matrix. This model was then used to analyze deterministic positioning using kinetic analysis. [SONG *et al.* 2005] established an anlytical criterion for the evaluation of deterministic locating using a locating matrix that is based on translations and rotations of 6 locating points. [BO LI 1999] presented a model that allows to reduce workpiece locating errors due to rigid body displacements. The optimization workpiece location was achieved using placement of locators and clamps around the workpiece based on elastic deformation of the workpiece at the fixturing points.

Another method used to evaluate the quality of a fixture is to use indicators. Nevertheless, very few studies have investigated this method. [BRISSAUD *et al.* 1998] presented an indicator on the locating quality. This indicator is called an acceptable locating quality. To evaluate the acceptable locating quality, the possible displacement of the part is calculated to ensure the geometry of the machining feature is within its tolerance interval. They then presented other indices for evaluating performance of a fixture, namely fixturing performance indices [PARIS *et al.* 2005]. In this paper, performance of a fixturing feature is characterized by indices ensuring the required quality. They classified the indices into three types: respect of the positioning quality of the machined surface on the whole workpiece reference, respect of workpiece stability throughout machining, and respect of cutter accessibility while machining. However, they provided the two first indices in this research:

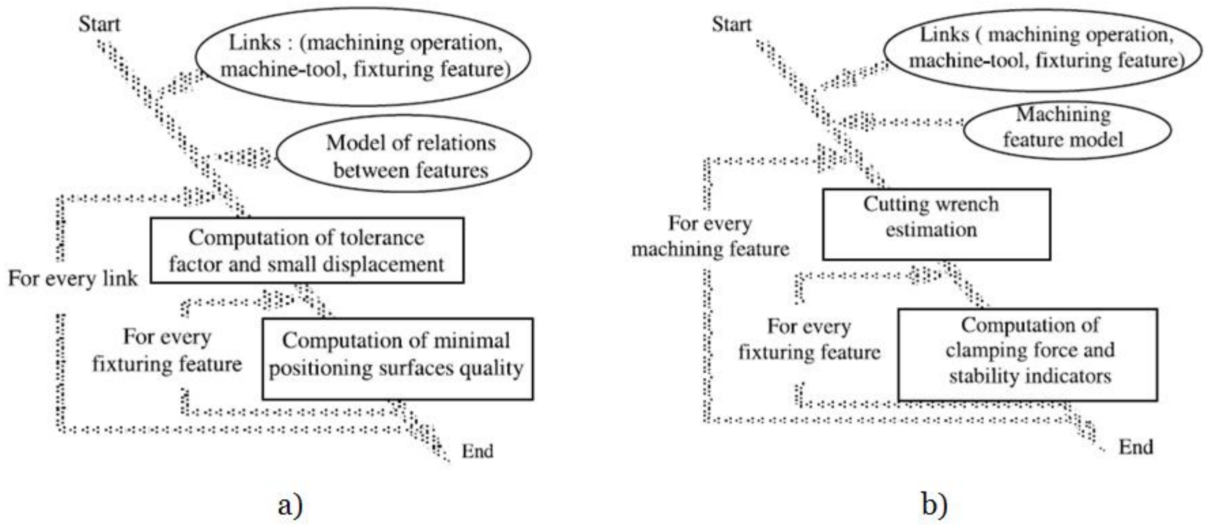


Fig. 1-6 Algorithm for determination a) index of positioning quality, and b) index of the workpiece stability [PARIS *et al.* 2005]

- *Index of the positioning quality* is used to specify the minimal quality of the locating surface to guarantee that the features machined while on the fixture under consideration meet the positioning required tolerance. The determination is implemented in three step in Fig. 1-6a.
- *Index of the workpiece stability* is used to ensure that the mechanical behaviour of the workpiece-fixture-cutter system is correct throughout the machining operation. The implement algorithm is presented in Fig. 1-6b.

Although several indicators have been presented, they are complicated for performing during a process plan. We will propose in this thesis simple indicators that allow evaluating the global quality of a fixture. This evaluation method can be used in fixture design for the preliminary estimate of fixture configurations.

The above research works provide comprehensive literature about the factors affecting the quality of products. Most studies considered one or several factors' effects on a machined part during machining or fixturing. As mentioned earlier, the methods used for identifying the positioning and machining deviation are considered in this thesis. Nevertheless, few studies have genuinely examined these methods. In the next section, several studies are elaborated.

1.2. Identifying manufacturing defects

1.2.1. Problems in quantification of manufacturing defects

Manufacturing defects are here considered including two principle categories: positioning deviation and machining deviation. To investigate these deviations, several studies on quantification of manufacturing defects have been conducted by [BOURDET *et al.* 1995b, DURET 1988, LEHTIHET *et al.* 1990]. Nevertheless, there still exist some difficulties. To clarify this, an example that is summarized by [TICHADOU 2005] is used to illustrate the difficulties as follows.

A cylindrical workpiece is located and machined in only one setup. A surface zero is used to locate the workpiece and two surfaces, 1 and 2, are then machined independently (Fig. 1-7).

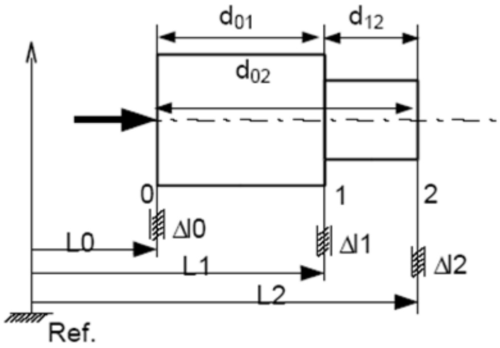


Fig. 1-7 A proposition of quantification of "Δl" [TICHADOU 2005]

The machined parts are then disassembled for measuring three dimensions: d_{01}, d_{02} and d_{12}. Variances of these dimensions can be obtained as $s_{01}^2, s_{02}^2, s_{12}^2$,respectively. The relationships of the dimensions are expressed as the following equation (1-1):

$$\begin{cases} d_{01} = L_1 - L_0 \\ d_{12} = L_2 - L_1 \\ d_{02} = L_2 - L_0 \end{cases} \tag{1-1}$$

It can be assumed that the lengths L_i are random variables with variances s_i^2. The objective is to find s_i^2 for quantifying the dispersions ΔL_i of each surface.

In consideration that the variations of each surface are independent, the relations of the variances are then expressed as follows (1-2):

$$\begin{pmatrix} s_{01}^2 \\ s_{12}^2 \\ s_{02}^2 \end{pmatrix} = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{pmatrix} \begin{pmatrix} s_0^2 \\ s_1^2 \\ s_2^2 \end{pmatrix} \tag{1-2}$$

It is rewritten (1-3) using inverse matrix:

$$\begin{pmatrix} s_0^2 \\ s_1^2 \\ s_2^2 \end{pmatrix} = 0.5 \begin{pmatrix} 1 & -1 & 1 \\ 1 & 1 & -1 \\ -1 & 1 & 1 \end{pmatrix} \begin{pmatrix} s_{01}^2 \\ s_{12}^2 \\ s_{02}^2 \end{pmatrix} \tag{1-3}$$

Alternatively, variation of each surface is obtained using (1-4):

$$\begin{cases} s_0^2 = 0.5(s_{01}^2 - s_{12}^2 + s_{02}^2) \\ s_1^2 = 0.5(s_{01}^2 + s_{12}^2 - s_{02}^2) \\ s_2^2 = 0.5(-s_{01}^2 + s_{12}^2 + s_{02}^2) \end{cases} \tag{1-4}$$

After the solution of (1-4), there are some negative signs which cause some problems for calculating variances. This is impossible because variance is definitely a positive value. Hence, we can affirm that $d_{01}, d_{12},$ and d_{02} are dependent variables. They have a relationship as follows (1-5).

$$d_{02} = d_{01} + d_{12} \tag{1-5}$$

If the relationship (1-5) is not satisfied by the measured values, it can be measurement errors that perturb the resolution of the calculation of variances.

Thus, there are just two independent equations in the system (1-1). It is rewritten as (1-6):

$$\begin{cases} d_{01} = L_1 - L_0 \\ d_{12} = L_2 - L_1 \end{cases} \quad (1\text{-}6)$$

The calculations of the variances are then expressed as the following relationships (1-7) or (1-8):

$$\begin{cases} s_{01}^2 = s_1^2 + s_0^2 \\ s_{12}^2 = s_1^2 + s_2^2 \end{cases} \quad (1\text{-}7)$$

Or

$$\begin{pmatrix} s_{01}^2 \\ s_{12}^2 \end{pmatrix} = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \end{pmatrix} \begin{pmatrix} s_0^2 \\ s_1^2 \\ s_2^2 \end{pmatrix} \quad (1\text{-}8)$$

It can be seen that there are three unknowns in two equations (1-8), so there is no solution to obtain s_0^2, s_1^2 and s_2^2.

1.2.2. A proposed method for quantification of manufacturing defects

In order to resolve the above problem, a supplementary measurement was proposed by [TICHADOU 2005]. It means that one more measurement will be done based on another reference. This measurement is carried out just after the final cutting step of a machined part. In other words, the measurements are carried out inside the machine tool without disassembling the machined parts out of a fixture.

This proposed method is illustrated in (Fig. 1-8) in which the machined parts are measured inside the machine tool using a measuring device just after the final cutting steps. Each machined surface is measured based on a reference of the machine tool. It can be seen that it is not available to measure the locating surfaces.

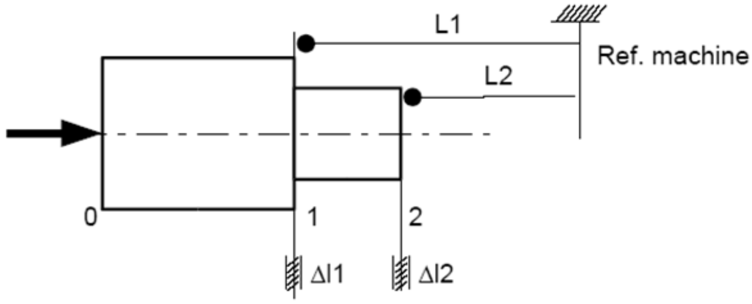


Fig. 1-8 Measurements of machined surfaces inside the machine tool [TICHADOU 2005]

In consideration of a batch of machined parts, variations of machined surfaces 1 and 2 are evaluated using the variances s_1^2 and s_2^2 obtained from the measurements of L_1 and L_2.

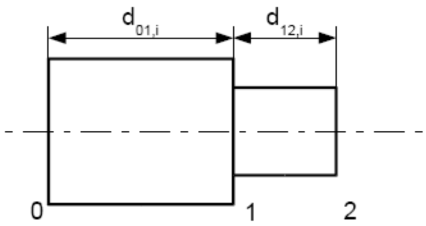


Fig. 1-9 Measurements of a machined part outside of the machine tool [TICHADOU 2005]

The machined parts are then measured individually outside of the machine tool as figure (Fig. 1-9). The variances s_{01}^2,s_{02}^2 of the dimensions d_{01}, d_{12} of the batch of the machined parts are then obtained in these measurements.

From the results of the measurements, the equations (1-8) are resolved easily.

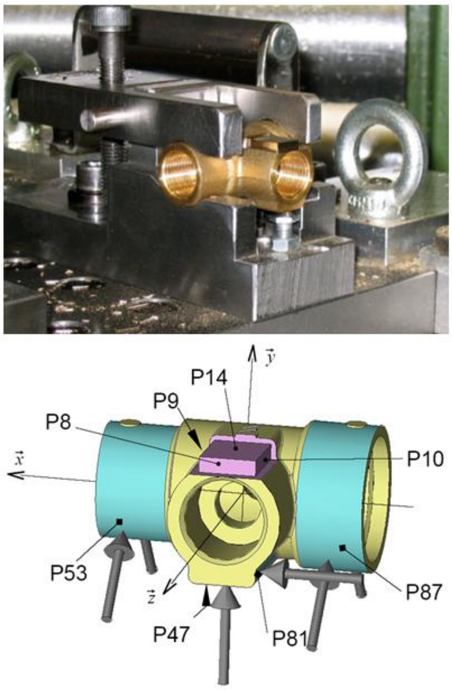
This proposition is then applied in two experiments that are conducted by [TICHADOU 2005] and [KAMALI NEJAD 2006]. These two studies are summarized below and their limitations are analyzed.

1.2.3. Tichadou (2005)

Tichadou conducted an experimental application of a batch of 70 milling parts. The double measurement method is applied in this study. The first measurement is carried out for measuring the parts fixed on a fixture. The second one is used to measure the parts in the end of production. The objective of this study is quantification of manufacturing defects in order to:

- compare machining dispersions/defects of identical machining operations with two different fixtures,
- compare positioning dispersions/defects of the two different fixtures.

To do this, two different fixtures are used for fixing the parts. The first fixture (Fig. 1-10a) fixes the parts using non-machined surfaces (raw surfaces) and the second one (Fig. 1-10b) fixes the parts using machined surfaces.



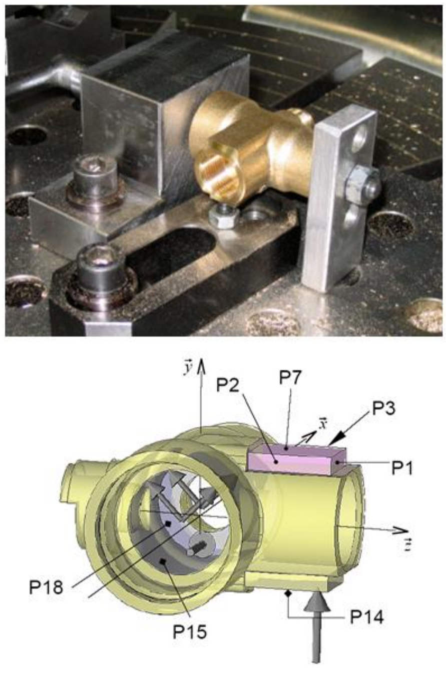


Fig. 1-10 Two different fixtures [TICHADOU 2005]

The same machining operation is then used to machine three surfaces of each part fixed on the two different fixtures. The measurement of each machined surface on the machine tool is analyzed and the results of machining deviations are expressed by a small displacement torsor (SDT). These results are used to compare machining dispersions/defects of the machined parts on the two different fixtures.

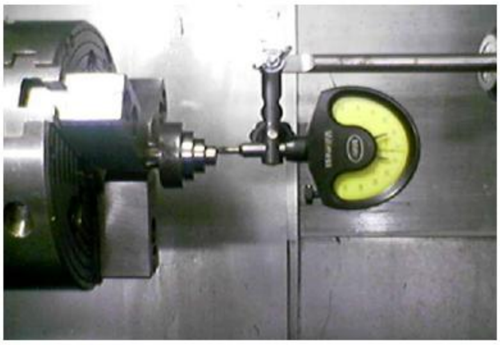
To compare positioning dispersions/defects, Tichadou used dispersions of a torsor deviation to describe the relations between machining dispersion/defect and positioning dispersion/defect in which they are expressed by variances of the torsor components.

This study shows that the differences between machining defects of two machined parts that were fixed by the two different ways of fixturing, are insignificant. It means that the machining defects are not dependent on the positioning defects.

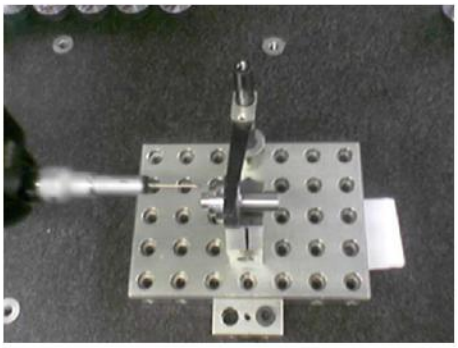
1.2.4. Tichadou and Kamali Nejad (2007)

Another experimental application of a batch of 55 turning parts was conducted by Kamali Nejad in [TICHADOU *et al.* 2007]. The objective of this study is also to quantify machining and positioning defects based on the double measurement method.

Workpieces are fixed and machined on a CNC turning machine. Each machined part is then measured inside the machine tool (Fig. 1-11a) after the last machining operation (without disassembling the machined parts). The second measurement is carried out on a CMM (Fig. 1-11b) for each machined part.



a)



b)

Fig. 1-11 a) Measurements on the CNC machine b) Measurements on the CMM

The results obtained from the two machines were used to compare and evaluate machining defects and positioning defects. They concluded that the machining defects of machined cylinders were insignificant in front of positioning defects.

1.2.5. Discussions

There are several limitations in the above studies:

- The machining defects are obtained using some measured points on machined surfaces, which cannot sufficiently represent defects of the analysed surfaces.
- The above studies have not proven that the results obtained from two different machines are comparable. For instance, a machined plane is measured by two different machines and measured results can be different because of the machine resolutions. Hence, results obtained from two different measurement means need proof that they are comparable.

To solve outstanding problems, we propose several solutions to develop the method that can be used to identify, analyse and estimate manufacturing defects [BUI *et al.* 2010, SERGENT *et al.* 2010].

- Using the double measure method to measure machined parts;
- Using the same method to associate a surface from a cloud of measured points, for instance, the least-square best-fit method is used to rebuild the geometric elements from measured data that are obtained from the two different machines. The advantage of this solution is to suppress deviations of the data processing;
- Two geometric elements of the machined parts are chosen and measured by the two measurement means. The measured points are then analysed using the least-square best-fit mentioned earlier. The measurement results are finally compared in order to

evaluate whether differences between the two measurement means are significant/insignificant;

- Determining defects of the machined parts as distributions and statistical parameters that can be used in simulations.

2. TOLERANCE ANALYSIS

Tolerance analysis is the general term for activities related to the study of accumulated variation in mechanical parts and assemblies. This includes two subcategories:

- The method used to determine the meaning of individual tolerancing specifications;
- The process used to determine the cumulative variation possible between two or more features. This is also known as a Tolerance Stack-up/Stack.

The cumulative variation of a tolerance stack-up has to satisfy dimensions and tolerances specified on an engineering drawing. Thus, before performing a tolerance stack-up, the dimensioning and tolerancing specifications applied to the drawing must be clearly understood. It means that the specifications must be translated into a form that can be used in a tolerance stack-up.

Tolerance analysis is commonly called tolerance stack-up when it is used to solve a given problem. [FISCHER 2004] explained that because dimensions and their tolerances are added together, they "stack up" to add to the total possible variation. Dimensions and tolerances are stacked up to form a chain of dimensions and tolerances, which can be followed head to tail from one end of the distance under consideration to the other.

There are two major types of tolerance analysis:

- *Worst-case tolerance stack-ups* are used to determine the absolute maximum variation possible for a selected distance or gap.
- *Statistical tolerance stack-ups* are used to determine the probable or likely maximum variation possible for a selected dimension.

In the earliest papers found in literature, [EVANS 1974, 1975] addressed the difference between two distinct tolerancing policies, statistical and worst-case, by the explanation of how the tolerancing activities are usually carried out. In these research works, the basic problem of statistical tolerancing is defined in terms of the following famous equation:

$$Y = f(X_1, X_2, \dots, X_n) \tag{1-9}$$

where

- Y is the response (final stack).
- X_i are component values (individual deviations).

The moments of Y is estimated as the fundamental approach to the problem. Different methods for estimating these moments are then reviewed such as: linear propagation of errors, approximation of nonlinear propagation by the extended Taylor series and by numerical integration, and Monte Carlo approach. It is interesting to note that the approaches presented by Evans have been applied in tolerancing research during 20 years with few changes. [NIGAM *et al.* 1995] presented the review of statistical approaches to tolerance analysis with some minor improvements.

It is interesting to note that in a manufacturing process as well as an assembly the geometrical deviation stacks up. Thus, tolerance analysis can be divided into two categories:

- *Tolerance analysis in assembly* aims to verify properly the function of an assembly in which each assembled part is comprised of tolerance (variable) features. Thus, the assembly is comprised of variable parts, and additional variation may occur as part of the assembly process. As more tolerance dimensions stack up, more and more variation is possible.
- *Tolerance analysis in manufacturing* is defined as the evaluation of a process plan in terms of functional tolerances. This verifies whether a given process plan satisfies functional tolerances of machined parts.

Different tolerance analysis models and three-dimensional propagations are quickly reviewed. Several studies on the tolerance analysis in multistage machining process are then elaborated.

2.1. Different tolerance analysis models

There are different models of performing a tolerance analysis: models for analyzing linear (one-dimensional) variation, several models for analyzing two- or three-dimensional variation by combined linear analyses. Numerous research works have been conducted on tolerance analysis models.

2.1.1. Tolerance analysis models

2.1.1.1. Models based on tolerance chain

The tolerance chain technique is a widely-used technique in tolerance analysis. A dimensional tolerance chain is used to represent the chain in which a conventional tolerance (plus/minus) is assigned on each element. Methods based on the dimensional tolerance chain are further classified into three approaches:

Linear tolerance accumulation models

[FORTINI 1967] presented the two most common models for the accumulation. The first one, worst case models, is used to predict assembly tolerance T by the accumulation of component tolerances T_i as equation (1-10).

$$T = \sum_{i=1}^{n} T_i \tag{1-10}$$

The second one, statistical models (root sum square), is used for tolerance estimation. This is expressed as follows:

$$T = \sqrt{\sum_{i=1}^{n} T_i^2} \tag{1-11}$$

Thanks to this approach, [GREENWOOD *et al.* 1988, 1990] applied in worst case tolerance and root sum square tolerance analysis. A similar analysis method is used for more complex mechanical assemblies and kinematic linkages based on the direct linearization method [CHASE *et al.* 1995, GAO *et al.* 1998, GLANCY *et al.* 1999, WITTWER *et al.* 2004].

Statistical tolerance analysis

This method assumes a probability distribution of system response as equation (1-9) for variation of tolerance and then uses this function to predict the assembly variability in the system. A standard procedure for tolerance analysis is to determine the first four moments of this function and use these to choose a distribution that describes the system variability [COX 1979]. Numerous reviews are available for these methods [CHASE *et al.* 1991, NIGAM *et al.* 1995].

The main techniques have been used for statistical tolerance analysis: Root Sum Squares method (RSS), estimated mean shift model, crofts method, method of moments, integration or quadrature technique, Taguchi's method, reliability index method, and Motorola six sigma model.

Monte Carlo simulation

Monte Carlo simulation can be used for both nonlinear assembly functions and non-normal distributions. In this simulation, a random number generator used to simulate the effects of manufacturing variations on assemblies. The disadvantage of Monte Carlo method is that to get accurate estimates it is necessary to generate very large numbers of samples, and if the distributions of variables change, the whole analysis must be done. Several studies have been conducted based on Monte Carlo simulation:[CVETKO *et al.* 1998, SKOWRONSKI *et al.* 1997, TURNER *et al.* 1987].

2.1.1.2. Variational dimension models

Variational dimension models are a kind of special variational geometry in which only the dimension (size) can vary [SHAH *et al.* 1995]. Several research works focus on tolerance sensitivity analysis in this area [DONG 1997, DONG *et al.* 1995, 1997, SHAH *et al.* 1995].

2.1.1.3. Variational solid models

Variation solid models are developed to overcome the problem of variational dimensional models with non-polygonal/polyhedral models and certain types of geometrical tolerances [BOYER *et al.* 1991]. Some recent works on the assembly of toleranced parts are available in literature [AKELLA *et al.* 2000, LI *et al.* 2001, WHITNEY *et al.* 1999a, 1999b]

The tolerance analysis models can be summarized in Fig. 1-12.

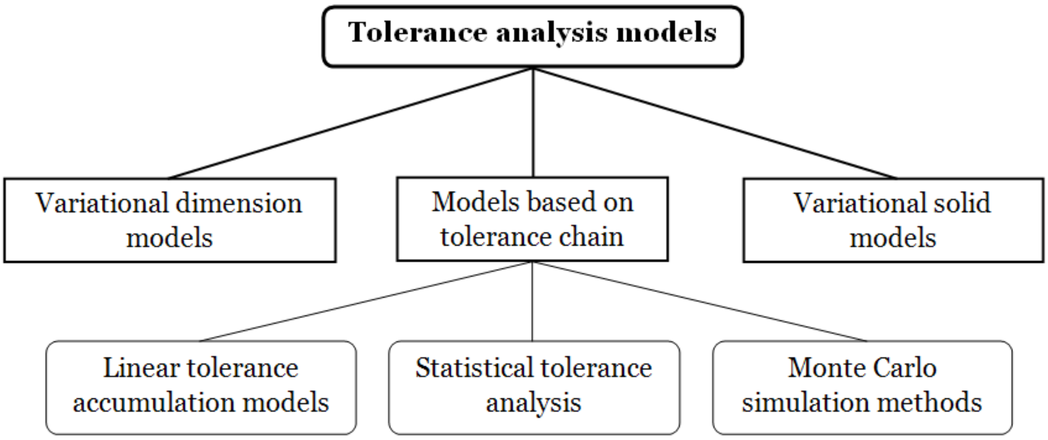


Fig. 1-12 Tolerance analysis models

2.1.2. Propagation of three-dimensional tolerances

In this section, different approaches that have been developed for analyzing how geometric tolerances are propagated in three-dimensional space, namely three-dimensional (3D) tolerance propagation, are reviewed. Most research works on 3D tolerance propagation are conducted based on kinematic formulation, small displacement torsor (SDT), matrix representation, and vectorial tolerancing. An analysis of 3D tolerance propagation passes through two related steps: representation of tolerance zone and spatial tolerance propagation mechanism.

2.1.2.1. 3D tolerance propagation based on kinematic formulation

As mentioned above, one of the steps in 3D tolerance propagation is representation of tolerance zones. To do this, [RIVEST *et al.* 1994] presented a model based on kinematic formulation. The description of the tolerance zone is summarized within the kinematic structure in Fig. 1-13. In this figure, each block and cylinder represents one degree of freedom, or a parameter, in translation (T) and rotation (R), respectively. The tolerance zone is determined by the region of space accessible to the point O_f of the structure when many parameters are set to fixed values, while the others are free to sweep within their given intervals.

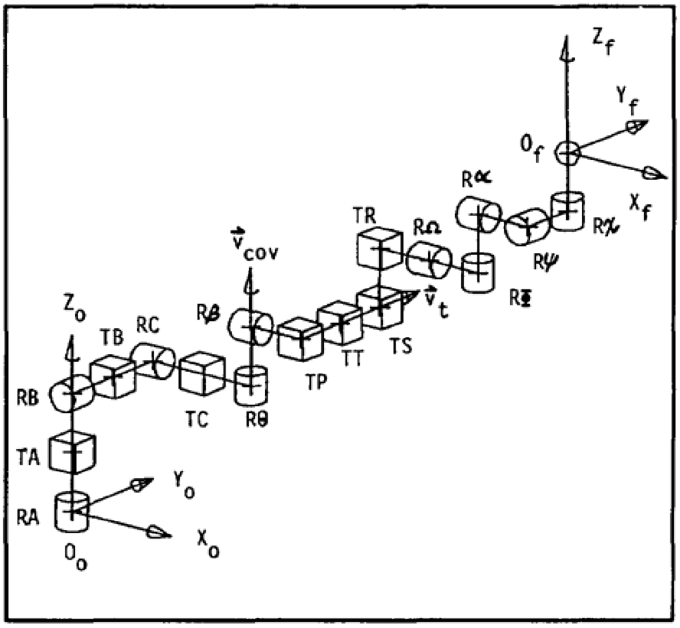
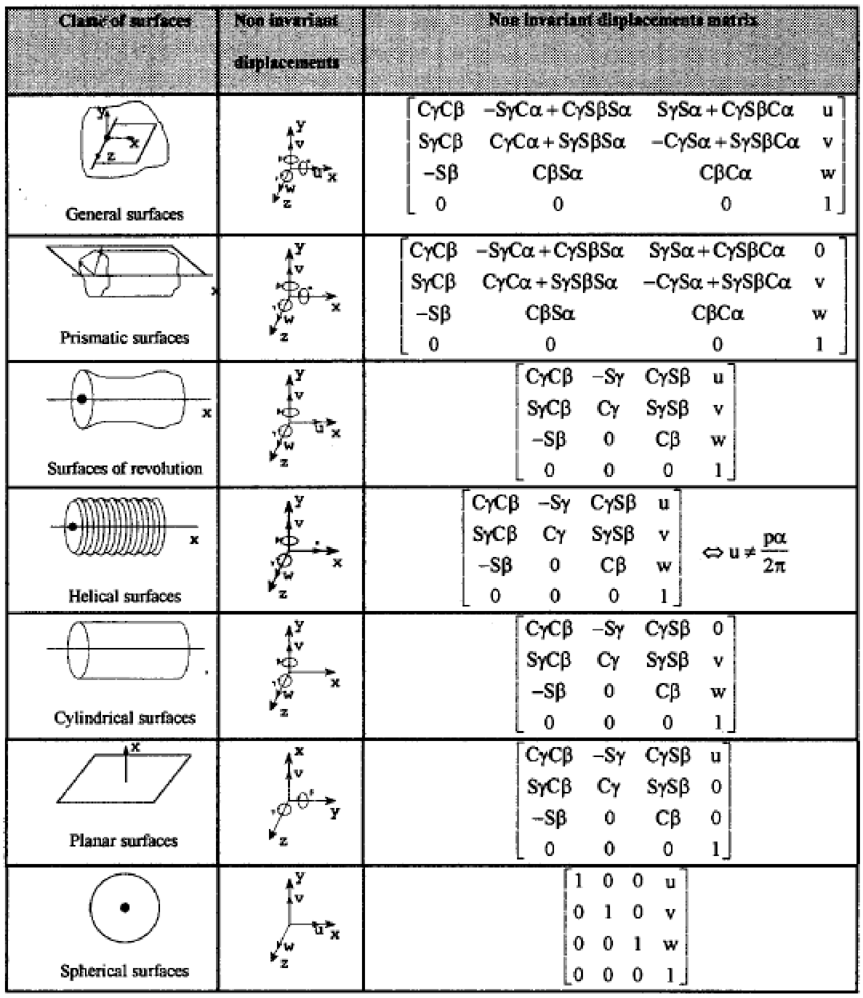


Fig. 1-13 Kinematic structure [RIVEST *et al.* 1994]

In a similar vein, a kinematic chain model associating a set of six virtual joints to every pair of functional elements in a tolerance chain is proposed by [LAPERRIÈRE *et al.* 1999]. The six virtual joints include three small translations and three small rotations. Jacobian transforms are also used for modeling the propagation of small dispersions along the tolerance chain in [LAFOND *et al.* 1999, LAPERRIÈRE *et al.* 2000, LAPERRIÈRE *et al.* 2001, LAPERRIÈRE *et al.* 1998].

2.1.2.2. 3D tolerance propagation based on matrix representation

In this section, the theory of matrices is used to model the tolerance zones, it is also known as matrix representation of tolerances. A methodology used to represent standard tolerances (ANSI Y14.5-1982) is based on homogeneous matrix transformations. Matrix transformations represent both nominal relations between parts and the variations caused by geometric deviation allowed by the tolerances. [WHITNEY *et al.* 1993] presented the statistical estimate of the location of the n^{th} part in an assembly starting from the first part or a fixture. Later, [DESROCHERS *et al.* 1997] presented a methodology allowing the representation of tolerance zones in which the degree of freedom associated with the tolerance zones are translated in terms of matrices. They presented displacement of seven types of elementary surface by matrix transformations (Fig. 1-14).



Class of surfaces	Non invariant displacements	Non invariant displacements matrix
General surfaces		$\begin{bmatrix} C\gamma C\beta & -S\gamma C\alpha + C\gamma S\beta S\alpha & S\gamma S\alpha + C\gamma S\beta C\alpha & u \\ S\gamma C\beta & C\gamma C\alpha + S\gamma S\beta S\alpha & -C\gamma S\alpha + S\gamma S\beta C\alpha & v \\ -S\beta & C\beta S\alpha & C\beta C\alpha & w \\ 0 & 0 & 0 & 1 \end{bmatrix}$
Prismatic surfaces		$\begin{bmatrix} C\gamma C\beta & -S\gamma C\alpha + C\gamma S\beta S\alpha & S\gamma S\alpha + C\gamma S\beta C\alpha & 0 \\ S\gamma C\beta & C\gamma C\alpha + S\gamma S\beta S\alpha & -C\gamma S\alpha + S\gamma S\beta C\alpha & v \\ -S\beta & C\beta S\alpha & C\beta C\alpha & w \\ 0 & 0 & 0 & 1 \end{bmatrix}$
Surfaces of revolution		$\begin{bmatrix} C\gamma C\beta & -S\gamma & C\gamma S\beta & u \\ S\gamma C\beta & C\gamma & S\gamma S\beta & v \\ -S\beta & 0 & C\beta & w \\ 0 & 0 & 0 & 1 \end{bmatrix}$
Helical surfaces		$\begin{bmatrix} C\gamma C\beta & -S\gamma & C\gamma S\beta & u \\ S\gamma C\beta & C\gamma & S\gamma S\beta & v \\ -S\beta & 0 & C\beta & w \\ 0 & 0 & 0 & 1 \end{bmatrix} \Leftrightarrow u \neq \frac{p\alpha}{2\pi}$
Cylindrical surfaces		$\begin{bmatrix} C\gamma C\beta & -S\gamma & C\gamma S\beta & 0 \\ S\gamma C\beta & C\gamma & S\gamma S\beta & v \\ -S\beta & 0 & C\beta & w \\ 0 & 0 & 0 & 1 \end{bmatrix}$
Planar surfaces		$\begin{bmatrix} C\gamma C\beta & -S\gamma & C\gamma S\beta & u \\ S\gamma C\beta & C\gamma & S\gamma S\beta & 0 \\ -S\beta & 0 & C\beta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$
Spherical surfaces		$\begin{bmatrix} 1 & 0 & 0 & u \\ 0 & 1 & 0 & v \\ 0 & 0 & 1 & w \\ 0 & 0 & 0 & 1 \end{bmatrix}$

Fig. 1-14 Description of the seven classes of elementary surfaces [DESROCHERS *et al.* 1997]

2.1.2.3. *3D tolerance propagation based on vectorial tolerancing*

Vectorial tolerancing was introducted by [WIRTZ 1991, WIRTZ *et al.* 1993]. Then [MARTINSEN 1993] showed all types of surfaces can be expressed by vectorial tolerancing. Vectorial tolerancing is a three dimensional mathematically unambiguous model for describing geometry and tolerances using vectors. It provides a clear distinction between the geometrical features' size, form, location and orientation for each surface of a workpiece. In vectorial tolerancing, the location of a surface is described with a position vector:

$$\vec{P}_0 = [X_0, Y_0, Z_0]^T \tag{1-12}$$

and a direction vector:

$$\vec{E} = [E_x, E_y, E_z]^T \tag{1-13}$$

Fig. 1-15 shows that the location and orientation of the substitute element are expressed by a pair constituted by a position vector and a direction vector in a workpiece coordinate system [YAU 1997].

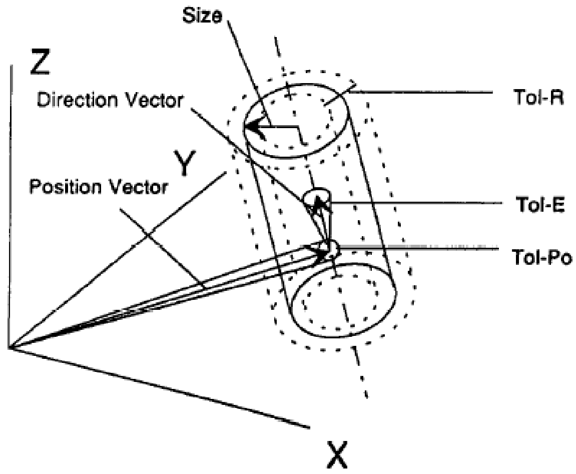


Fig. 1-15 Illustration of vectorial tolerancing [YAU 1997]

2.1.2.4. 3D tolerance propagation based on small displacement torsor

3D tolerance propagation models based on the small displacement concept [BOURDET *et al.* 1996] have been conducted by many researchers. This concept is based on an assumption of small displacements of a rigid body. It allows solving a general problem of the fit of a geometrical surface model to a set of points. Based on the assumption that the displacements are small, the linearization is used to express the final form of a torsor at point O of X, Y, Z coordinate.

$$\mathcal{T} = \begin{Bmatrix} r_X & t_X \\ r_Y & t_Y \\ r_Z & t_Z \end{Bmatrix}_{OXYZ} \tag{1-14}$$

Based on the approaches traced in the SDT concept, [GIORDANO *et al.* 1992] presented a new concept called clearance and deviation spaces. This concept is then used for tolerance analysis and synthesis [GIORDANO *et al.* 2003]. This paper presents notions of clearance torsor and deviation torsor in which the clearance torsor represents a small relative displacement between two parts and the deviation torsor represents a small deviation between a datum frame attached to the part and the frame attached to the feature. [GIORDANO *et al.* 1993] extended this theoretical work for another concept called deviation volume (see Fig. 1-16), and investigated the topological operations on those volumes.

[BOURDET *et al.* 1995b] showed limitations of traditional tolerance chain models and proposed a model that uses a set of different kinds of torsor: deviation torsor, variation torsor, gap torsor, and small displacement torsor per part. They defined two operators, intersection and union, which are used in 3D tolerance propagation. This tolerance propagation is refined in [BALLOT *et al.* 1998], where the composition and aggregation methods for deviations are formalized.

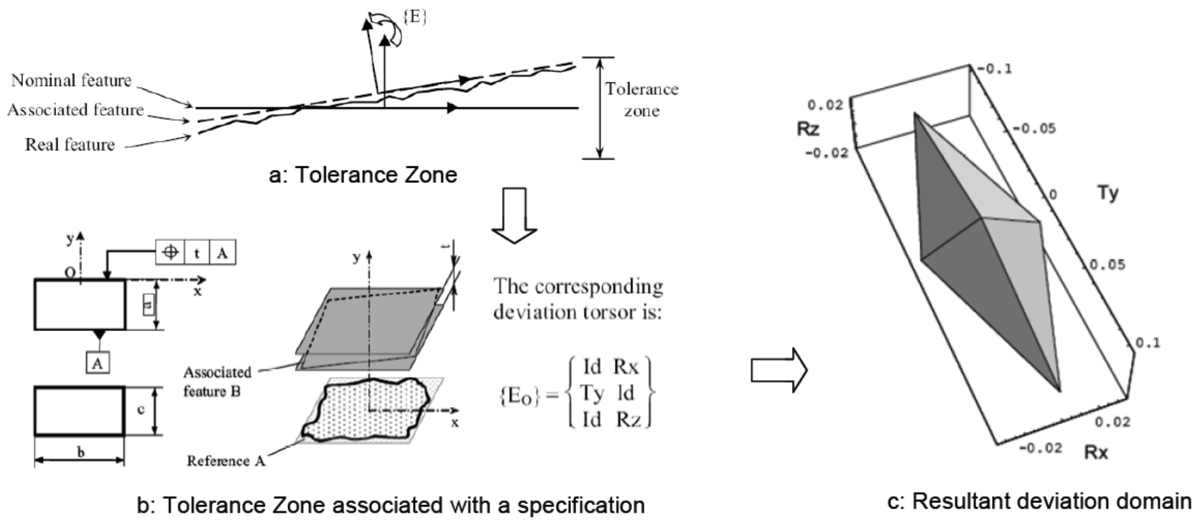


Fig. 1-16 Deviation domain example [SAMPER *et al.* 2006]

Another tolerancing model based on the SDT concept, namely proportioned assembly clearance volume (PACV), is developed by [TEISSANDIER *et al.* 1998, 1999a] in order to create a 3D tolerancing analysis tool takes into account only standardized specifications. By modelling fabricated surfaces, they compute the limits of small displacements of a fabricated surface inside a tolerance zone. The values of these limits define a PACV. Using a graph, they showed how PACV can be associated in series or in parallel between any surfaces in an assembly to create 3D dimension-chains. Two operations on polytopes, the Minkowshi and the intersection, can be used for calculating PACVs [TEISSANDIER *et al.* 1999b].

The above comprehensive reviews show that there are many different approaches, different models can be used for analysing propagation errors in three-dimensional space. Most of the above research works have been performed on tolerance analysis in assembly. In the next section an overview of tolerance analysis in multistage machining process is provided.

2.2. Tolerance analysis in multistage machining process

Generally, a machined part passes through several machining operations, also known as a multistage machining process. Errors of each stage will be accumulated on the workpiece that will affect the machining accuracy at a subsequent stage. The machining errors of previous stages are known as error propagation or error stack-up.

Numerous research works on error propagation in a multistage machining process have been conducted; meanwhile little research has been done in evaluation of a process plan.

2.2.1. Error propagation/stack-up analysis

Various error-propagation analyses in a multistage machining process have been conducted based on state space model [DING *et al.* 2002, DJURDJANOVIC *et al.* 2001, HUANG *et al.* 2003a, HUANG *et al.* 2003b, ZHOU *et al.* 2003].

The state space model is a different form of the standard kinematic analysis model. It provides analytical tools for system evaluation and synthesis. The basic task solved in the state space model environment is an estimation of unobserved states based on observed values.

[HUANG *et al.* 2003b] used a state space model to describe part error propagation in multistage machining processes. In this model, not only the machining operation, but also the setup operation affects part quality. The induced part errors propagate in the subsequent stage. Thus, the part deviation for each operation includes the datum errors, fixture errors, and machine tool errors. The error propagation model is here expressed by a state vector x(k) (Fig. 1-17).

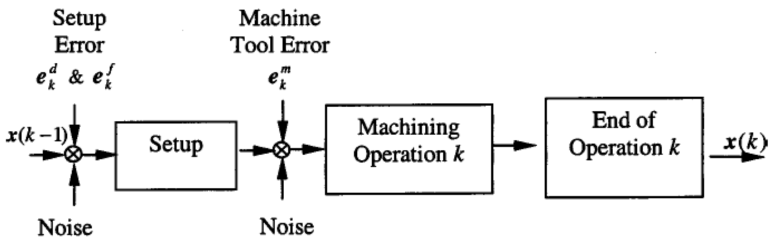


Fig. 1-17 Error propagation [HUANG *et al.* 2003b]

where e_k^f, e_k^d, e_k^m are fixture error, datum error, and machine tool error in operation k, respectively.

[ZHOU *et al.* 2003] used Differential Motion Vector (DMV) as the state vector to represent the geometric deviation of the workpiece. DMV is an approach whereby the orientation vector is based on the three Euler rotating angles instead of using a unit direction vector. To understand DMV representation, let us consider an ideal workpiece surface plane defined by its local coordinate system (LCS_1) in Fig. 1-18. The location vector $\vec{t}^R{}_1 = \begin{bmatrix} \frac{L}{6} & \frac{L}{2} & L \end{bmatrix}^T$ and the orientation vector $\vec{w}^R{}_1 = \begin{bmatrix} \frac{\pi}{2} & 0 & 0 \end{bmatrix}^T$ are used to locate LCS_1. These vectors are defined with respect to RCS, denoted as R. The deviation of surface plane from the ideal one due to certain errors in a machining process, so the actual local coordinate system is deviated by a location

$\vec{d}^R{}_1 = [\Delta x \quad \Delta y \quad \Delta z]^T$ and an orientation $\vec{s}^R{}_1 = [\Delta\alpha \quad \Delta\beta \quad \Delta\gamma]^T$. Hence, the variation feature representation is defined by the stacked DMV $\boldsymbol{x}^R{}_1 = [\Delta x \quad \Delta y \quad \Delta z \quad \Delta\alpha \quad \Delta\beta \quad \Delta\gamma]^T$.

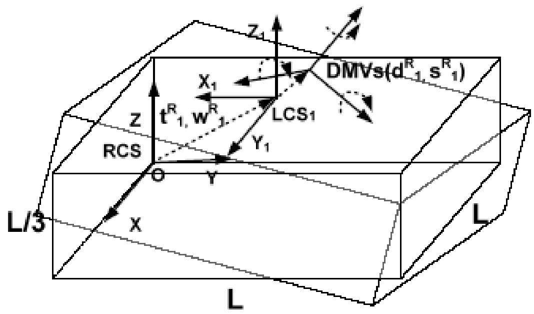


Fig. 1-18 Workpiece feature representation [ABELLAN-NEBOT *et al.* 2009]

The deviation propagation at stage k+1 is expressed by linear discrete state space format (1-15) with the workpiece deviation at stage k comes from three sources: the datum-induced deviation cause in the previous stage, the machining inaccuracy at the current stage, and unmodeled noise.

$$
\begin{aligned}
x(k+1) &= A(k)x(k) + B(k)u(k) + w(k) \\
y(k) &= C(k)x(k) + v(k)
\end{aligned}
\tag{1-15}
$$

where

- $A(k)x(k)$ represents the deviation of previously machined features and the deviation of newly machined features that is only contributed by the datum error.
- $B(k)u(k)$ represents the deviation introduced within stage k due to fixture errors.
- $w(k)$ is the unmodeled system noise.
- $y(k)$ is the measurement.
- $C(k)x(k)$ represents the deviation of key product characteristics that are the linear combination of the deviation of features on workpieces.
- $v(k)$ is the measurement noise.

This formulation (1-15) does not include specific machining operation variations, such as geometric/kinematic errors, cutting tool wear errors or spindle thermal errors.

The above models are limited to an orthogonal 3-2-1 fixture layout. [LOOSE *et al.* 2007b] then developed a linear model to describe the dimensional variation propagation of machining processes through kinematic analysis of the relationships among fixture errors, datum errors, machine geometric errors, and the dimensional quality of the product. This model can handle general fixture layouts. However, locating errors are considered based on a punctual contact that is not practical for a fixture that has a contact plane/plane or cylinder/cylinder.

Due to the above limitations, [VILLENEUVE *et al.* 2001] have applied the SDT concept for error stack-up analysis in machining. This approach was then followed up by Vignat in turning operation [VIGNAT *et al.* 2003] and then more globally for machining process by the Model of Manufacture Part (MMP) [VIGNAT *et al.* 2005, VIGNAT *et al.* 2009, VILLENEUVE *et al.* 2007]. These works are summarized in chapter 4 and this method is continued to be developed in this thesis.

2.2.2. Simulation manufacturing defects for tolerance analysis

Simulation of manufacturing defects is used to model geometrical defects during manufacture. This allows identifying all sources of deviations that affect functional conditions and manufacturing. It is based on the prediction of the influences of defects.

The simulation can be used to analyze a preliminary process plan by indentifying and classifying the defects, which occur during the manufacturing process. This allows validating a provisional process plan, namely tolerance analysis.

Different models have been used in tolerance analysis. These models can be classified from different points of view, such as:

- According to dimension, there are three approaches: one-dimensional (1D), two-dimensional (2D) and three-dimensional (3D) approach.
- According to objectives, two techniques can be used: worst-case tolerance analysis and statistical tolerancing analysis.

The following studies are cited based on the first point of view.

2.2.2.1. 1D and 2D approach

A one-dimensional method (Δl) is developed by [BOURDET 1973]. In this method, manufacturing dimensions of a machined part are defined, following one direction chosen, with optimal tolerances based on the shortest distance between two surfaces of the condition. This method is conducted by the following steps:

- Identification of geometric elements, which are considered in the chosen direction, by numbering from left to right in Fig. 1-19.
- Simulation of the data in a table for each setup: the machined surfaces need to be in a zone Δl_i in width, the locating surfaces need to be in a zone Δl_i^j in width (where i is number of surface and j is number of setup)

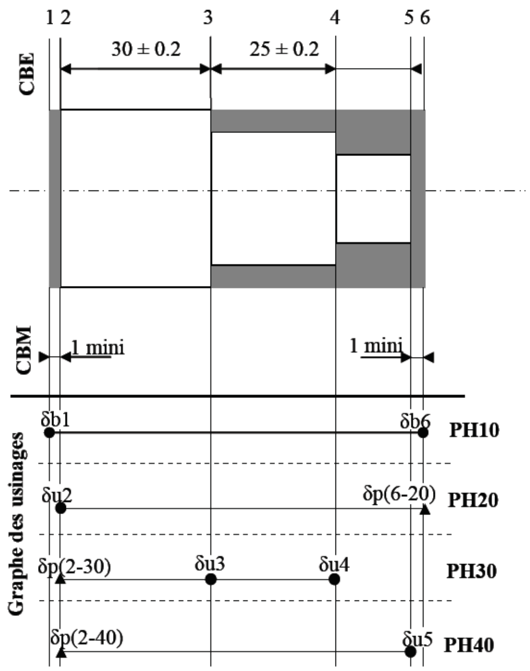


Fig. 1-19 One-dimensional method Δl [BOURDET 1973]

- Analysis of the functional specifications in the manufacturing specifications
- Formulation of inequalities from the above table
- Solution of the inequalities at the end to obtain all dimensions and tolerances by a calculation algorithm

[ANSELMETTI 1983] presented a two-dimensional approach that can be applied in Numerical Control machining. Nevertheless, this approach does not take into account the effects of angular deviation in the workpiece setup.

Due to limitations in 1D and 2D approaches, studies in the 3D approach have been developed as follows:

2.2.2.2. 3D approach

[HUANG *et al.* 2004] proposed a method to verify a process plan by predicting machining tolerance via Monte Carlo simulation. Workpiece geometry is represented by a set of discrete sample points. Each point has an ideal coordinate $p_m(x_m, y_m, z_m)$ in the workpiece coordinate system (WCS) in Fig. 1-20.

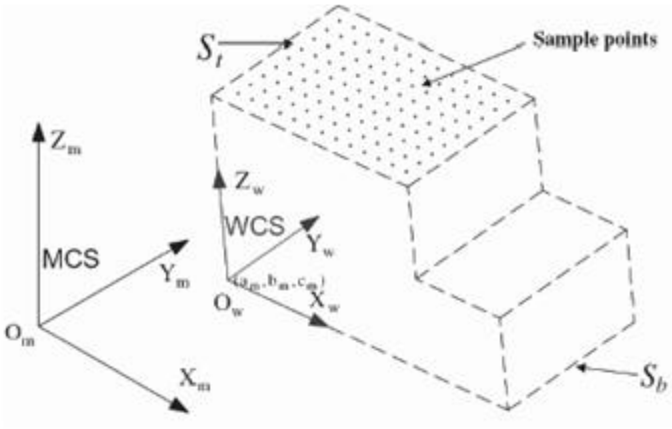


Fig. 1-20 Sample points [HUANG *et al.* 2004]

Error synthesis mechanism is used to evaluate the effect of a process plan on component quality. This error is determined as follows:

- Sample points generation
- Manufacturing error source modelling
- Error synthesis and tracking
- Virtual inspection

The simulation procedure passes through the following steps:

Step 1. *Establish Workpiece Coordinate System (WCS)*

A vertex of a prismatic part or centre of a circle is chosen as the origin O_w of WCS.

Step 2. *Represent the raw workpiece*

Sample points are generated based on the WCS in Step 1 to represent each surface of the raw workpiece according to its quality as specified using surface flatness.

Step 3. *Locate the workpiece*

Translation and rotation error components are generated based on given fixture accuracy. Datum feature error is expressed by the sample points on the datum feature. The errors are then combined as the work-holding error and calculated the transformation matrix that maps WCS to MCS (Machining Coordinate System).

Step 4. *Simulate machining operation*

All workpiece sample points are mapped on the MCS. At each sample point, machine tool errors and cutting tool errors are sampled from their respective models and synthesized to compute the simulated cutting tool position. If $p_m(x_m, y_m, z_m)$ is a nominal point, it will be $p'_m(x'_m, y'_m, z'_m)$ after machining because of manufacturing errors. The resultant coordinates (in MCS) $P'_m(u)$ replace those of the sample point $P'_m(u-v)$.

where, once a machining operation u is completed, a set of new sample points $P'_m(u) = \{p_m(i): i = 1, \dots, N_s\}$ for the machined surface will replace the sample points $P'_m(u-v)$ generated in the vth process previously in MCS to describe the newly developed surface; $i = 1, \dots, N_s$ is the number of ideal sample points.

Step 5. Store and propagate workpiece accuracy information

The new sample point coordinates obtained in Step 4 are mapped back to the WCS $\left(P'_m(u) \rightarrow P'_w(u)\right)$ as equation (1-16). At this point, the accuracy of the workpiece surfaces that are machined has changed. These surfaces might be used as locating datum in the subsequent machining operations.

$$P'_w = P'_m . T_t^{-1}(a_m, b_m, c_m) . T_r^{-1}(\theta_x) . T_r^{-1}(\theta_y) . T_r^{-1}(\theta_z) \tag{1-16}$$

in which $P'_m = [x'_m, y'_m, z'_m \quad 1]$, where (x'_m, y'_m, z'_m) is the coordinate of a point with error p'_m in MCS, and $P'_w = [x'_w, y'_w, z'_w \quad 1]$, where (x'_w, y'_w, z'_w) is the coordinate of a point with error p'_w in WCS.

Step 6. Inspect the finished component

CMM inspection algorithms are used to evaluate the dimensional and geometric accuracy of the machined component based on tolerance requirements.

Step 7. Perform statistical analysis

The iteration of simulation n is chosen to determine n values of a particular tolerance. A histogram and statistical parameters can be built for evaluation.

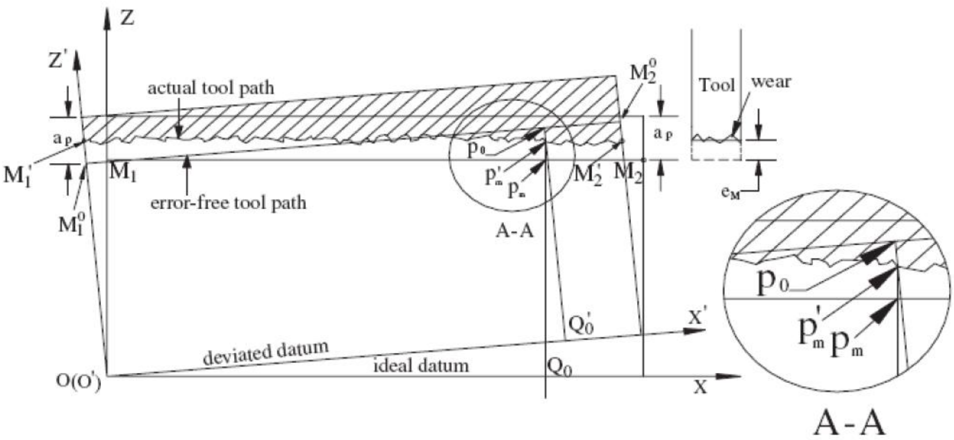


Fig. 1-21 Error synthesis [HUANG *et al.* 2004]

This study shows a method that can be used to verify a process plan. The machined surfaces are represented by the sample points that are close to reality. However, there are some limitations in this work, for example:

- The workpiece is treated as having theoretically perfect geometry, so a vertex of a prismatic or the centre of a circle that is chosen is not significantly to represent the workpiece position on a fixture.
- Just a plane/plane contact between a fixture and workpiece is considered. This is a limit for another fixture that has a cylinder/cylinder contact. And Monte Carlo simulation takes much time for generation of numerous sample points.

[TICHADOU *et al.* 2005] followed the work of Villeneuve to propose a graphic representation of the manufacturing process. The graph models the successive machining setups along with the positioning surface, hierarchical order and machined surfaces for each setup. This graph makes it possible to highlight the most influential paths in terms of functional tolerance. More specifically, the authors propose two analysis methods. The first model uses a small displacement torsor model. The second one is based on the use of a CAD/CAM system to virtually manufacture the part while integrating the manufacturing errors. The authors then virtually measure the parts made and check their compliance with design requirements.

Another work on this area is conducted by [LOUATI *et al.* 2006] for 3D modelling of geometrical errors propagation in machining to optimize a manufactured part setting. This work is followed by [AYADI *et al.* 2008].

However, several limitations of these last works are:

- defects of workpieces and machined parts are not experimental results;
- a link between two surfaces (fixture surfaces and workpiece surfaces) cannot be measured in reality;
- it is difficult to create a workpiece with its surface defects by CAD (time consuming);
- positioning defects of workpiece on the fixture that cause contacts between them are just assumptions.

Due to the limitations of the above studies, in this thesis, we propose a mathematical model for tolerance analysis based on Model of Manufactured Part (MMP). In this model, the experimental results or simulated results presented are used as input variables for simulating manufacturing defects (machining and positioning defects) during design phases of a process plan. This allows to:

- verify the process plan in term of functional tolerances
- or determine minimum tolerances of a batch of machined part based on an existing process plan and number of rejected parts per million (ppm).

3. CONCLUSIONS

In the context of production, manufacturing defect is the amount of various error sources. K. Whybrew and G. A. Britton in [ZHANG 1997] have summarized 27 error resources in a machining process for the following 8 items in machining: machine tool, cutting tool, fixture, workpiece, coolant, operator, environment conditions, and process variable. Numerous studies have been conducted on the four first items. Although researchers have recognized the important role of one or more source errors effects on the quality of machined parts, little research has been done on manufacturing defects on final part quality. For this reason we consider the manufacturing defects are divided in two categories: positioning defects and machining defects in which different source errors can affect them. With limitations of some research on quantification of positioning and machining defects, we propose several solutions to develop the method that can be used to identify, analyse and estimate manufacturing defects [BUI *et al.* 2010, SERGENT *et al.* 2010]:

- Using the double-measurement method to measure machined parts;
- Using the same method to associate a surface from a cloud of measured points, for instance, the least-square best-fit method is used to rebuild the geometric elements from measured data that are obtained from the two different machines. The advantage of this solution is to suppress deviations of the data processing;
- Two geometric elements respectively of the machined parts are chosen and measured by the two measurement means. The measured points are then analysed using the least-square best-fit mentioned earlier. The measurement results are finally compared in order to evaluate whether differences between the two measurement means are significant/insignificant;
- Determining defects of the machined parts as distributions and statistical parameters that can be used in simulations.

In addition, an essential step in manufacturing is to evaluate the quality of products in terms of functional tolerances. During manufacturing, a product can pass through one or several processes, machine tools, fixtures, cutting tools and the sequencing of operations that are called the process plan. Consequently, evaluation of a process plan in terms of functional tolerance is a direct step to control the quality of products, which is also known as tolerance analysis. In other words, controlling the quality of products is verifying whether the design tolerance requirements meet a given process plan with specified manufacturing deviations. To do this, we present a mathematical model for tolerance analysis based on Model of Manufactured Part (MMP). The two following diagrams can express some of objectives of this thesis. The first diagram shows the summary of previous works with their limitations. The second diagram shows our works in this study.

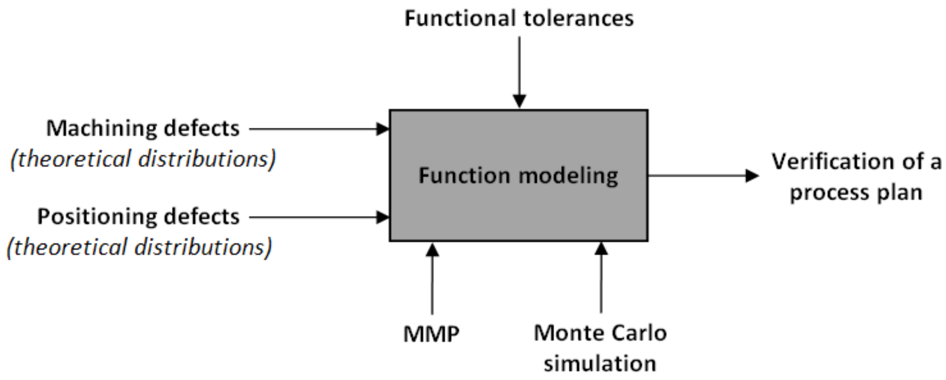


Fig. 1-22 The summary of previous works

Fig. 1-22 shows a tolerance analysis in which the input variables are theoretical distributions such as normal distribution or uniform distribution. This analyze used the MMP and Monte Carlo simulation to verify a process plan in term of functional tolerances.

Our objectives are to use experimental results as input variables in the tolerance analysis. In order to obtain the experimental results, a method is developed to separate and identify machining defects and positioning defects. These results are then expressed by different forms, such as variances of SDT components, distributions of SDT components, parallelism defects, flatness defects, and perpendicular defects, etc. Additionally, functional tolerances and a ppm are used as constrained parameters to create deviation domains. The deviation domains are then used to verify a process plan in term of functional tolerance or to identify achievable tolerances based on an existing process plan and number of rejected parts per million (ppm).

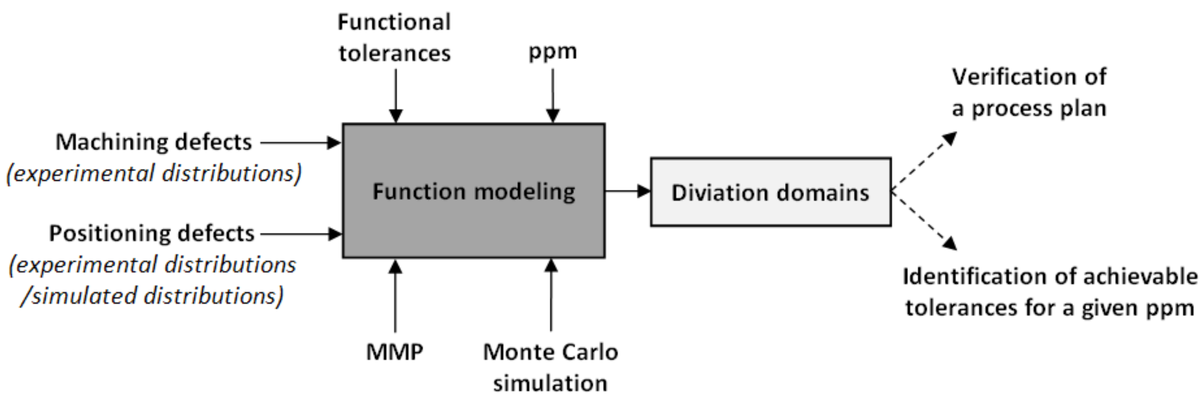


Fig. 1-23 The summary some of our works in this study

Moreover, the experimental results can be obtained by measuring experiments. Nevertheless, these are very costly and time-consuming; on the other hand, appropriate equipment is essential. For instance, a machined tool must be equipped with a measurement mean in order to measure a workpiece on a fixture inside this machine. Developing accurate models and methodologies for simulating the positioning defects and evaluating the factors that affect these defects can overstep these drawbacks. Hence, an investigation of influences of different factors on positions of a workpiece fixed on a fixture is presented. Three treatment factors are proposed: geometrical error of workpiece locating plane, clamping force and coefficient of friction at contact surfaces between the workpiece and the fixture. For this purpose, a number of finite element simulations based on statistical two-level full factorial design of experiments method are conducted in order to determine mathematical models. The models are then used in Monte Carlo simulation for evaluating positions of the workpiece.

The simple indicators are also proposed in this research for evaluating the global quality of a fixture. These indicators can be integrated into software in order to choose quickly a fixture configuration from different geometric parameters.

CHAPTER 2

IDENTIFICATION OF MANUFACTURING DEFECTS

1. INTRODUCTION

An essential step to succeed in evaluation of the quality products is the method used to identify defects in manufacture. This chapter presents a method that allows distinguishing between machining defects and positioning defects. To do this, some previous studies in quantification of manufacturing defects are provided. The limitations of these studies are discussed and then several propositions are given to solve the outstanding problems.

An experimental application is applied to show the necessity of the propositions. The results of the experiment are expressed in different ways, such as components of a SDT, form and orientation defects, or distributions and statistical parameters. The results can be used as input variables in some simulations that are used to predict manufacturing defects.

2. LIMITATIONS OF PREVIOUS WORKS AND PROPOSITIONS

In chapter literature review, we provided the limitations on quantifying the manufacturing defects in studies of [BOURDET *et al.* 1995b, DURET 1988, LEHTIHET *et al.* 1990]. Concerning the quantification of the manufacturing defects (the positioning and machining defects), the limitations can be summarized by the fact that we cannot use the only measurement on the machine tool. This does not provide enough data for analyzing the positioning defects.

In order to resolve the above problem, a supplementary measurement was proposed by [TICHADOU 2005]. It means that one more measurement will be done based on another reference. This measurement is carried out just after the final cutting step of a machined part.

In other words, the measurements are carried out inside the machine tool without disassembling the machined parts out of the fixture then inside a CMM after disassembling the machined part out of the fixture. This proposition can be shown in Fig. 2-1.

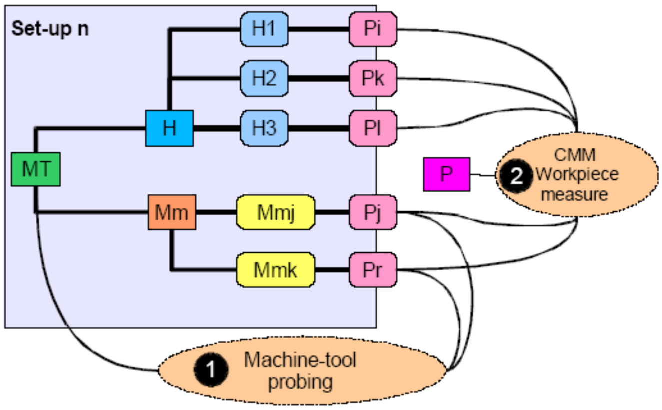


Fig. 2-1 Principle of double measurement method [TICHADOU *et al.* 2007]

The double measurement method is then applied in two experimental applications [TICHADOU *et al.* 2007] that are detailed in chapter literature review. There still exist several limitations for quantifying the manufacturing defects. Therefore, we proposed the following solutions to develop the method for analyzing the positioning and machining defects [BUI *et al.* 2010, SERGENT *et al.* 2010].

- Using the double measure method to measure machined parts
- Using the same method to associate a surface from a cloud of measured points. For instance, the least-square best-fit method is used to rebuild the geometric elements from measured data that are obtained from the two different machines. The advantage of this solution is to suppress deviations due to data processing
- Two geometric elements of the machined parts are chosen and measured by the two measurement means. The measured points are then analysed using the least-square best-fit mentioned earlier. The measurement results are finally compared in order to evaluate whether differences between the two measurement means are significant/insignificant
- Determining defects of the machined parts as distributions and statistical parameters that can be used in simulations

3. IDENTIFICATION OF MACHINING DEFECTS

As mentioned earlier, manufacturing defects can be divided into two categories: machining and positioning defects. In the present section, machining defects of a batch of 50 machined parts are determined based on the SDT concept. The objectives of this section are to:

- Determine machining defects of machined surfaces

- Estimate influence of different tool-paths on machined surfaces

3.1. Method

3.1.1. Reminder of the SDT concept

The methods that are used for determining the manufacturing defects are based on the Small Displacement Torsor (SDT) concept, which has been developed since the seventies by Bourdet and Clément [BOURDET *et al.* 1988, BOURDET *et al.* 1996]. This concept is based on an assumption of small displacements of a rigid body. It allows solving a general problem with the fit of a geometrical surface model to a set of points. A SDT is represented using two vectors: vector $\vec{R}$ includes three small rotations (r_X, r_Y, r_Z) and vector $\vec{T}$ includes three small translations (t_X, t_Y, t_Z). Thanks to the SDT concept, [VILLENEUVE *et al.* 2001] have extended the concept to the manufacturing process where machining defects were obtained using measurement of relationships between a nominal part (perfect surfaces) and a real part. A SDT can be used to express defects of different surfaces. For instance, two rotations and one translation (along a normal vector of a plane) in a SDT are used for a plane, or two rotations and two translations (along two axes, which are perpendicular with cylinder axis) are used for a cylinder SDT, etc.

Fig. 2-2 illustrates a plane SDT used to represent the small defects between an associated plane, which is built from a real plane, and its nominal one. Let (OXYZ) be the origin system of a plane, which has a normal vector along Z axis. A SDT of this plane is expressed using three components: two rotations around the X and Y axis and one translation along the Z axis. The plane SDT is shown as equation (2-1).

$$\mathcal{T}_{Pl} = \{\vec{R} \quad \vec{T}\}_{OXYZ} = \begin{Bmatrix} r_X & 0 \\ r_Y & 0 \\ 0 & t_Z \end{Bmatrix}_{OXYZ} \tag{2-1}$$

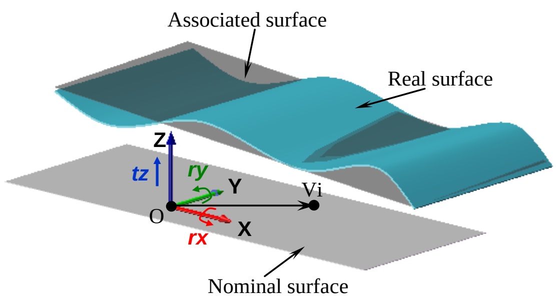


Fig. 2-2 A plane SDT [KAMALI NEJAD 2009]

3.1.2. Reconstruction of geometrical elements from 3D measuring points

SDT components are determined based on associated surfaces that are obtained from measured points of geometric elements on a machined part by measurement equipment (e.g. touch probe of a CNC machine tool or a CMM).

In order to reconstruct a measured surface from a cloud of measured points, several approaches can be used, for example: B-spline [PATRIKALAKIS *et al.* 2002], best-fitting geometrical features with Maximum Likelihood criterion [ARANDA *et al.* 2010], least-squares best-fit. [FORBES 1989] presented algorithms for finding least-squares best-fit geometric to data, the specific geometries considered are: lines in a specified plane, lines in 3 dimensions, planes, circle in a specified plane, circles in 3 dimensions, spheres, cylinders, and cones. In the following analysis, the least-squares best-fit are used to reconstruct machined planes and workpiece cylinders from 3D measuring points (for more detail see in Appendix 2).

Thanks to the least-square best-fit method, an associated plane is specified by a centroid on the plane and a direction cosine of the normal to the plane. An associated cylinder is specified by a point on its axis, a vector pointing along the axis and its radius (for more detail of the raw data see in Appendix 3).

3.2. Description of workpieces and designed part

A batch of 50 workpieces in aluminum (2017 A) that has diameter of 30mm and a length of 50mm (Fig. 2-3) is used in this study. A long aluminum bar is used for cutting of the 50 workpieces by a sawing machine. Two sawed surfaces and a cylinder surface of a workpiece are considered as an initial state of our production.

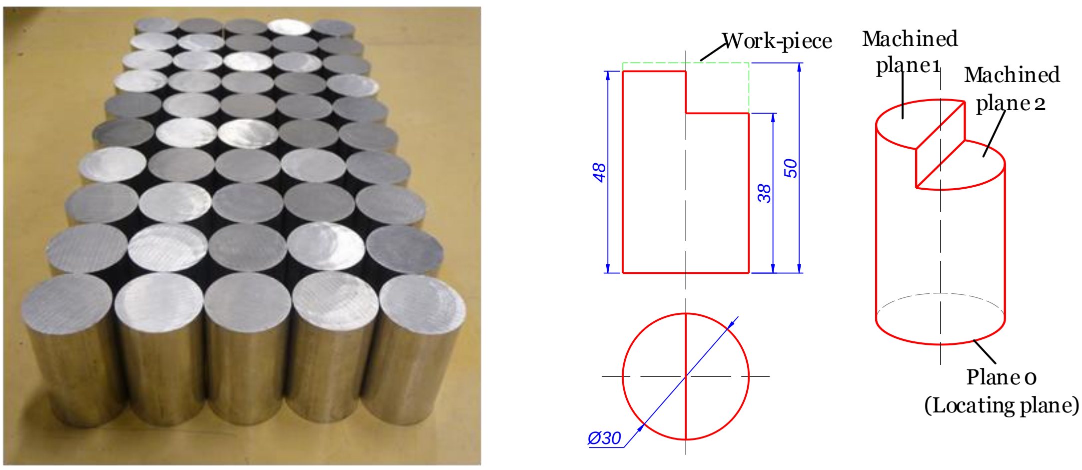


Fig. 2-3 Workpieces and designed part

The designed part consists of two machined planes (1 and 2) which are parallel. Distance between these two planes is 10mm (Fig. 2-3).

3.3. Manufacturing and measuring processes

3.3.1. Fixturing of workpieces on the CNC machine

A workpiece is located and fixed on the table of the CNC milling machine (DMG Dekel Maho - DMU50) by a three-soft-jaw chuck. In theory, the three soft jaws come into contact with the workpiece by a cylinder surface and a plane surface. The surfaces of the three soft jaws are machined on this machine (for more detail see in Appendix 4), namely locating surfaces of fixture. The locating surfaces of fixture are then measured by the touch probe of this machine to create a coordinate system that is used as an origin of machining and measuring programs (Fig. 2-4). This coordinate system will be detailed in the next section.

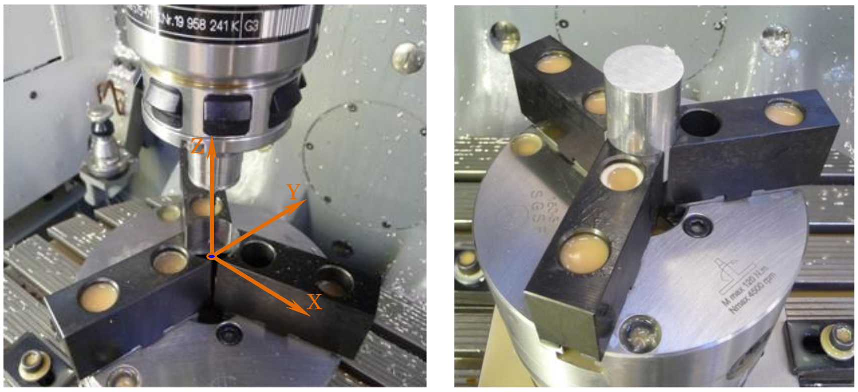


Fig. 2-4 Coordinate system of machining and measuring processes

3.3.2. Machine coordinate system (MCS)

Machine coordinate system of the machine tool is created based on the locating surfaces of fixture. First, locating plane of the fixture is measured to set a zero offset for Z axis of the machine. This is considered as the plane O_MXY of the MCS. The locating cylinder of the fixture is then measured to define a center of this cylinder that is finally used for establishing the Z axis of the MCS.

3.3.3. Machining processes

Planes 1 and 2 of the workpiece are machined by an end mill (ϕ20) with two different tool-paths (Fig. 2-5). These allow us to evaluate influences of the different tool-paths on the machined planes. A circle path is used for machining plane 1 with only one pass of the end mill. A straight-line path is used to machine plane 2 with five passes of the tool. It can be seen that the tool path on the plane 1 is the up milling. Tool path 1 and 2 have been chosen to produce a greater difference in machining defects.

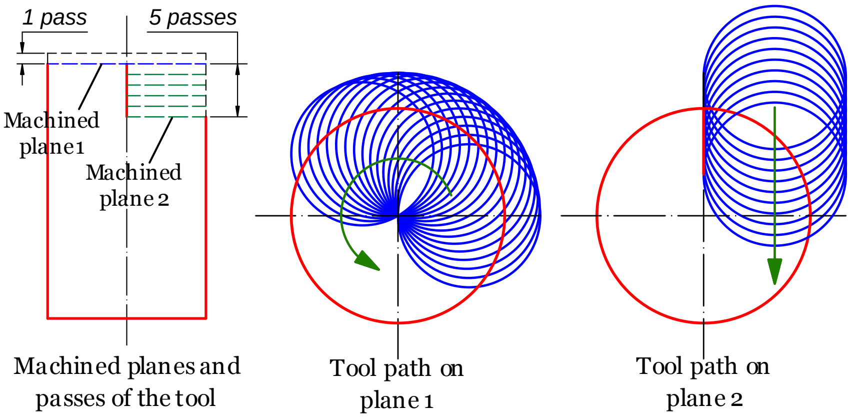


Fig. 2-5 Different tool paths of machined planes

The cutting conditions are used as the following table:

Parameters	**Machined plan 1**	**Machined plan 2**
Cutting speed	300 (mm/min)	300 (mm/min)
Feed rate	0.1 (mm/rev/teeth)	0.05 (mm/rev/teeth)

Tab. 2-1 Cutting conditions

3.3.4. Measuring processes

Each machined plane is measured just after the final cutting steps by the touch probe of this machine. A measuring point pattern that is created for each machined plane has parameters as in Fig. 2-6. Each measuring point is expressed by three components x, y and z in the Cartesian coordinate system.

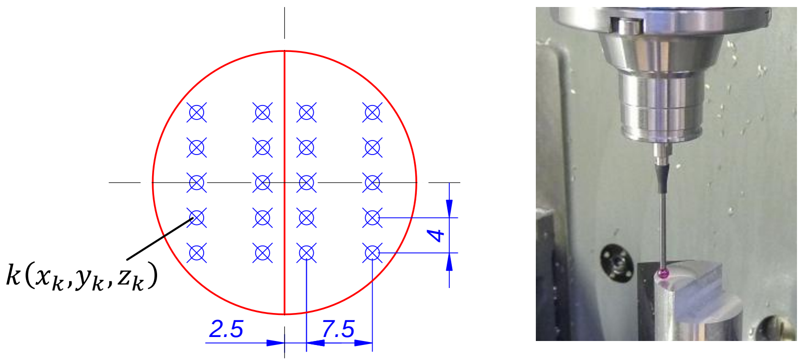


Fig. 2-6 Measuring point pattern

To ensure that noises of measurement system (measurement noises) do not influence measurement results, a square gage block (class 0) was measured 100 times repeatedly by [SERGENT *et al.* 2008] to estimate the dispersion of measurement. The results show that the standard deviation of a measured length on this machine is about 0.27E-03 mm. This is insignificant compared with the standard deviations of the machining defects obtained in this study ($s_{t_{Z1}} = 1.92E-03mm$ or $s_{t_{Z2}} = 1.77E-03mm$).

In addition, [RAMESH *et al.* 2000b] said that "*continuous usage of a machine tool causes heat generation at the moving elements and this heat causes expansion of the various structural elements of the machined tool*". To reduce variation of the heat between the moving elements in the machine tool during machining and measuring processes, a warm-up program is carried out before machining and measuring processes.

3.4. Exploitation of results

From the measuring data, defects of the machined surfaces can be analyzed and expressed in different forms.

3.4.1. Machining defects expressed by SDT

From the measurements of the machined planes on the CNC machine, two SDTs are obtained for each machined part (2-2).

$$\mathcal{T}_{Pl_i} = \begin{Bmatrix} r_{Xi} & 0 \\ r_{Yi} & 0 \\ 0 & t_{Zi} \end{Bmatrix}_{O_M XYZ} \tag{2-2}$$

where

- r_{Xi}, r_{Yi}, t_{Zi} are rotation and translation deviations of machined plane i around x, y axis and along z axis, respectively.
- i is the number of machined plane (i = 1 or 2).
- $O_M XYZ$ is the machine coordinate system (MCS).

The machining defects of every one of the 50 machined planes are expressed using variances of rotations and translations. We proposed the following method to calculate rotation and translation deviation of the machined part.

3.4.1.1. Deviations of a machined plane

Deviations of a machined plane are defined by the differences between its associated plane and its nominal one. In the recent study, the deviations of a machined plane are divided into two types: rotation and translation deviation.

3.4.1.2. Rotation defect

Here, a rotation defect is determined by the variance of 50 rotation deviations corresponding to 50 machined parts. Meanwhile, a rotation deviation of a machined plane is obtained by the difference between normal vector n_i of the associated plane i and the Z-axis of the MCS (Fig. 2-7).

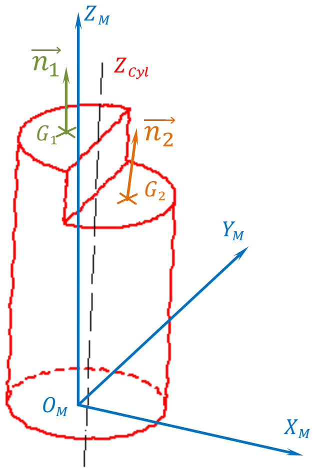


Fig. 2-7 Normal vectors of associated surfaces and the MCS

Rotation deviations of machined planes are calculated in the MCS as equations (2-3).

$$\begin{cases} r_{Xi,MCS} = arctan\left(\frac{n_{Yi}}{n_{Zi}}\right) \\ r_{Yi,MCS} = arctan\left(\frac{n_{Xi}}{n_{Zi}}\right) \end{cases} \tag{2-3}$$

where

- $r_{Xi,MCS}, r_{Yi,MCS}$ are rotations of the machined plane i in the MCS.
- i is the number of the machined plane (i = 1 or 2).
- $\vec{n}_i(n_{Xi}, n_{Yi}, n_{Zi})$ is the normal vector of the machined plane i.

The rotation deviation and rotation defects ($s^2_{r_{Xi}}$, $s^2_{r_{Yi}}$) of the machined planes are shown in Fig. 2-8 and Tab. 2-2, respectively.

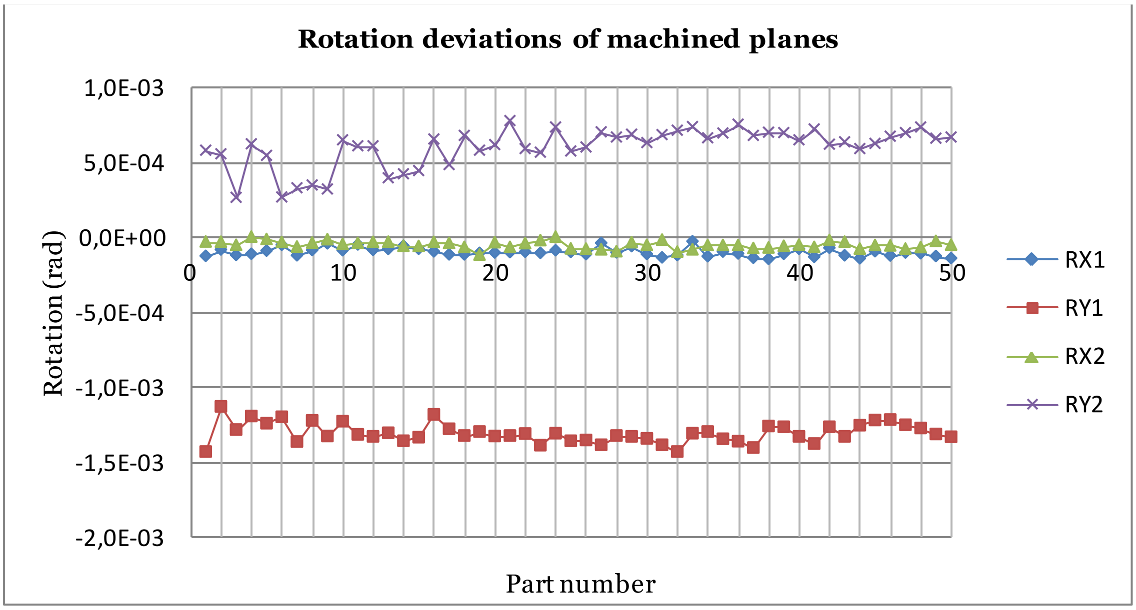


Fig. 2-8 Rotation deviations of machined planes

Components	CNC (MCS)	
	Machined planes 1	Machined planes 2
m_{r_X} *(rad)*	-0.978E-04	-0.479E-04
$s^2_{r_X}$ *(rad²)*	7.907E-10	6.356E-10
m_{r_Y} *(rad)*	-13.05E-04	6.036E-04
$s^2_{r_Y}$ *(rad²)*	41.714E-10	163.72E-10

Tab. 2-2 Rotation defects of the machined planes in the MCS

where

- m_{r_X}, m_{r_Y} are means of rotation defects of the machined planes around x and y-axis.
- $s^2_{r_X}, s^2_{r_Y}$ are variances of rotation defects of the machined planes around x and y-axis.

3.4.1.3. Translation defect

Similarly, a translation deviation of each machined plane is obtained in the MCS. This is calculated by a difference between a centroid of an associated plane and its nominal one along the Z axis (Fig. 2-9).

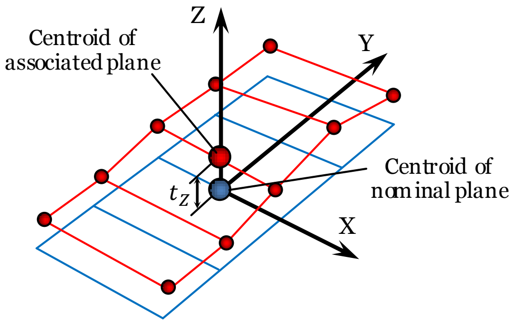


Fig. 2-9 Translation deviation of a machined plane

Translation defect is then obtained by the variance of the 50 translation deviations corresponding to the 50 machined parts.

The translation deviations and translation defects of the machined planes are shown in Fig. 2-10 and Tab. 2-3, respectively.

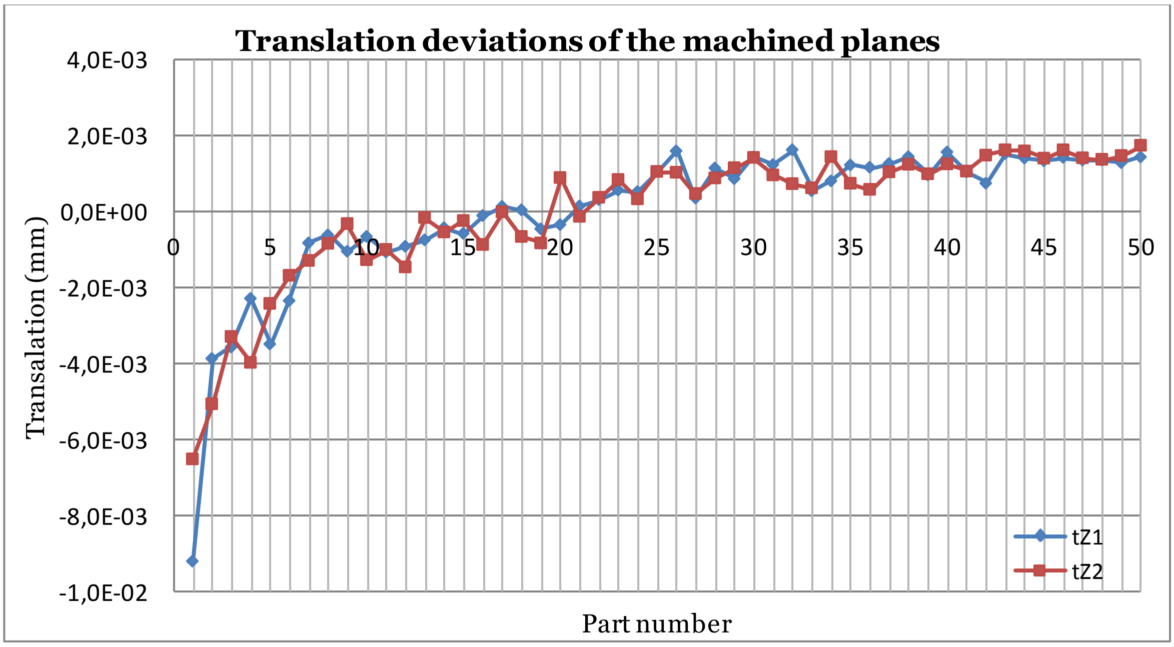


Fig. 2-10 Translation deviations of machined planes

Components	CNC (MCS)	
	Machined planes 1	Machined planes 2
$s_{t_Z}^2 (mm^2)$	3.711E-6	3.123E-6

Tab. 2-3 Translation defects of the machined planes in the MCS

In Fig. 2-10, t_{Z1} and t_{Z2} represent the translation defects along Z-axis of the two milled planes. It can be seen that the translation defects of the two machined planes increase together during machining times (from the 1st part to the 50th part), these are drifts. The increases of the two drifts are similar. It means that translation defects of the two machined planes are dependent. To verify this assumption, the correlation coefficient between two variables (translation deviations of the two machined planes) is analysed (for more details see Appendix 1). The results show that these variables are dependent. We try to explain in section 3.4.5 as systematic errors.

The translations and rotations defects of the machining defects are summarized in Tab. 2-4.

Components		CNC (MCS)	
		Machined planes 1	Machined planes 2
Rotation	s_{rX}^2 (rad^2)	7.907E-10	6.356E-10
	s_{rY}^2 (rad^2)	41.714E-10	163.72E-10
Translation	s_{tZ}^2 (mm^2)	3.711E-6	3.123E-6

Tab. 2-4 Machining defects

Based on the probability distributions (section 3.4.3) of the translation and rotation components, a test for equality of two variances is proposed to evaluate machining defects of the two machined planes. The objective of this test is to show that the proposed method allows us to detect the difference of the defects on the machined planes using the different tool paths.

Components		CNC (MCS)		
		Machined planes 1	Machined planes 2	Test for equality of variances
Rotation	s_{rX}^2 (rad^2)	7.907E-10	6.356E-10	Δ = 1.551E-10 Test is not OK
	s_{rY}^2 (rad^2)	41.714E-10	163.72E-10	Δ = 122.006E-10 Test is OK
Translation	s_{tZ}^2 (mm^2)	3.711E-6	3.123E-6	Δ = 0.5881E-06 Test is not OK

Tab. 2-5 Test for equality of two variances

There are some different tests for equality of two variances, e.g. Levene's test and Bartlett test. The Bartlett test is used when the data sets come from a normal, or nearly normal distribution. Conversely, Leneve's test is used (for more details see Appendix 1). If the statement of the test for equality of variances is "OK", it means that the difference between the two observed variances is insignificant and vice versa.

The results in Tab. 2-5 show that the difference between two rotation defects s_{rX}^2 or two translation defects s_{tZ}^2 obtained on the two machined planes is significant. Meanwhile, the difference between two rotation defects s_{rY}^2 obtained on the two machined planes is insignificant.

3.4.2. Machining defects expressed by flatness, perpendicular, and parallelism defects

From measured points of the machined planes, the machining defects can be expressed by form defects (flatness defects) or orientation defects (perpendicular and parallelism defect).

It is important to note that the form and orientation defects are calculated based on the associated plane using the least square best fit method. Thus, the defects that are obtained are not based on ISO.

3.4.2.1. Flatness defects

We propose that flatness of the machined plane can be calculated as follows:

Let A be an associated plane created from a cloud of measured points of the machined plane. A_{min} and A_{max} are two planes that are parallel to the plane A and pass through two points which have minimum and maximum coordinates along the normal vector of the plane A. The flatness of the machined plane is defined by the distance t between the plane A_{min} and A_{max}.

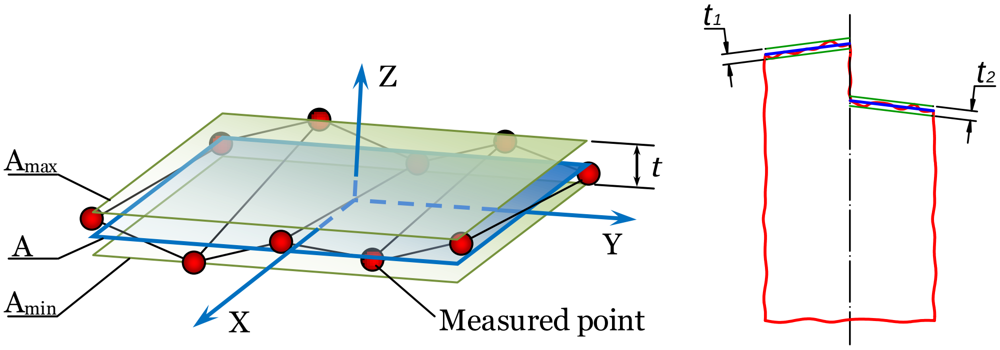


Fig. 2-11 Flatness of machined planes

This method is used to obtain the flatness of each machined plane. An average of the 50 flatnesses of each machined plane is calculated to evaluate the flatness of the two machined planes; the results are shown as Tab. 2-6.

	Machined planes 1	Machined planes 2
Average *(mm)*	9.517×10^{-3}	1.943×10^{-3}
Standard deviation *(mm)*	0.458×10^{-3}	0.462×10^{-3}

Tab. 2-6 Flatness defects

Comparing the results of the flatness in Tab. 2-6, it can be seen that the defects of the machined planes 1 are more significant than the machined planes 2. This confirms that the results obtained in section 3.4.1 are correct.

3.4.2.2. Parallelism defect

Let the machined plane 1 be the reference plane. The parallelism of the machined plane 2 compared to the reference plane is the distance between two planes which are parallel to the associated plane of the machined plane 1 and pass through two points which have a maximum and minimum coordinates along the normal vector of the machined plane 1 (Fig. 2-12).

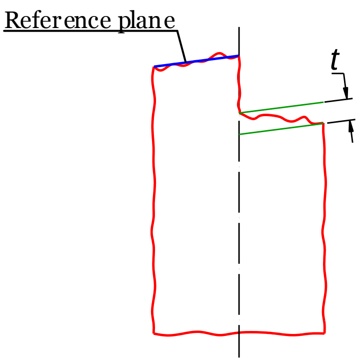


Fig. 2-12 Parallelism

The parallelism of the machined planes 2 compared to the machined planes 1 is expressed by an average of the 50 values obtained from the above method. The results are shown as Tab. 2-7.

	Machined planes 2 Vs. Machined planes 1
Average *(mm)*	16.013×10^{-3}
Standard deviation *(mm)*	1.268×10^{-3}

Tab. 2-7 Parallelism defect

3.4.2.3. Perpendicular defects

Let the workpiece cylinder axis be the reference line. The perpendicularity of a surface compared to the reference line is defined as the distance between two planes which are perpendicular to the reference line and pass through two points that have minimum and maximum coordinates along the cylinder axis.

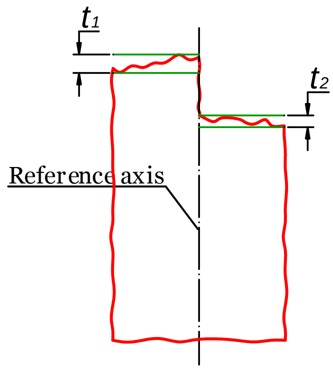


Fig. 2-13 Perpendicularity

Thanks to the proposed method, the perpendicularities of the two machined planes are obtained and expressed by an average value of the each of the 50 machined planes. The results are shown in Tab. 2-8.

	Machined planes 1	Machined planes 2
Average *(mm)*	17.30×10^{-3}	14.03×10^{-3}
Standard deviation *(mm)*	4.935×10^{-3}	5.637×10^{-3}

Tab. 2-8 Perpendicular defects

The results show that the perpendicular defects of the machined planes 1 are greater than those of the other one. Therefore, this confirms, again, that the results obtained in section 3.4.1 are correct. In other words, the method based on the SDT concept allows us not only to identify machining defects, but also to detect whether there is a difference on the machined surfaces.

3.4.3. Machining defects expressed by distributions and statistical parameters

Experimental results that are analysed and expressed as probability distributions and statistical parameters are very useful for simulations (e.g. Monte Carlo simulation). Hence, the defects of the machined planes are here expressed by probability distribution types.

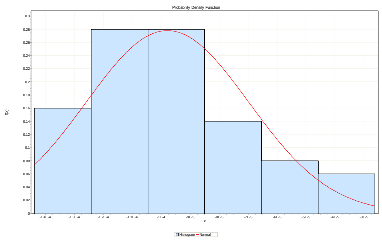
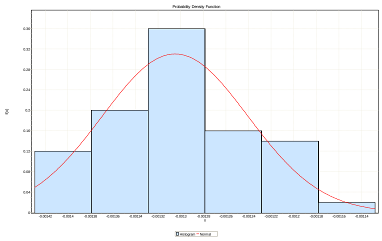
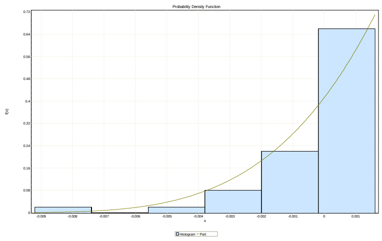
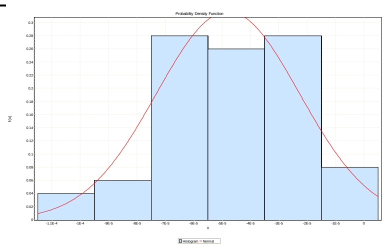
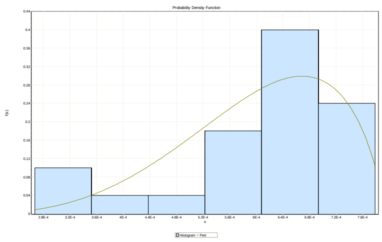
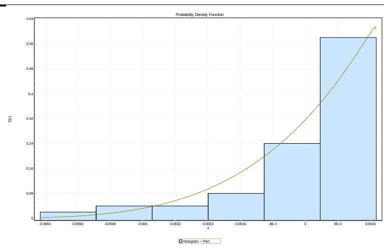
To determine probability distribution type for a data set, the following steps can be conducted [WEB-PAGE 2011].

1. Plot data for a visual representation of the data type.
2. One of the first steps to determining what data distribution one has is to rule out what it cannot be:
    - The data set cannot be a discrete uniform distribution if there are any peaks in the set data.
    - The data set is not Poisson or binomial if the data has more than one peak.

- If it contains a single curve, no secondary peaks, and has a slow slope on each side, it may be Poisson or a gamma distribution, but it cannot be a discrete uniform distribution.
- If the data is regularly distributed, and it is without a skew toward one side, it is safe to rule out a gamma or Weibull distribution.
- If the function has an even distribution or a peak in the middle of the graphed results, it is not a geometric distribution or an exponential distribution.
- If the occurrence of a factor varies with an environmental variable, it probably is not a Poisson distribution.

3. After the probability distribution type has been narrowed down, do an R squared analysis of each possible type of probability distribution. The one with the highest R squared value is most likely correct.
4. Eliminate all outliers in the data point. Then recalculate R squared. If the same probability distribution type comes up as the closest match, then it is a very likely that this is the correct probability distribution to use for the data set.

From the above steps we can determine probability distribution of a data set. To save time, a statistical software (Easy-Fit) is used for the first estimation of probability distribution of data sets. The parameters of a probability distribution are then obtained in the Mathematica software. This allows synchronizing analyzed results for simulation in chapter 5. Probability distributions of the machining defects are obtained as in Tab. 2-9.

Components	Distribution	Parameters	Histogram
r_{X_1} *(rad)*	Normal	s=2.784E-5 m=-9.784E-5	
r_{Y_1} *(rad)*	Normal	s =6.394E-5 m =-1.305E-3	
t_{Z_1} *(mm)*	Pert	min=-1.123E-2 max=0.162 E-2 c=0.162 E-2	
r_{X_2} *(rad)*	Normal	s =2.496E-5 m =-4.792E-5	
r_{Y_2} *(rad)*	Pert	min=-66.378 E-2 max=73.903 E-2 c=1.465 E-2	
t_{Z_2} *(mm)*	Pert	min=-4.544 E-2 max=3.287 E-2 c=3.287 E-2	

Tab. 2-9 Probability distributions of the machining defects

The results show that the machining defects are neither always normal distributions nor uniform distributions as propositions in simulations.

3.4.4. Evaluating form defects of machined surfaces

To clarify the differences of the defects on the two machined planes obtained from the method based on the SDT concept, the defects are evaluated by the different tool paths and tool deflections occurred on these surfaces.

To assess form defects of the machined surfaces, an average surface of the 50 machined surfaces is used to represent their defects. For instance, the average surfaces 1 and 2 represent the defects of the 50 machined planes 1 and 2, respectively. The average surface includes 10 average points. The two average surfaces are shown as Fig. 2-14.

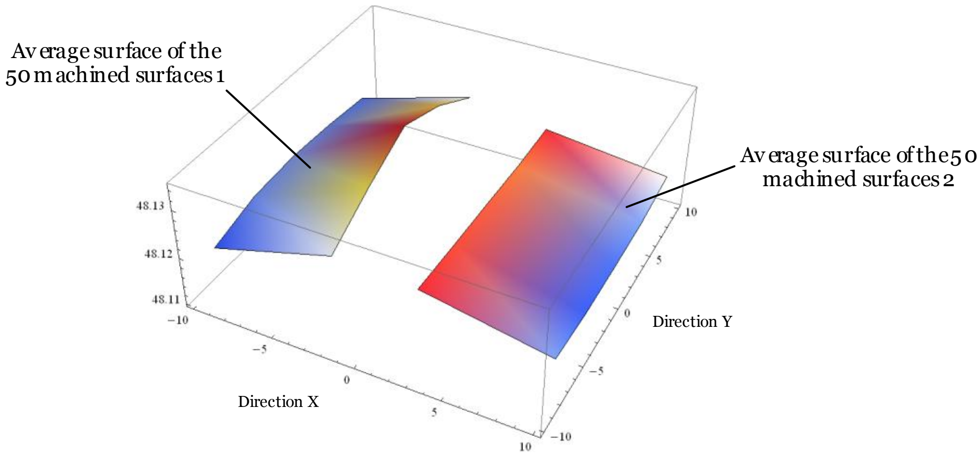


Fig. 2-14 Average surfaces

It can be seen that the average surface 1 looks like a piece of a cone where the materials near the centre is higher than the outside. On the average surface 2, the materials on the left side are higher than on the right side. To evaluate the phenomena of the two machined surfaces, we reconsider the tool paths used on these machined surfaces.

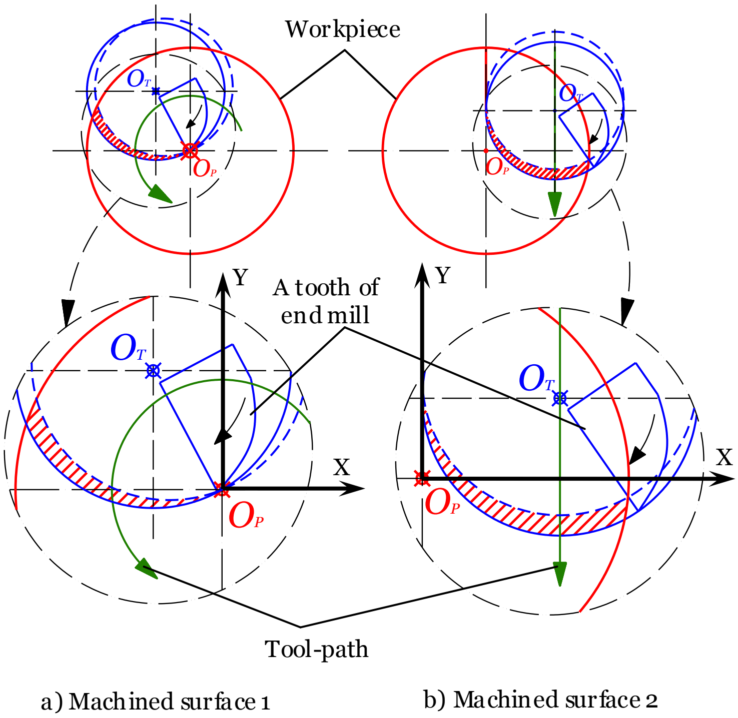


Fig. 2-15 Tool paths on the machined surfaces

From Fig. 2-15, there are some remarks on the machined surfaces as follows.

- On the machined surface 1 (Fig. 2-15a), the direction of rotation of the cutter around its axis and the direction of feed of the cutter around the workpiece axis are opposite to each other. Hence the cutting increases from zero to maximum per tooth cutter movement. It means that the thickness of the chip will be minimum at the beginning and maximum at the termination of the cutter.
- On the machined surface 2 (Fig. 2-15b), the direction of rotation of the cutter around its axis and the direction of the feed of the cutter along the Y axis are also opposite. But the cutting decreases from maximum to zero per tooth cutter movement. Hence the thickness of the chip will be maximum at the beginning and minimum at the termination of the cutter.

Consequently, the cutting on the surface 1 can be considered as up milling, where the quality of surface generated will be wavy. Whereas, the cutting on the surface 2 can be considered as down milling in which it is used to improve surface finish. These can be used to evaluate the roughness on the two machined planes, e.g. calculation of flatness.

Furthermore, to explain the phenomena of form defects on the machined surfaces, a model of deflection error is proposed to determine deflections of the milling tool and the workpiece.

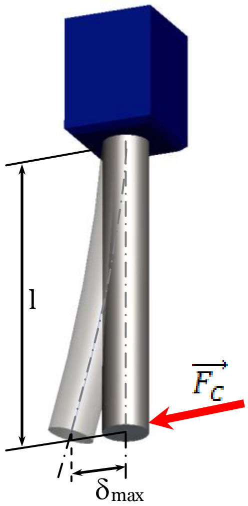


Fig. 2-16 Model of deflection error

In this model, the milling tool and the workpiece are considered as a cylindrical cantilever beam with the concentrated load at the free end. Maximum deflection of the cylinder can be expressed as Fig. 2-16 and equation (2-4).

$$\delta_{max} = \frac{F_C l^3}{3EI} \qquad (2\text{-}4)$$

where

- δ_{max} is the maximum deflection of the cylinder.
- F_C is the cutting force that is considered as the concentrated load at the free end.
- l is the length of the cylinder.
- E is young's modulus of the cylinder material.
- $I = \frac{\pi d^4}{64}$ is the moment of inertia of a circle diameter d.

In our case, the milling tool and the workpiece are considered as two cylinder contacts at their free ends with the same concentrated load F_C (Fig. 2-17). Thus, deflections of these two cylinders need determining to evaluate their deflections.

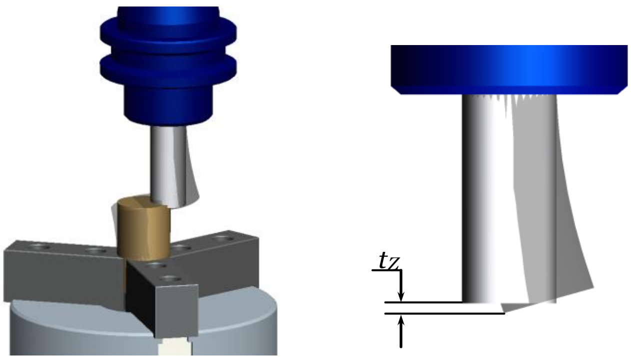


Fig. 2-17 Deflections of the milling tool and the workpiece

If the milling tool and the workpiece are considered at a moment, the cutting force F_C will be constant. Thus, the deflections of the milling tool and the workpiece are obtained as follows.

	Parameters				Deflection maximum	Ratio of deflections
	F_C (N/mm^2)	D (mm)	l (mm)	E (N/mm^2)	δ_{max} (mm)	$\left(\delta_{maxT}/\delta_{maxWP}\right)$
Workpiece (WP)	F_C	30	30	74.10^3	$3.059 \times 10^{-3}.F_C$	2.67
Tool (T)	F_C	20	50	650.10^3	$8.162 \times 10^{-3}.F_C$	

Tab. 2-10 Deflections of the milling tool and the workpiece

Results in Tab. 2-10 show that the deflection of the cutting tool is more significant than the workpiece.

On the other hand, in order to determine variations of cutting forces F_C on the two machined planes, the thickness of the chip during machining on these two planes is analyzed as Fig. 2-15. As we know the cutting force is directly proportional to thickness of the chip as in the following equation:

$$F_C = K_C.a.f \tag{2-5}$$

where

- K_C is the specific cutting force. This is chosen in the table based on the average chip thickness.
- a is the depth of cut. This is constant for each pass of the milling tool (2mm).
- f is the feed rate. Feed rate is defined as relative velocity at which the cutter is advanced against the workpiece. This is directly proportional to the chip thickness as in (2-6):

$$f = h.Z.n \tag{2-6}$$

where

- Z is the number of teeth on the cutter.
- n is the spindle speed.
- h is the chip load or the feed per tooth. It changes during the machining as in Fig. 2-15.

In consideration of the tool deflection due to the cutting force, the form defects of the two machined planes can be explained as follows:

- On the machined plane 1, the deflection of the cutting tool at the centre of the workpiece is less than at the border of the machined surface. As Fig. 2-17 shows that the greater the deflection of the end mill, then the lower the end mill will cut the workpiece. Thus, the end mill cuts the workpiece at the centre less than at the border of the workpiece. As the results (Fig. 2-14) show the machined plane looks like a piece of cone.

- On the machined plane 2, the deflection of the cutting tool at the right border of the machined surface is greater than at the left border (on the Y axis) of the workpiece. Similarly, the greater the deflection of the end mill is, the lower the end mill will cut the workpiece. Thus, the end mill cuts the workpiece at the left border of the surface less than at the right one. As the results (Fig. 2-14), the machined plane inclines along the Y axis.

The analysed results confirm, once again, that the difference of the defects between the two machined surfaces obtained from the method based on the SDT concept is correct.

3.4.5. Explanation of the drifts in the machining defects

Here, we try to explain the drifts of the machined planes obtained in section 3.4.1.3. The results show that the drifts of the two machined surfaces are similar (Fig. 2-18). In metrology, measurement errors can be subdivided in two classes, namely random errors and systematic errors. The division of measurement errors into systematic and random errors is important, because these components are manifested differently and different approaches are required to estimate them [RAKSIRI *et al.* 2004].

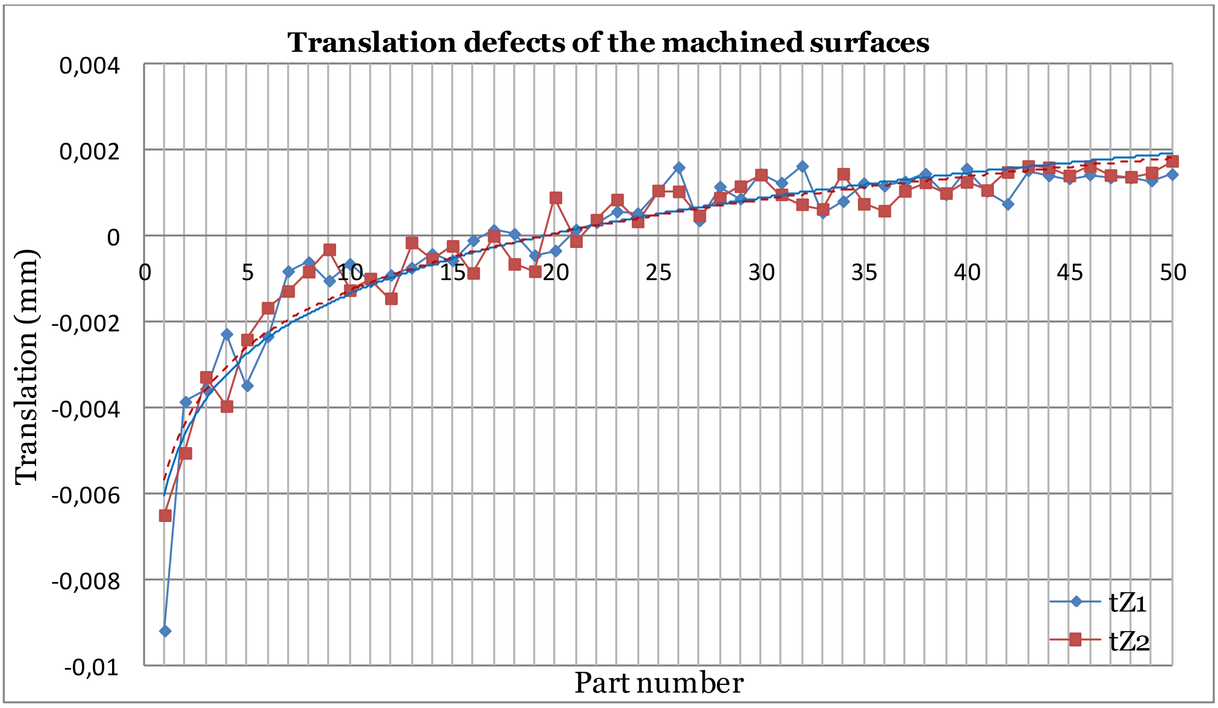


Fig. 2-18 Drifts of the translation defects on the machined surfaces

Here, the drifts of the two translation defects are considered as systematic errors because of their similarity to each other. It can be seen that the systematic errors are more significant than the random errors. Sources of these systematic errors may be changes of machining/measuring conditions during machining/measuring operations, i.e. thermal error interferes with the measurement process. [RAMESH *et al.* 2000b] investigated a temperature variation at critical elements on machine tools, which is a major source of inaccuracy.

Considering the same conditions of manufacturing (type of machined surfaces, machine tool, milling tool, environment of manufacture, etc), translation defects can be reduced significantly if errors of the drift are compensated for in the machining program. For that purpose, regression analysis is used for modelling and analyzing the drifts of translation defects. There are two types of regression analysis: linear regression where the data are approximated using a straight line; and otherwise, non-linear regression. From Fig. 2-18, the drifts of the two machined planes are non-linear. Thus, non-linear regression is applied in this case. There are different functions of non-linear regression, e.g. power, polynomial, and logarithmic … According to scatter plots of the variables, two functions, polynomial and logarithmic regression, are selected to describe fitting functions. A correlation coefficient R^2 is used to estimate deviations between the variables and the fitting functions. The final fitting function is then selected based on comparisons of R^2.

Components	Polynomial regression		Logarithmic regression	
$t_{Z_1}(mm)$	Fitting function	R^2	Fitting function	R^2
Machined planes 1	$-5 \times 10^{-6}.n^2 + 0.005$	0.83	$0.002\ln(n) - 0.007$	0.96
Machined planes 2	$-5 \times 10^{-6}.n^2 + 0.005$	0.95	$0.002\ln(n) - 0.006$	0.88

Tab. 2-11 Fitting functions

As results in Tab. 2-11 show the logarithmic regression is selected to model the systematic errors of the translation defects. In order to evaluate corrections of the systematic errors, a relationship analysis of the variables obtained from the measurement on the CNC machine is carried out. Both the variables before and after the corrections are analysed.

According to properties of variance and covariance in probability theory and statistics, two random variables x and y can be expressed in equation (2-7).

$$s^2_{(a-b)} = s^2_a + s^2_b - 2Cov(a,b) \qquad (2\text{-}7)$$

where

- $s^2_{(a-b)}$ is variance of the sum of the two random variables a and b.
- s^2_a, s^2_b are variances of a and b, respectively.

- $Cov(a,b)$ is covariance of a and b.

As mentioned previously, the 0 in t_{Z01CNC},t_{Z02CNC} is plane O_MXY, which is a perfect plane. Thus, the t_{Z01CNC},t_{Z02CNC} are considered as translation deviations of the machined planes 1 and 2. These deviations can be rewritten as t_{Z1CNC}, t_{Z2CNC}. Let $s^2_{t_{Z12CNC}}$ be variance of the sum of the two variables t_{Z1CNC},t_{Z2CNC}. These three variables can be expressed as the following equation:

$$s^2_{t_{Z12CNC}} = s^2_{t_{Z1CNC}} + s^2_{t_{Z2CNC}} - 2Cov(t_{Z1CNC}, t_{Z2CNC}) \qquad (2\text{-}8)$$

Equation (2-8) is used to verify the translation defects before the correction as follows.

Components	$s^2_{t_{Z1}}$ *(mm²)*	$s^2_{t_{Z2}}$ *(mm²)*	$s^2_{t_{Z12}}$ *(mm²)*
CNC	37.108E-07	31.233E-07	4.352E-07
$Cov(t_{Z1CNC}, t_{Z2CNC})$	31.354E-07		

Tab. 2-12 Variance and covariance of translation defects before the corrections

$$\begin{cases} s^2_{t_{Z12CNC}} = 4.352 \times 10^{-7} \\ s^2_{t_{Z1CNC}} + s^2_{t_{Z2CNC}} - 2Cov(t_{Z1CNC}, t_{Z2CNC}) = 5.632 \times 10^{-7} \end{cases} \qquad (2\text{-}9)$$

The difference between $s^2_{t_{12CNC}}$ obtained from the measurements (4.352×10^{-7}) and the calculation of the relations (5.632×10^{-7}) is significant (2-9).

Hereafter, the results of the variables (t_{Z1CNC},t_{Z2CNC}, and t_{Z12CNC}) are obtained after the corrections of the systematic errors.

Components	$s^2_{t_{Z1}}$ *(mm²)*	$s^2_{t_{Z2}}$ *(mm²)*	$s^2_{t_{Z12}}$ *(mm²)*
CNC	4.438E-07	2.228E-07	4.352E-07
$Cov(t_{Z1CNC}, t_{Z2CNC})$	1.134E-07		

Tab. 2-13 Variance and covariance of translation defects after the corrections

$$\begin{cases} s^2_{t_{Z12CNC}} = 4.352 \times 10^{-7} \\ s^2_{t_{Z1CNC}} + s^2_{t_{Z2CNC}} - 2Cov(t_{Z1CNC}, t_{Z2CNC}) = 4.398 \times 10^{-7} \end{cases} \qquad (2\text{-}10)$$

The difference between $s^2_{t_{Z12CNC}}$ obtained from the measurements (4.352×10^{-7}) and the calculation of the relations (4.398×10^{-7}) is insignificant (2-10).

The analysis results show that the systematic errors play a significant role in the machining defects. If these errors can be corrected the translation defects will reduce significantly (comparisons of results in Tab. 2-12 and Tab. 2-13). Nevertheless, in practice, systematic error is difficult to correct.

It is necessary to note that the above analysis seeks to explain the errors which occurred during machining, but the correction results will be not used for analysing the positioning defects in section 4 of chapter 2.

3.5. Evaluation of machining defects using the form parameterization method

The method based on the SDT concept is used to identify manufacturing defects. As assumption of the SDT concept, form defects are neglected. We therefore propose to use another method, namely the form parameterization method, for completing analysis of form defects on the machined surfaces.

This method has been often used to analyse form defects of a part measured on a CMM with hundreds of measurement points [FAVRELIERE *et al.* 2009, FORMOSA *et al.* 2007]. In this section, we try to apply the form parameterization method for analysing machining defects with restricted number of measurement points (only 10 points on a machined surface).

3.5.1. Reminder of the form parameterization method

The concept of this method is based on a discrete modal decomposition (DMD) [ADRAGNA *et al.* 2007]. The DMD decomposes a signal in a set of discrete functions, like a discrete Fourier transform. These signals of geometrical elements are measured, for example: measured plane, measured cylinder, and measured sphere. The measured geometrical surfaces are searched for and described in the set of discrete functions; some steps of this method can be shown as follows:

- The measured surfaces (measured points of the surfaces) are discretized using a finite element approach.
- These surfaces are expressed by a displacement vector $(\vec{V})$
- A modal analysis is used to obtain the modal basis (Q_i) that is then used to decompose the vector $\vec{V}$ and calculate the λ_i coefficients, which represent the form deviation in the basis of modal shapes.
- Finally, the decomposition operation consists in projecting the vector $\vec{V}$ in the modal basis (Q_i).

$$Q^{*}.V = ((Q^{T}.Q)^{-1}.Q^{T}).V = \lambda \qquad (2\text{-}11)$$

where the basis of matrix Q is made of the vectors $\vec{Q}_i$. The projection that is performed in the basis Q is not orthonormal, consequently the dual basis Q^* has to be used.

It therefore becomes a new expression of vector $\vec{V}$:

$$V = \sum_{i=1}^{m} \lambda_i . Q_i + \varepsilon \tag{2-12}$$

where m is the number of the modes which are chosen to present the measured vector $\vec{V}$ and the residual vector $\vec{\varepsilon}$. The modes Q_i are assessed through the resolution of a classic problem of mechanical vibration.

$$M.\ddot{q} + K.q = 0 \tag{2-13}$$

where

- M is the matrix of the generalized mass.
- K is the matrix of the generalized stiffness.
- $\vec{q}$ is the displacement vector.

The resolution of this problem can be analyzed using the finite element method. Thus, in the present method, the most significant modes of the modal basis are considered as representing the finite element of the machined planes.

3.5.2. Machining defects expressed by form parameterization method

This method focuses on form errors of the machined planes. Theoretically, the residual vector ε equals zero if the number of modes equals the number of measured points multiplied by the number of the degree of freedom. In order to reconstruct exactly the form errors of a surface, the number of modes (N_{mode}) is used to analyse different defects of the surface.

$$N_{mode} = n_p \times n_{dof} \tag{2-14}$$

where

- N_{mode} is a number of calculated modes.
- n_p is a number of measured points.
- n_{dof} is a number of degrees of freedom of each measured point.

Here, only one degree of freedom of the measured points (translation along Z) is considered. The finite element mesh is built up using the same number of nodes than the number of measured points (Fig. 2-19). The measuring pattern of this model has 10 nodes; therefore, 10 modes can be obtained.

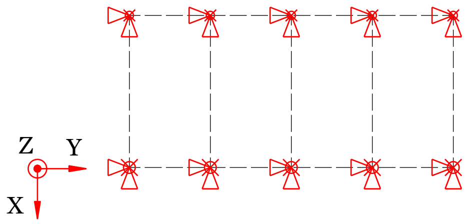


Fig. 2-19 Finite element mesh is based on measured points

As mentioned previously, the significant modes are considered to estimate the form defects. Thus, the eight significant modes are shown as Fig. 2-20.

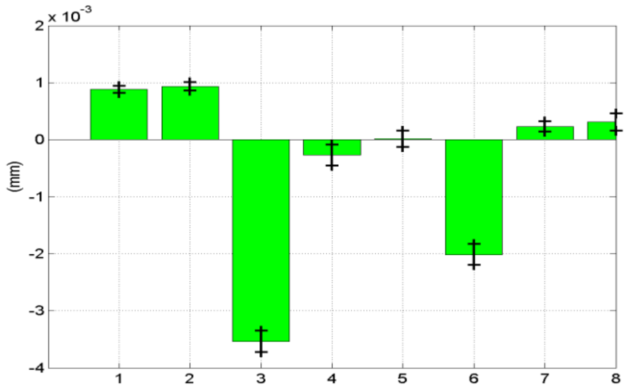


Fig. 2-20 Eight important modes of the machined planes 1

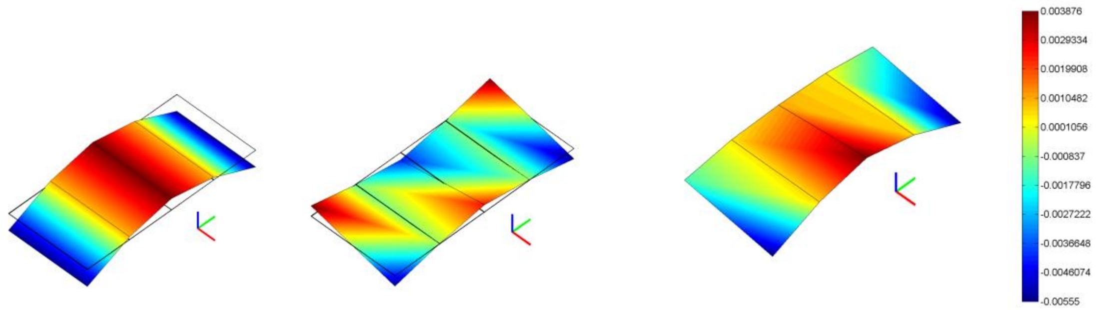
They are named as follows.

- Mode 1 and 2: translations and rotations
- Mode 3: comber
- Mode 4: 1st twist
- Mode 5: undulation
- Mode 6: 2nd twist
- Mode 7: 1.5 undulation
- Mode 8: 3rd twist

This method focuses on the form errors of the machined planes, consequently the mode 3 (comber –Fig. 2-21a) and the mode 6 (2nd twist –Fig. 2-21b) of the machined planes 1 are the most significant modes of the form errors. Means and standard deviations of these two modes are calculated and shown in Tab. 2-14.

Components	Machined planes 1	
	Mode 3	Mode 6
Mean *(mm)*	-3.535×10^{-3}	-2.015×10^{-3}
Variance *(mm²)*	3.548×10^{-8}	3.437×10^{-8}

Tab. 2-14 The most significant modes of the planes 1



a) Mode 3 b) Mode 6 c) Mean of reconstructed

Fig. 2-21 Form errors of machined planes 1

The two most significant modes are then used to obtain the final form errors (Fig. 2-21c) of the machined planes 1.

Similarly, the significant modes of the machined planes 2 are obtained and shown in Fig. 2-22; means and standard deviations of the three most significant form modes are shown in Tab. 2-15.

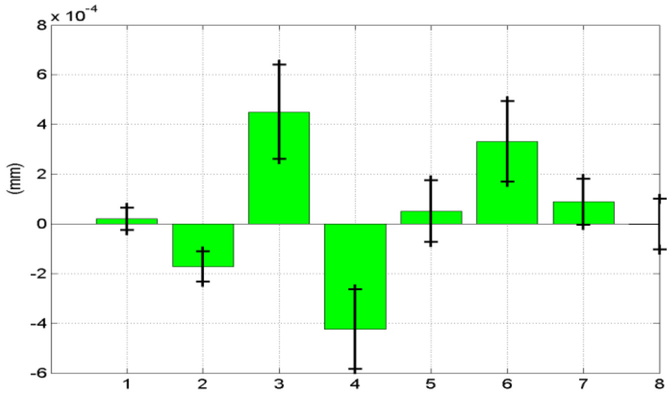
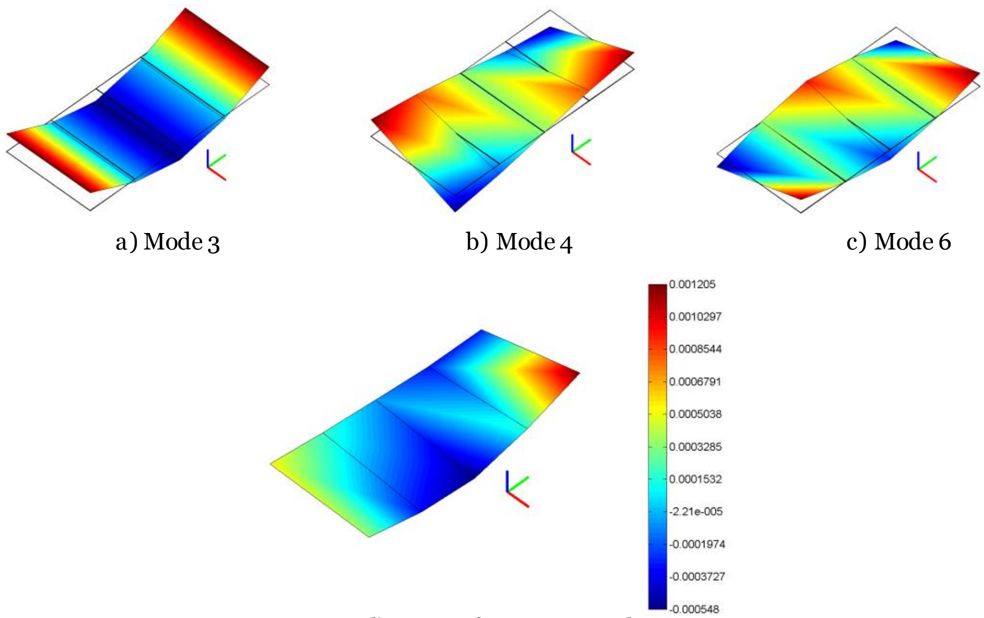


Fig. 2-22 Eight significant modes of the machined planes 2

Components	Machined planes 2		
	Mode 3	Mode 4	Mode 6
Mean *(mm)*	0.451×10^{-3}	-0.423×10^{-3}	0.331×10^{-3}
Variance *(mm²)*	3.601×10^{-8}	2.579×10^{-8}	2.647×10^{-8}

Tab. 2-15 The most significant modes of the planes 2

Fig. 2-23 shows the form defects of the machined planes 2. It is obtained using the sum of the three most significant modes (3, 4 and 6 –Fig. 2-23a, b, c).



a) Mode 3 b) Mode 4 c) Mode 6

d) Mean of reconstructed

Fig. 2-23 Form errors of the machined planes 2

Let us now compare the form errors of the two machined surfaces. They show that:

- Comparing mode 3 and mode 6 of the two machined surfaces, it can be seen that the form errors are more significant on the surfaces 1 than on the surfaces 2.
- The form error of the surfaces 1 (Fig. 2-21c) looks like a part of a cone in which the material near the centre is higher than the outside.

The results show that although the restricted number of measured points used for the form parameterization method, the modes of the form defects are almost obtained. The most important modes are then chosen to analyze the form defects of the machined surfaces. The results of this method are used to complete the results obtained from the method based on the SDT concept.

3.6. Conclusion on the identification of machining defects

We proposed in this section two methods that are used to identify the machining defects. The first method based on the SDT concept allows determining the components of deviation torsor of the machined surfaces. The results obtained from this method are expressed by the statistical parameters: mean variance (standard deviation) and type of distribution (normal, pert). They can then be used for simulating a process plan and detecting the differences of the machining defects that may be due to tool paths, tool and workpiece deflection, etc. The second one is used to complete the results of the first one. This determines the form defects and identifies the types of the form defects (comber, undulation, twist, etc.).

As mentioned earlier, the manufacturing defects are divided into two categories: machining and positioning defects. The machining defects have just been obtained by the experimental application. In the next section, we identify the positioning defects based on the results of this section and a completed measurement conducted on a CMM.

4. IDENTIFICATION OF POSITIONING DEFECTS

4.1. Method

4.1.1. Principle of measuring process

As mentioned earlier, the 50 workpieces are located and fixed on the CNC milling machine by a three soft jaw-chucks (fixture). The objective of the present section is to determine positioning defects of the workpieces on the fixture. For that purpose, besides the measurements of the two machined planes, we measure the workpiece cylinder on the CNC machine and a second measurement on the CMM (MarVision MS222). The two measurements on the two different machines are illustrated as the following figure (Fig. 2-24).

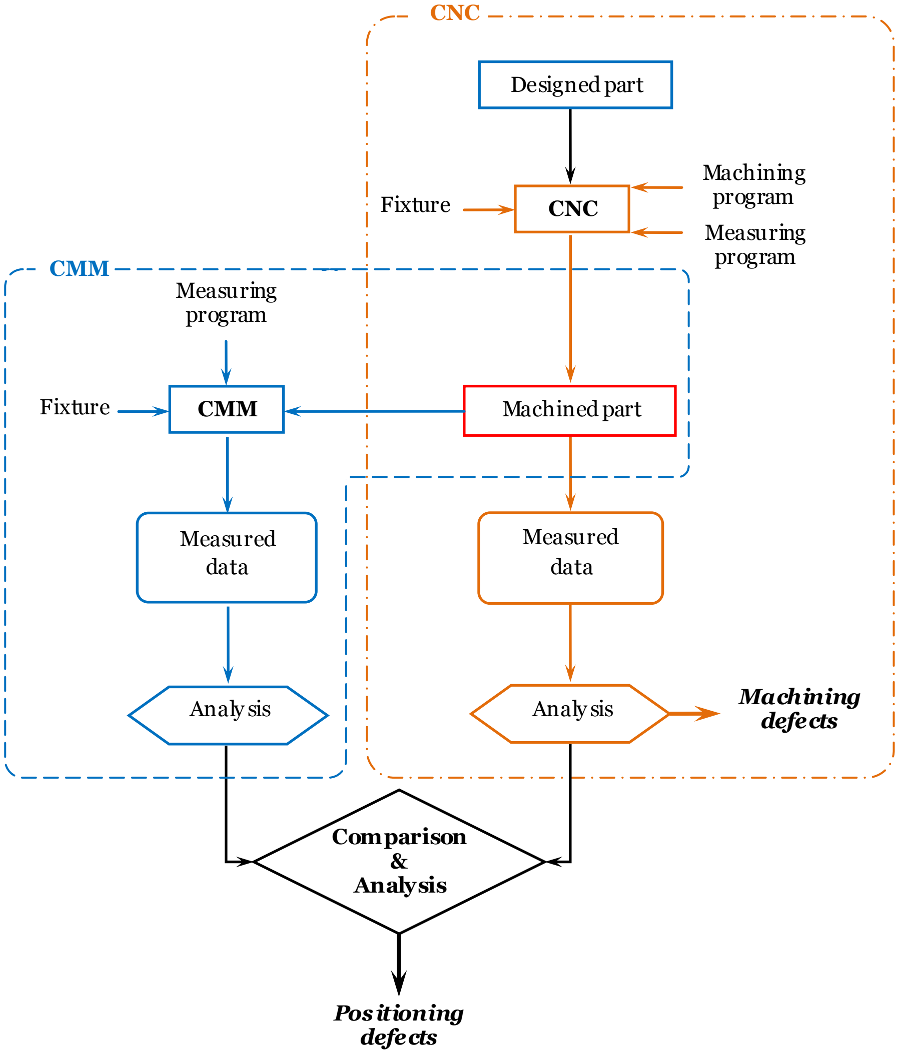


Fig. 2-24 Principal measuring processes

4.1.2. Fixturing and measuring processes

One of the advantages of the measurements on the CMM is a complementary measurement of plane 0 (locating plane) on a workpiece. For this purpose, the fixture that is used to locate and fix the workpieces on the CMM is two V-blocks. Fig. 2-25 shows the two types of fixture and the surfaces that can be measured on the two machines.

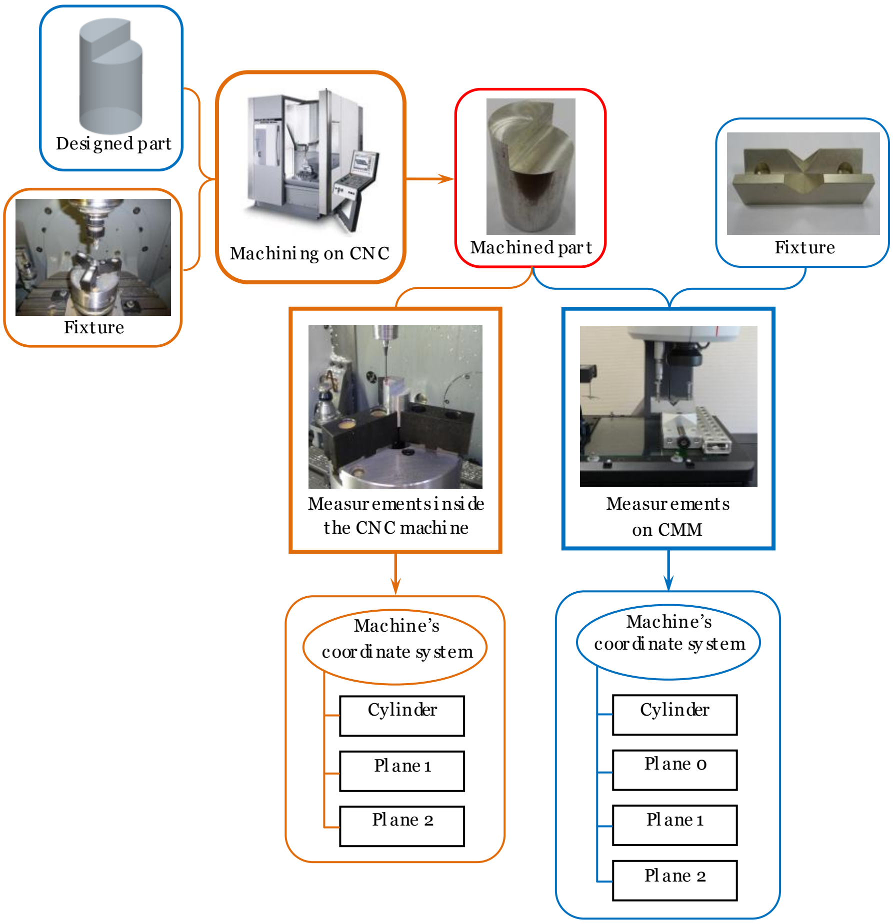


Fig. 2-25 A double measure

4.2. Evaluating comparable capability of measurement results obtained from two measurement means

In order to prove the comparable capability of the measurement results obtained from the two different machines, we compare the machined defects that are obtained by the two machines. To satisfy this need, we need to use the same reference for comparison, for instance, the part's coordinate system (PCS). In fact, we have two different coordinate systems; they are illustrated as Fig. 2-26.

Generally, measured points obtained from a measurement mean are given in the machine coordinate system (MCS). In this study, the MCS is used for analyzing the machining defects on the CNC machine (section 3.3). Whereas, the PCS is used to compare the measurement results obtained from the two measurement means as well as to quantify the positioning defects. Hence, we will focus on two objectives in this section:

- Evaluation of the measurement results obtained from the two measurement means;
- Quantification of the positioning defects based on the measurement results of the two measurement means.

For that purpose, the PCS is considered as a common reference for the two measurement means because it is created from a part's cylinder axis which does not change. However, a PCS created on the CNC machine and the CMM are different (Fig. 2-26). It is explained clearly as follows.

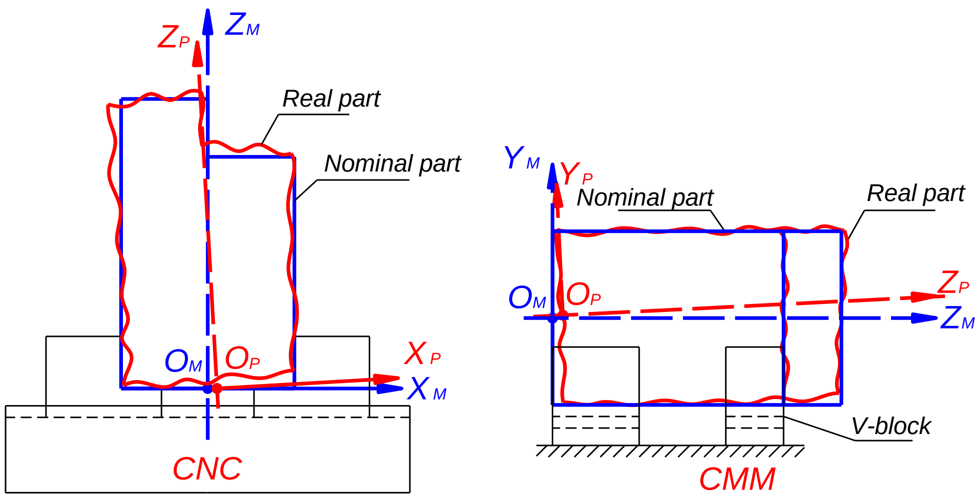


Fig. 2-26 Coordinate systems on CNC and CMM

4.2.1. A PCS on the CNC machine

A PCS is created on the CNC machine by the following steps:

- Locating plane of a workpiece cannot be measured on the CNC machine. Thus, to define the origin of a PCS, the machine plane O_MXY is needed here. This plane is defined by two following steps (without workpiece on the fixture): 1 - Measure locating plane of the fixture; 2 - Set a zero offset for Z axis of the machine on this plane. In other word, the fixture locating plane is considered as the machine plane O_MXY which is a perfect plane.
- A part cylinder is measured to define the Z axis of the PCS (workpiece is fixed on the fixture).
- Intersection point between Z axis and the plane O_MXY defines an origin of the PCS (O_P).

- X and Y axis of the PCS pass through O_P, lie in the planes O_PXZ and O_PYZ that are parallel to planes O_MXZ and O_MYZ of the CNC machine, and are perpendicular to Z axis of the PCS.

4.2.2. A PCS on the CMM

A PCS is created on the CMM machine as follows.

- A machined part cylinder is measured to define the Z axis of the PCS.
- Locating plane of the machined part is measured to define a bottom plane of the part.
- Intersection point between Z axis and the bottom plane defines an origin of the PCS (O_P).
- X and Y axis of the PCS pass through O_P, lie in the planes O_PXZ and O_PYZ that are parallel to planes O_MXZ and O_MYZ of the CMM, and are perpendicular to Z axis of the PCS.

4.2.3. Comparing rotation components of machining defects

Here, we compare the rotation defects of the machined planes that are measured on the CNC machine and the CMM.

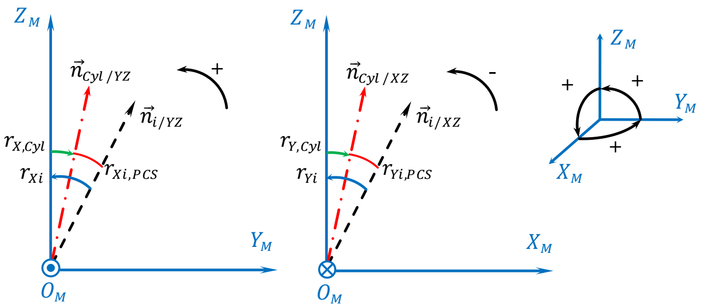


Fig. 2-27 Rotation defects

From Fig. 2-27, the angle between the cylinder axis and the normal vectors of the machined planes is calculated. In other words, rotations of machined planes can be calculated in PCS by the following formula (2-15):

$$\begin{cases} r_{Xi,PCS} = arctan\left(\dfrac{n_{Yi}}{n_{Zi}}\right) - arctan\left(\dfrac{n_{Y,Cyl}}{n_{Z,Cyl}}\right) \\ r_{Yi,PCS} = arctan\left(\dfrac{n_{Xi}}{n_{Zi}}\right) - arctan\left(\dfrac{n_{X,Cyl}}{n_{Z,Cyl}}\right) \end{cases} \qquad (2\text{-}15)$$

where

- $r_{Xi,PCS}, r_{Yi,PCS}$ rotations of machined plane i in PCS
- $\vec{n}_{Cyl}(n_{X,Cyl}, n_{Y,Cyl}, n_{Z,Cyl})$ direction vector of workpiece cylinder axis

The results obtained are shown as Tab. 2-16.

Components		CNC (PCS)	CMM (PCS)	Test of variance
Machined plane 1	$s^2_{r_{X1}}$ *(rad²)*	3.057 E-07	2.558E-07	Δ = 0.498E-07 The test is OK
	$s^2_{r_{Y1}}$ *(rad²)*	2.067 E-07	2.317 E-07	Δ = 0.25E-07 The test is OK
Machined plane 2	$s^2_{r_{X2}}$ *(rad²)*	3.101E-07	2.407 E-07	Δ = 0.693E-07 The test is OK
	$s^2_{r_{Y2}}$ *(rad²)*	2.469E-07	2.379E-07	Δ = 0.095E-07 The test is OK

Tab. 2-16 Rotation defects of the machined planes in the PCS

For each component, a test for equality of variance is used to affirm whether the difference between the two variances obtained from the two machines is significant (or insignificant). If the statement of the test for equality of variances is "OK", it means that the difference between the two observed variances is insignificant and vice versa.

The last column in Tab. 2-16 shows that the difference between two variances ($s^2_{r_{X1}}$, $s^2_{r_{Y1}}$, $s^2_{r_{X2}}$, $s^2_{r_{Y2}}$) obtained from two machines is insignificant. In other words, the results of measurements obtained from the two machines can be combined to determine positioning defects of the workpieces on the CNC machine's fixture. The comparison of translation defects will be considered in the section below.

4.3. Determining translation component of positioning defects

In the present case, translation deviation of a workpiece on the fixture is defined by a deviation along Z-axis between a real workpiece and its nominal one. For the ease of analysis, a translation deviation is determined by a difference between a centroid of the workpiece's locating plane and its nominal one. However, the locating plane of the workpieces cannot be measured on the CNC machine and that is why a supplementary measurement is carried out on the CMM. Comparisons of the measurements (CNC and CMM) are then executed to obtain translation defects. To do this, first, we propose to compare a translation relationship between the two machined surfaces. If it is comparable the translation component of the positioning defects is then determined.

Because of the machining imperfection, the two machined planes may be not parallel. Thus, a translation relationship is used to evaluate a relationship along the part's Z axis between these two planes.

Let $t_{Z01CNC}, t_{Z02CNC}, t_{Z12CNC}$ and $t_{Z01CMM}, t_{Z02CMM}, t_{Z12CMM}$ be translation relationships between plane 0 and plane 1, plane 0 and plane 2, plane 1 and plane 2 measured on the CNC machine and the CMM, respectively.

As mentioned previously, the 0 in t_{Z01CNC},t_{Z02CNC} is a perfect plane so the t_{Z01CNC},t_{Z02CNC} are considered as translation deviations of the machined planes 1 and 2. These deviations can be rewritten as t_{Z1CNC},t_{Z2CNC}. Let $s^2_{t_{Z12CNC}}$ be variance of the sum of the two variables t_{Z1CNC},t_{Z2CNC}.

To illustrate these relationships, two methods are proposed to compare the measurements of the two different machines.

In general, different calculation methods may give different results, namely uncertainty of calculation method. In case the differences between proposed methods are insignificant, they are accepted. Hereafter, two methods are proposed in order to consider the translation relationship of two planes [BUI *et al.* 2011]. They are then assessed to allow us to choose a suitable method.

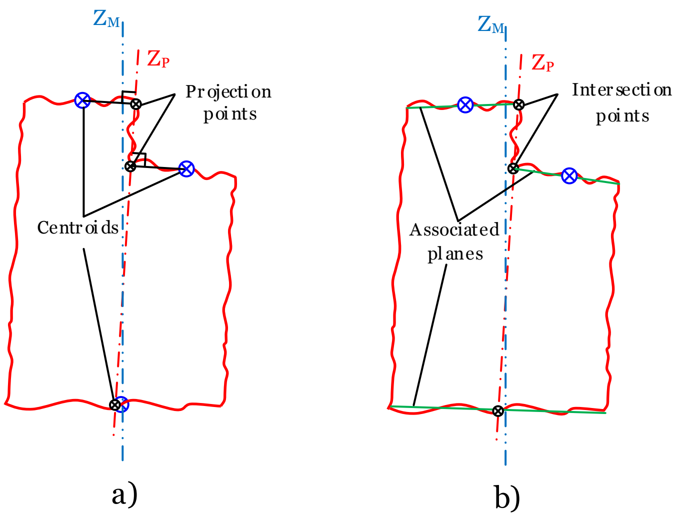


Fig. 2-28 Two proposed methods

4.3.1. *Projection Z_P method*

In this method, the centroid of each associated plane is projected on the part's Z-axis (Fig. 2-28a), namely projection Z_P method. It means that the translation relationship between the two machined planes is expressed by a distance between two projection points of the centroid of the associated planes on the Z-axis of the PCS. The variance of each 50 translation relationship is finally calculated.

4.3.2. *Intersection Z_P method*

The translation relationship of the two machined planes is here expressed by a distance between two intersection points that are intersections of associated planes and the part's Z-axis (Fig. 2-28b), namely intersection Z_P method.

4.3.3. Interpreting results

Hereafter, results of the two different methods are shown and evaluated for choosing the method that will be used in analyzing positioning defects. To assess the results obtained from the methods, the difference (Δ) between $s^2_{t_{Z12CNC}}$ and $s^2_{t_{Z12CMM}}$ determined from the two measurement means is firstly compared. Bartlett's test is then used to test if two samples (t_{Z12CNC}, t_{Z12CMM}) have equal variances.

4.3.3.1. Projection Z_P method

In the first method, the difference (Δ) between $s^2_{t_{Z12CNC}}$ and $s^2_{t_{Z12CMM}}$ obtained from the two machines is insignificant. Additionally, the difference between two variances $s^2_{t_{Z12CNC}}$ and $s^2_{t_{Z12CMM}}$ is significant (Tab. 2-17).

Components	$s^2_{t_{Z01}}$ (mm²)	$s^2_{t_{Z02}}$ (mm²)	$s^2_{t_{Z12}}$ (mm²)
CNC	1.338E-05	1.049E-05	3.450E-05
CMM	4.136E-04	4.578E-04	4.409E-05
Δ	----------		0.096E-05
Test for equality of variances	----------		The test is not OK
$Cov(t_{Z01CNC}, t_{Z02CNC})$	-5.21E-06		

Tab. 2-17 The projection Z_P method

The fifty values of t_{Z01CNC} and t_{Z02CNC} are plotted in Fig. 2-29 in order to verify the results.

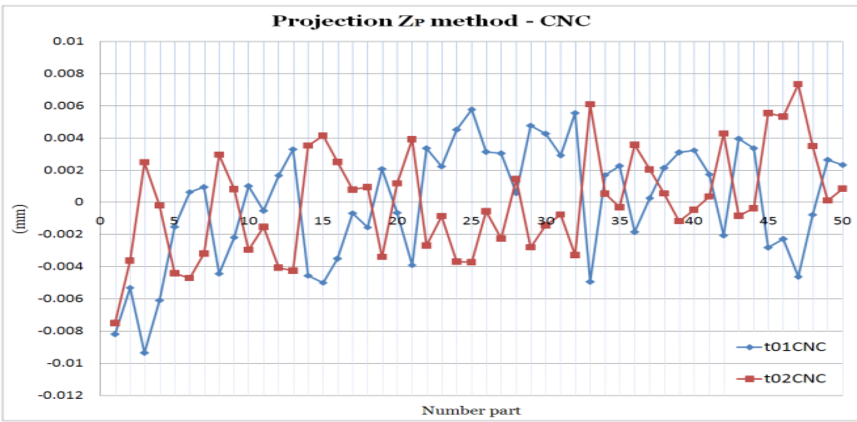


Fig. 2-29 Relationships of two planes using projection Z_P method

It can be seen that the scatter plots of t_{01CNC} and t_{02CNC} appear as two graphics that seem to be symmetrical. It means that when translation defects of the machined planes 1 increase the translation defects of the machined plane 2 will decrease and vice versa. To explain this phenomenon, an assumption is proposed that the machining defects are insignificant with positioning defects of the workpiece cylinders. Thus, centroids of machined planes do not change. Changes of the workpiece cylinders therefore are taken into account as in Fig. 2-30.

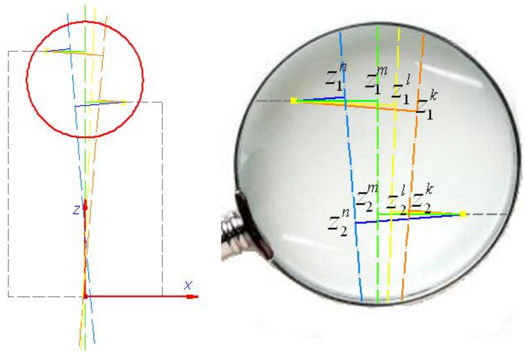


Fig. 2-30 Influences of changes of the workpieces' cylinder on the projection Z_P method (CNC)

Four rotations of the workpieces' cylinder axes (k, l, m, and n) are taken from measurements of the workpieces on the fixture, and the results are shown in Tab. 2-18.

Components	k	l	m	n
t_{Zo1} (mm)	47.27	47.67	48	48.53
t_{Zo2} (mm)	38.48	38.27	38	37.26

Tab. 2-18 Four examples

The results show that the values of the distances t_{Z01CNC} and t_{Z02CNC} are symmetrical. Hence, changes of the workpieces' cylinder axes are the cause of the symmetrical phenomenon in the projection Z_P method.

According to the above analysis, the results obtained from the projection Z_P method can be used for comparison of the measurements results obtained from two measurement means, but it cannot be used for quantification of positioning defects.

4.3.3.2. Intersection Z_P method

Tab. 2-19 shows that the difference (Δ) between $s^2_{t_{Z12CNC}}$ and $s^2_{t_{Z12CMM}}$ obtained from the two machines is insignificant. On the other hand, the difference between two variances is significant.

Components	$s^2_{t_{Z01}}$ (mm²)	$s^2_{t_{Z02}}$ (mm²)	$s^2_{t_{Z12}}$ (mm²)
CNC	40.56E-07	49.59E-07	7.720E-07
CMM	4.229E-04	4.183E-04	7.864E-07
Δ	----------		0.144E-07
Test for equality of variances	----------		The test is not OK
$Cov(t_{Z01CNC}, t_{Z02CNC})$	40.39E-07		

Tab. 2-19 Relationships of two planes using intersection Z_P method

The fifty values of t_{Z01CNC} and t_{Z02CNC} obtained from this method are plotted in Fig. 2-31 to verify the results.

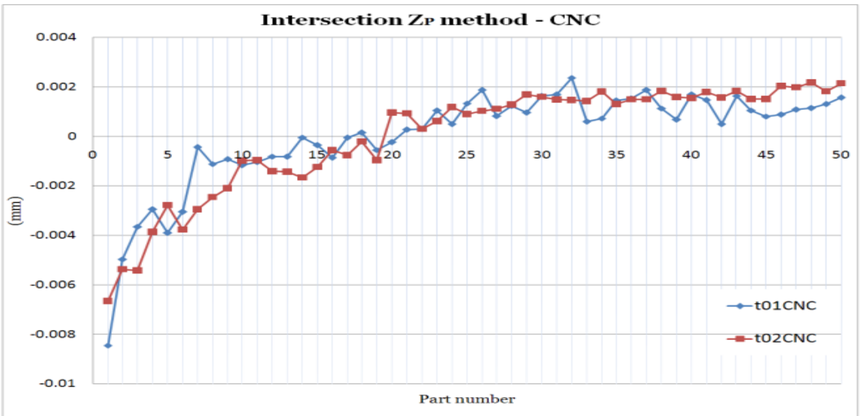


Fig. 2-31 Relationships of two planes using intersection Z_P method

Fig. 2-31 shows the relationship between two planes (plane 0 and 1 or 2) have the same phenomenon (the drift) of the translation deviations of machining defects obtained in section 3.

From comparison of rotations and translation (in the intersection Z_P method) of the two machined planes, we can conclude that the measurement results obtained from the two different measurement means are comparable. Thus, this method can be used for quantification of translation components in positioning defects.

Moreover, Tab. 2-19 shows differences between $s^2_{t_{Z01CNC}}$ and $s^2_{t_{Z01CMM}}$ or between $s^2_{t_{Z02CNC}}$ and $s^2_{t_{Z02CMM}}$. These differences will be considered in the next section to determine the translation defects in the positioning defects.

4.3.4. Translation component of positioning defects

Let $s^2_{t_{Z1}} = s^2_{t_{z01CNC}}$ and $s^2_{t_{Z2}} = s^2_{t_{z02CNC}}$ be translation defects of the machined planes 1 and 2 (because there is no defect of the workpieces' locating plane in this case). In addition, translation defects between two planes ($s^2_{t_{Z01CMM}}, s^2_{t_{Z02CMM}}$) measured on the CMM include defects of the machined planes 1 and 2 (s_1^2,s_2^2) and the workpieces' locating plane (planes 0). They can be expressed in the following equation:

$$\begin{cases} s^2_{t_{Z01CMM}} = s^2_{t_{Z0}} + s^2_{t_{Z1}} & (I) \\ s^2_{t_{Z02CMM}} = s^2_{t_{Z0}} + s^2_{t_{Z2}} & (II) \end{cases} \tag{2-16}$$

where

- $s^2_{t_{Z0}}$ variance of translation defect of the workpieces' locating plane along its Z-axis
- $s^2_{t_{Z1}} = s^2_{t_{z01CNC}} = 4.056 \times 10^{-6}$
- $s^2_{t_{Z2}} = s^2_{t_{z02CNC}} = 4.959 \times 10^{-6}$

So,

$$\begin{cases} s^2_{t_{Z0}} = 4.189 \times 10^{-4} & (I) \\ s^2_{t_{Z0}} = 4.134 \times 10^{-4} & (II) \end{cases} \tag{2-17}$$

Difference between $s^2_{t_{Z0}}$ obtained from (2-16-I) and (2-16-II) is insignificant. Hence, average of (2-17-I) and (2-17-II) can be used to obtain the $s^2_{t_{Z0}}$ as follows:

$$s^2_{t_{Z0}} = 4.161 \times 10^{-4} \tag{2-18}$$

The translation defect of the workpieces' locating plane is here considered as the translation defect of the workpieces on the fixture of the CNC machine. In other words, this is the translation component of the positioning defects. However, the translation defect of the workpieces on the fixture may include contact defects between the workpieces' locating plane and the locating plane of the fixture or errors of the three clamps of the fixture in its base that have not yet been considered in the recent study.

4.4. Determining rotation components of positioning defects

A rotation deviation of a workpiece on the fixture is defined as an angle deviation between axis of real workpiece and its nominal one on the fixture. Variance of the 50 angle deviations is then obtained for a rotation component of the positioning defects.

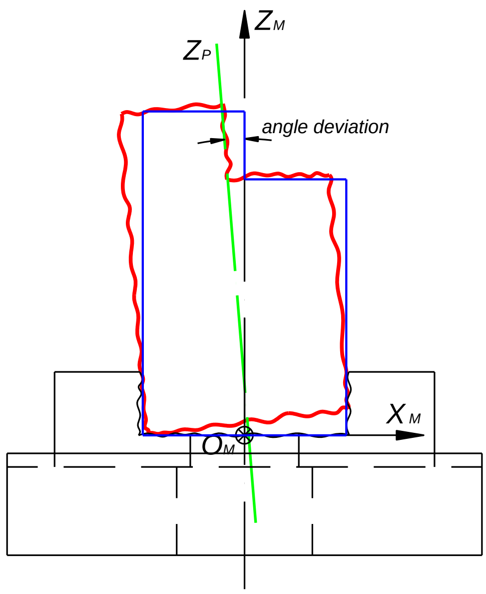


Fig. 2-32 Rotation deviation of a workpiece

The following table shows the results of the rotation component of the positioning defects.

$s^2_{r_X}$ (*rad²*)	$s^2_{r_Y}$ (*rad²*)
3.141E-07	2.235E-07

Tab. 2-20 Rotation component of the positioning defects

4.5. Characterization of positioning defects

The positioning defects of the workpieces on the fixture, which are expressed by the variances of the two rotations around X-axis, Y-axis and the translation along Z-axis, are summarized in Tab. 2-21.

$s^2_{r_X}$ (*rad²*)	$s^2_{r_Y}$ (*rad²*)	$s^2_{t_Z}$ (*mm²*)
3.141E-07	2.235E-07	4.161E-04

Tab. 2-21 Positioning defects of the workpieces on the fixture

The three components of the positioning defects can be also expressed by probability distributions and their statistical parameters in Fig. 2-33.

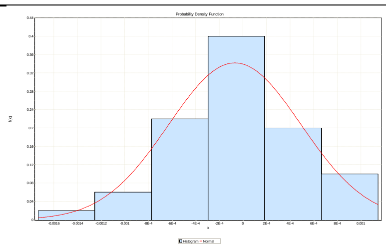
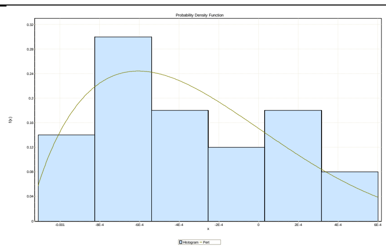
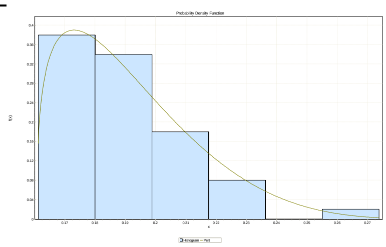
Components	Distribution	Parameters	Histograms
r_X	Normal	s=6.712E-5 m=-5.548E-4	
r_Y	Pert	min=-0.116 E-2 max=0.140 E-2 c=-0.059 E-2	
t_Z	Pert	min=-35.709 E-2 max=124.782 E-2 c=14.301 E-2	

Fig. 2-33 Probability distribution of the positioning defects

4.6. Evaluating position of the workpiece locating plane on the fixture

The objective of this section is to evaluate the position of the workpiece locating planes on the fixture. This allows us to determine whether there is a contact between the workpiece locating planes and the fixture locating plane.

For this purpose, it is assumed that the workpiece locating planes do not have form defects. Thus, geometric defects (two rotations and a translation) of these planes obtained from the measurements on the CMM will be considered in the following analysis. In addition, as mentioned before, the locating plane of the fixture is considered as a perfect plane.

Based on the measurement results obtained from the CNC machine and the CMM, a distance between a workpiece locating plane and the fixture locating plane is obtained as follows.

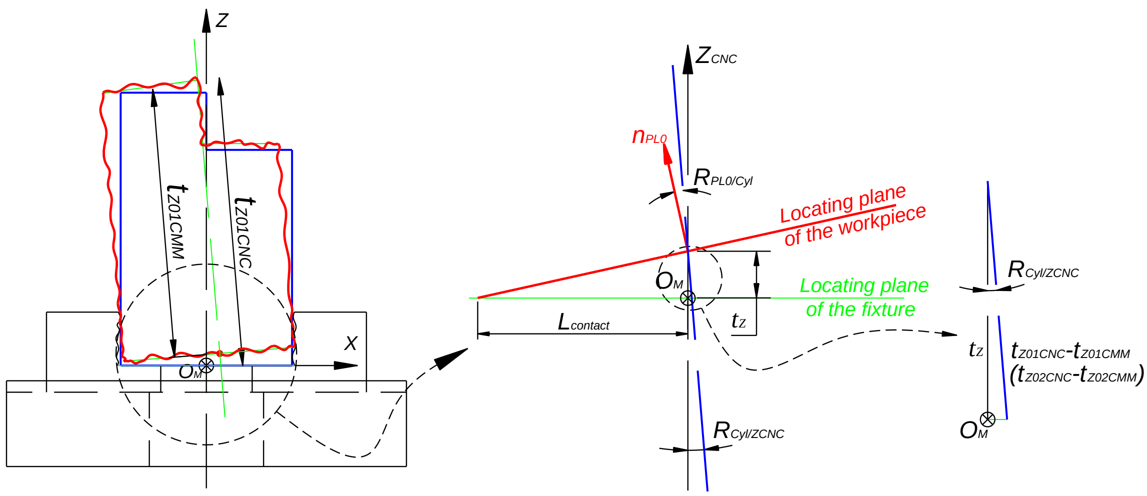


Fig. 2-34 two measurement results of a workpiece

Let t_Z be a translation of centroid of a workpiece locating plane (PL0) compared with the origin of the MCS (O_M). From Fig. 2-34, t_Z can be expressed as

$$t_Z = \delta_1.\cos(R_{Cyl/Z_{CNC}}) \tag{2-19}$$

where:

- $\delta_1 = t_{Z01CNC} - t_{Z01CMM}$ is the difference of translation relationships obtained from the two machines.
- $R_{Cyl/Z_{CNC}}$ is the rotation of the workpiece on the fixture.

Let $L_{contact}$ be a distance from a contact point of the workpiece locating plane and the fixture locating plane to the O_M (centroid of the fixture locating plane). This can be calculated by the following equation:

$$L_{contact} = \frac{t_Z}{\tan\left(R_{PLO/Z_{CNC}}\right)} \tag{2-20}$$

where

- $R_{PLO/Z_{CNC}}$ is the angle between the normal vector of the workpiece locating plane and the Z axis of the MCS.

If $L_{contact} > 15mm$ (radius of the fixture locating plane), the workpiece locating plane does not come into contact with the fixture locating plane. Conversely, the workpiece locating plane come into contact with the fixture locating plane.

The results of this analysis show that there are **61% contacts** and **39% non-contacts** of the 50 workpieces between the workpiece locating planes and the fixture locating plane. It means that during locating and fixing the workpieces on the fixture, the two locating planes of the workpiece and the fixture may not come into contact with each other.

4.7. Conclusion of the identification of the positioning defects

This section shows the method that is used to identify the positioning defects from the results obtained by the two different measurement means. The results are expressed by the statistical parameters: mean, variance (standard deviation), and type of distribution (normal, pert). The positioning defects can then be used to simulate a process plan.

5. ASSESSMENT OF THE MACHINING AND POSITIONING DEFECTS

The following table (Tab. 2-22) synthesizes the machining and positioning defects obtained from the experimental application. Variance of each component in machining and positioning defects is used to assess these defects. In theory probability and statistics, the variance is used as a measure of how far a set of numbers are spread out from each other. Here, a set of numbers is a set of deviations, e.g. rotation or translation deviations of machined surfaces. On the other hand, the machining and positioning defects can also be expressed by a probability distribution and its parameters.

Components	**Machining defects**		**Positioning defects**
	Machined planes 1	Machined planes 2	
s^2_{rX} (rad²)	7.907E-10	6.356E-10	3.141E-07
s^2_{rY} (rad²)	41.714E-10	163.72E-10	2.235E-07
s^2_{tZ} (mm²)	3.711E-6	3.123E-6	4.161E-04

Tab. 2-22 the machining and positioning defects

The results in Tab. 2-22 show that the positioning defects are more significant than the machining defects. Consequently, the proposed method can be used not only to identify machining and positioning defects, but also to detect whether there are differences between the machined surfaces or between machining and positioning defects.

6. SUMMARY

This chapter presented the method based on the SDT concept that can be used to identify the machining and positioning defects. The parameterization method is also presented to complete the analysis of the form defects [SERGENT *et al.* 2010].

Several solutions are proposed in the double measurement method. These provide a simple and effective means:

- Analysing the measurement results obtained from two different measurement means. The advantage of this analysis allows suppressing deviations of the data processing of two different measurement means.
- Evaluating the comparable capability of the measurement results obtained from two different measurement means.
- Determining a translation relationship between two machined planes that may not be parallel because of machining imperfection.
- Explaining the form defects of the machined planes based on the deflection errors of the milling tool and the workpiece.
- Calculating the form defects and orientation defects of the machined planes to complement the explanations of the form defects.
- Verifying whether there is a contact between the workpiece locating plane and the fixture's locating plane.

Furthermore, the experimental results obtained from the application are expressed by the statistical parameters: mean, variance (standard deviation), type of distribution.

The results obtained in this chapter are useful for the simulation (e.g. Monte Carlo simulation) that can be used to predict machining or positioning defects. But in the design of the fixture, it is interesting to estimate the quality of a fixture. This is the reason why we propose simple indicators to estimate the quality of a fixture in the next chapter (chapter 3). In chapter 4, the experimental results will be used to evaluate the capability of a process plan to produce parts compatible with the functional tolerances.

CHAPTER 3

EVALUATING THE QUALITY OF A FIXTURE BY INDICATORS

1. INTRODUCTION

In the previous chapter, we carried out the measurements on the different machines (machine tool, measuring machine) and analyzed the measurement data in order to determine the machining and the positioning defects. The experimental results show that the positioning defects play an important role in the manufacturing process. Nevertheless, the results only showed the positioning defects of the workpieces on the fixture, but the quality of the fixture has not been estimated yet. In this chapter, two simple indicators are proposed for evaluating the quality of a fixture [DURET *et al.* 2010].

Two types of fixture are applied to show analyzed results of the indicators. An experimental fixture is then employed to evaluate the quality of this fixture and compare the results of the two proposed indicators.

In general, different sources can influence the quality of a fixture such as workpiece errors, fixture errors, clamping force, etc. In this chapter, the quality of a fixture is considered based on displacements of the workpiece and/or sensibility of reacting forces at contact points. These will be detailed in the next section.

2. DEFINITION OF FIXTURE QUALITY

In this study, the quality of a fixture is defined by:

- dispersion of the workpiece localization repeatability on a fixture, which is obtained by displacements of the n^{th} workpiece localization in comparison to the 1^{st} workpiece localization (or the theoretical localization of the workpiece on the fixture);
- sensitivity of the fixture that depends on the reacting forces at the contacts between the workpiece and the fixture under an application of external forces (e.g. clamping force).

Consequently, the two following indicators are proposed:

- The first indicator is used to evaluate the quality of a fixture based on the analysis of displacements of a workpiece on a fixture. In this case, a fixture will be considered to be a good fixture if the dispersion of the displacements of the workpiece on the fixture are small.
- The second indicator is used to evaluate the quality of a fixture based on the influences of the reacting forces at the contacts between the workpiece and the fixture. Thus, a fixture will be considered to be a good fixture if the influences of an external force (e.g. clamping force), which is applied to the workpiece, are insignificant on the reacting force at the contacts.

The two proposed indicators are detailed in the next section.

3. PROPOSITION OF INDICATORS

The geometric model of a fixture is established based on the Small Displacement Torsor (SDT) concept. As mentioned previously, a SDT is represented using two vectors: vector $\vec{R}$ includes three small rotations (r_X, r_Y, r_Z) and vector $\vec{T}$ includes three small translations (t_X, t_Y, t_Z).

The following assumptions are used in the investigation of the proposed indicators.

3.1. Assumptions of the study

Three assumptions are considered:

- The fixturing system is not over constrained.
- Contacts between the workpiece and the fixture are considered to be contact points. There are 6 contact points M_i (i = 1, 2... 6) in our application.
- The workpiece and the fixture are considered to be rigid parts (no-deformation).

For each localization of the workpiece, the coordinate of 6 contact points M_i are measured by a CMM. A displacement of each point M_i is considered along its normal vector $\vec{n}_i$ (Fig. 3-1).

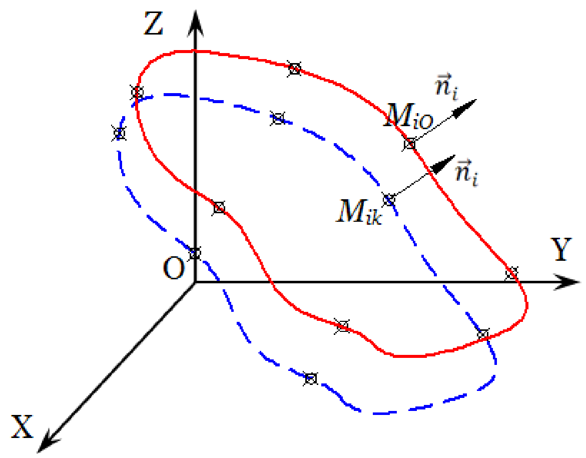


Fig. 3-1 Measured points M_i and its normal vector

3.2. First indicator proposed "Determinant"

3.2.1. Model of displacements

In this model, only one workpiece is used for investigating the workpiece displacements on the fixture. The workpiece is disassembled and reassembled for each new measurement. The displacements of the workpiece on the fixture are defined by the differences between the nth workpiece localization and the 1st workpiece localization (or the theoretical localization which is obtained by a CAD model). If the workpiece displacements are small (compared with its geometric dimension), the workpiece displacements can be modelled by a SDT (3-1).

$$\mathcal{T} = \{\vec{R} \quad \vec{T}\}_{OXYZ} = \begin{Bmatrix} r_X & t_X \\ r_Y & t_Y \\ r_Z & t_Z \end{Bmatrix}_{OXYZ} \tag{3-1}$$

A CAD model is created to initialize the measurement points M_i on the workpiece. The first localization of the workpiece on the fixture (or the theoretical localization that is obtained by a CAD model) is used as a reference OXYZ (Fig. 3-1), it corresponds to:

$$\overrightarrow{OM}_{iO} = (X_{iO}, Y_{iO}, Z_{iO}) \tag{3-2}$$

The new workpiece localization is measured using the same program (six measurement points: from M_1 to M_6). The kth measurement of the workpiece localization is shown as follows:

$$\overrightarrow{OM}_{ik} = (X_{ik}, Y_{ik}, Z_{ik}) \tag{3-3}$$

The kth workpiece localization compares with the reference (0) as follows:

$$\vec{T}_{MiOk} = \overrightarrow{M_{iO} M_{ik}} = (X_{ik} - X_{iO}, Y_{ik} - Y_{iO}, Z_{ik} - Z_{iO}) \tag{3-4}$$

Let δ_{ik} be the displacement of a contact point i at the k[th] measurement along the normal vector $\vec{n}_i$, it can be expressed as the following equation:

$$\delta_{ik} = \vec{n}_i \cdot \vec{T}_{MiOk} \tag{3-5}$$

$$\delta_{ik} = n_{X_i}(X_{ik} - X_{iO}) + n_{Y_i}(Y_{ik} - Y_{iO}) + n_{Z_i}(Z_{ik} - Z_{iO}) \tag{3-6}$$

Following torsor properties, translations of the contact point M_i at the k[th] measurement are expressed:

$$\vec{T}_{MiOk} = \vec{T}_{Ok} + \vec{R}_k \times \overrightarrow{OM}_{iO} \tag{3-7}$$

$$\Leftrightarrow \; \vec{n}_i \cdot \vec{T}_{MiOk} = \vec{n}_i \cdot \vec{T}_{Ok} + \vec{n}_i \cdot (\vec{R}_k \times \overrightarrow{OM}_{iO}) \tag{3-8}$$

$$\Leftrightarrow \; \vec{n}_i \cdot \vec{T}_{MiOk} = \vec{n}_i \cdot \vec{T}_{Ok} + \vec{R}_k \cdot (\overrightarrow{OM}_{iO} \times \vec{n}_i) \tag{3-9}$$

$$\Rightarrow \; \delta_{ik} = \vec{n}_i \cdot \vec{T}_{Ok} + \vec{R}_k \cdot \vec{g}_O \tag{3-10}$$

with $\vec{g}_O = \overrightarrow{OM}_{iO} \times \vec{n}_i$

The equation (3-10) shows how to determine the displacement of a contact point i of the k[th] measurement based on the normal vector of the contact point (translations and rotations of a workpiece on a fixture). It can be rewritten in matrix form as follows:

$$\delta_{ik} = \begin{bmatrix} n_{X_i} & n_{Y_i} & n_{Z_i} \end{bmatrix} \begin{bmatrix} t_{X_k} \\ t_{Y_k} \\ t_{Z_k} \end{bmatrix} + \begin{bmatrix} r_{X_k} & r_{Y_k} & r_{Z_k} \end{bmatrix} \begin{bmatrix} g_{O,X_i} \\ g_{O,Y_i} \\ g_{O,Z_i} \end{bmatrix} \tag{3-11}$$

where $t_{X_{Ok}}, t_{Y_{Ok}}, t_{Z_{Ok}}$; r_{Xk}, r_{Yk}, r_{Zk} (translations and rotations) are unknowns.

so,

$$\delta_{ik} = n_{Xi}.t_{X_k} + n_{Yi}.t_{Y_k} + n_{Zi}.t_{Z_k} + g_{O,X_i}.r_{Xk} + g_{O,Y_i}.r_{Yk} + g_{O,Z_i}.r_{Zk} \tag{3-12}$$

From equation (3-12), displacements of the 6 contact points between the workpiece and the fixture can be expressed in the following matrix:

$$\begin{bmatrix} \delta_1 \\ \delta_2 \\ \delta_3 \\ \delta_4 \\ \delta_5 \\ \delta_6 \end{bmatrix} = \begin{bmatrix} g_{O,X_1} & g_{O,Y_1} & g_{O,Z_1} & n_{X_1} & n_{Y_1} & n_{Z_1} \\ g_{O,X_2} & g_{O,Y_2} & g_{O,Z_2} & n_{X_2} & n_{Y_2} & n_{Z_2} \\ g_{O,X_3} & g_{O,Y_3} & g_{O,Z_3} & n_{X_3} & n_{Y_3} & n_{Z_3} \\ g_{O,X_4} & g_{O,Y_4} & g_{O,Z_4} & n_{X_4} & n_{Y_4} & n_{Z_4} \\ g_{O,X_5} & g_{O,Y_5} & g_{O,Z_5} & n_{X_5} & n_{Y_5} & n_{Z_5} \\ g_{O,X_6} & g_{O,Y_6} & g_{O,Z_6} & n_{X_6} & n_{Y_6} & n_{Z_6} \end{bmatrix} \begin{bmatrix} r_X \\ r_Y \\ r_Z \\ t_X \\ t_Y \\ t_Z \end{bmatrix} \tag{3-13}$$

It can be rewritten:

$$[\boldsymbol{\delta}] = [\boldsymbol{A}][\boldsymbol{\alpha}] \tag{3-14}$$

where

- Matrix $[\boldsymbol{\delta}]$ represents the displacements of the contact points along their normal vectors (obtained by the calculations of the measurement data, equation (3-6));
- Matrix $[\boldsymbol{A}]$ represents a configuration of a fixture. Each configuration of a fixture has an unique matrix $[\boldsymbol{A}]$. (obtained by the initial configuration of the fixture: positions and normal vectors of the contact points);
- Matrix $[\boldsymbol{\alpha}]$ represents the displacements of the workpiece on the fixture (translations and rotations) (obtained by the calculations).

Each matrix $[\boldsymbol{A}]$ has a determinant. Thus, each fixture configuration has an absolute value of the determinant of the matrix $[\boldsymbol{A}]$. In other words, $|\det[\boldsymbol{A}]|$ is constant for each fixture configuration.

From the equation (3-14), if we consider the displacements of the contact points $[\boldsymbol{\delta}]$ are constant (by the given displacements of the contact points) for different fixture configurations, the fixture configuration that gives the smallest displacements of the workpiece $[\boldsymbol{\alpha}]$ will be the best. This fixture is characterized by the maximal value of the $|\det[\boldsymbol{A}]|$.

Following the model of the displacements, the two types of fixture below are employed for evaluating their quality.

3.2.2. Applications

A configuration of a fixture is defined by positions and normal vectors of the contacts between the workpiece and the fixture. Thus, a fixture can have different configurations if the positions and the normal vectors of the contacts are different.

To evaluate the quality of different configurations of a fixture, we propose to analyze this by the following steps:

1. Using the constant displacements of the contact points $[\boldsymbol{\delta}]$ for both fixture configurations;
2. Defining the matrix $[\boldsymbol{A}]$ of different fixture configurations;
3. Calculating the displacements of the workpiece using the matrices $[\boldsymbol{\delta}]$ and $[\boldsymbol{A}]$;
4. Finding the best fixture that has the smallest absolute value of the determinant of the matrix $[\boldsymbol{A}]$.

To do this, two types of the fixture are used in this investigation:

- The first type of fixture is often used to fix a workpiece on a machine tool during manufacturing. The contacts between the workpiece and the fixture are based on three types of contact (plane-line-point). This fixture is called Kelvin fixture.

- The second one is a special fixture that is often used in measurement instruments, for example it is used in probes of CMMs. In general, the quality of this fixture is good and is called Boys fixture.

3.2.2.1. Kelvin fixture

The basic fixture (plane-line-point or 3-2-1) is used to consider the influences of positions of locating points on its quality. Fig. 3-2 shows that the fixture provides a deterministic location for the prismatic workpiece when the workpiece is pushed towards the locators. Rest of the directions for motion would be stopped by a set of clamps. The first five locators (1, 2, 3, 4 and 5) are used to investigate their influences on the quality of the fixture. A coefficient d used to represent the positions of the five locators as in Fig. 3-2. The lateral locating points (4, 5, and 6) are halfway up to the workpiece thickness.

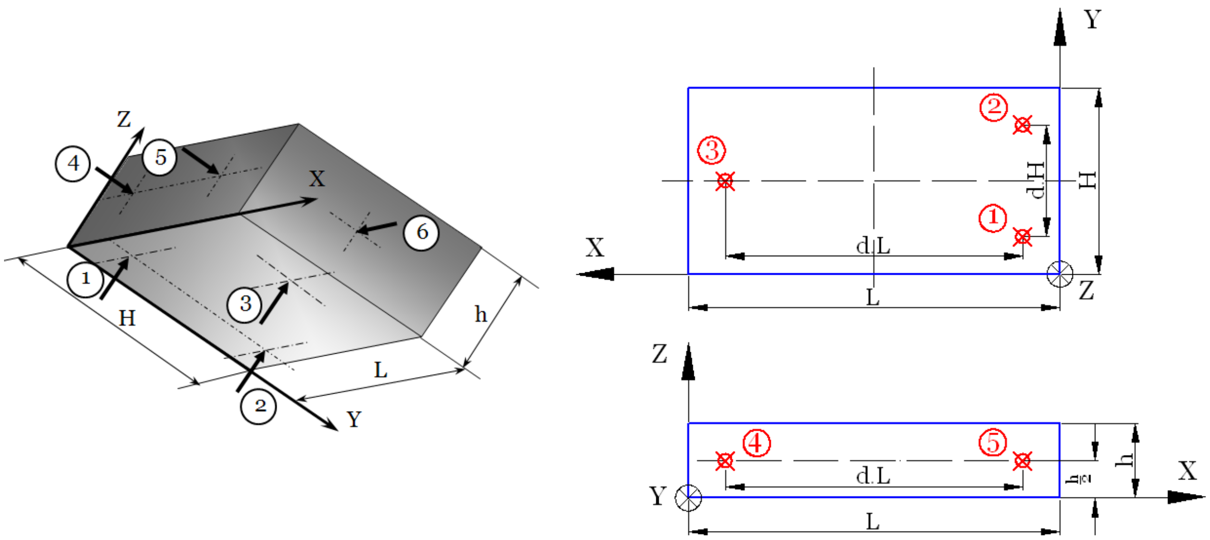


Fig. 3-2 Fixture and coefficient d

Let H = 50 mm, L = 100 mm, h = 20 mm be the dimensions of the workpiece. Now, if the coefficient d is changed, the fixture configuration will change. In other words, the absolute value of the determinant of the matrix $[\boldsymbol{A}]$ will change. If we assume the displacements of the contact points $[\boldsymbol{\delta}]$ are constant for all fixture configurations, the relation of the coefficients d and the $|\det[\boldsymbol{A}]|$ is therefore obtained as in Fig. 3-3.

For an extreme fixture configuration, if d equals zero the fixture will have three locators (1 ≡ 2 ≡ 3, 4 ≡ 5, and 6). The fixture now becomes a spherical joint (ball joint). The $|\det[\boldsymbol{A}]|$ equals zero and this is a bad fixture because it is easier to displace.

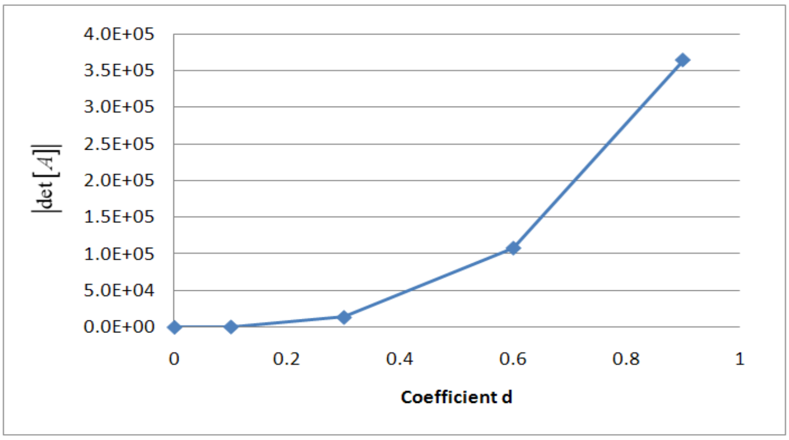


Fig. 3-3 The relation between the coefficient d and the $|\det[\boldsymbol{A}]|$

Fig. 3-3 shows that the larger the three locating points (1, 2, and 3), the greater the absolute value of the determinants.

From the relation of the equation (3-14) and the assumption of the displacements of the contact points $[\boldsymbol{\delta}]$, we conclude that if the absolute value of the determinants of the matrix $[\boldsymbol{A}]$ is great, the displacements of the workpiece on the fixture are small and vice versa.

In conclusion, the first indicator "determinant" can be used to estimate the global quality of a fixture (from the point of view of the displacements). This indicator will be investigated in the second type of fixture as follows.

3.2.2.2. Boys fixture

Another type of fixture is investigated in this example, namely Boys fixture (Fig. 3-4). This fixture is widely used in industry, for example it is used in probes of CMMs. A tri-axes workpiece is located on a fixture that includes three V-blocks. Two geometric parameters of this fixture are used to investigate the influences of different configurations on the fixture quality.

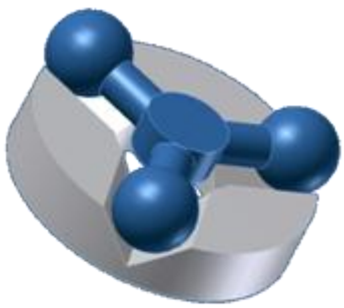


Fig. 3-4 Boys fixture

The two geometric parameters of this fixture are angles of V-blocks and a radius of contact points (Fig. 3-5), in which the radius is measured from the workpiece centre to the contact points, notation V and R (or diameter D).

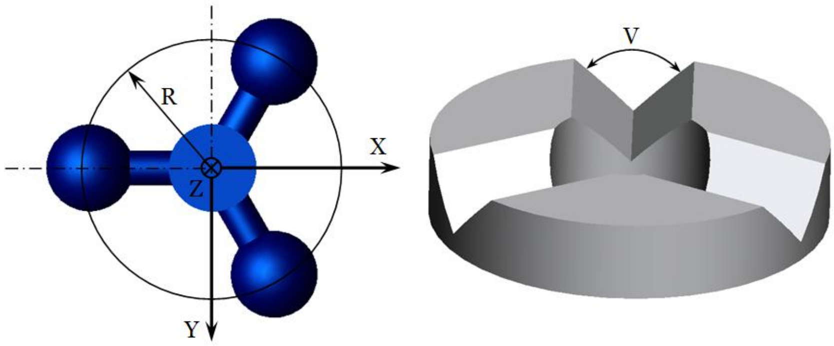


Fig. 3-5 Two geometric parameters of the fixture

Six fixture configurations are used to calculate the absolute values of the determinants of the matrix $[\boldsymbol{A}]$ and the results are shown as in Fig. 3-6. There are two parameters of the angles of the V-blocks (90°, 120°) and three parameters of the radius of the contact points (30mm, 40mm, and 50mm).

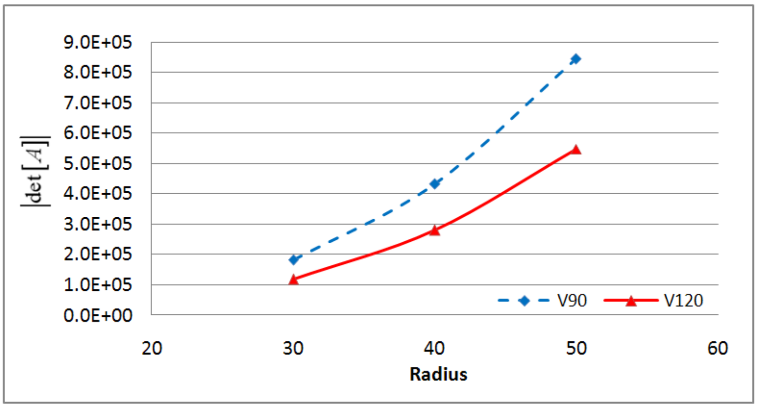


Fig. 3-6 $|\det[\boldsymbol{A}]|$ of different fixture configurations

The results show that the values of the $|\det[\boldsymbol{A}]|$ increase when the distances (radius) of the contact points increase. Inversely, the values of the $|\det[\boldsymbol{A}]|$ decrease when the angles of the V-blocks increase. We can explain as follows:

- The workpiece fixed on the fixture is more stable if the locating points are larger.
- The workpiece is easier to rotate around the Z-axis (Fig. 3-5) if the angles of the V-blocks are larger (e.g. 180°).

3.2.3. Conclusions

In conclusion, the first indicator "determinant" is used to evaluate the global quality of a fixture based on the displacements of the workpiece. The results obtained from the two types of fixture can be summarized by saying that the greater the absolute value of the determinant of the matrix $[\boldsymbol{A}]$, the higher the quality of the fixture.

In the next section, another indicator is proposed to evaluate the quality of the fixture based on the influences of the forces at the contacts between the workpiece and the fixture.

3.3. Second indicator proposed "Coefficient K"

The first indicator investigated the influences of the fixture geometric parameters on the displacements of the workpiece. Here, another indicator is proposed to evaluate the influences of an external force and the fixture geometric parameters on the reacting forces at the contact points. This indicator is applied in the Boys fixture (Fig. 3-4) because the angles of the V-blocks influence the reacting forces at the contact points. The six contact points of this fixture are expressed by Plücker coordinates quickly presented below.

3.3.1. Plücker coordinates

Plücker coordinates are a representation of lines in 3-space. The Plücker coordinates specify lines in 3D space by six-dimensional vectors. Here is common Plücker 3D line computations in which a line is given by:

- two distinct points
- two distinct homogenous points
- two distinct planes
- a 3D point and a direction vector

In this study, a 3D point M_i and a direction vector $\vec{n}_i$ is given to represent lines (L_i) in 3-space. From a contact point M_i of a workpiece and a fixture as in Fig. 3-7a, we can construct a normal line (L_i) which passes through the contact point. Let R(O, X, Y, Z) be a reference of "machine-fixture" as in Fig. 3-7b, the normal line (L_i) can be defined in the Plücker coordinates as follows:

$$L_i = \{\vec{n}_i \quad \vec{n}_i \times \overrightarrow{OM}_i\} \tag{3-15}$$

where, $\vec{n}_i$ is the direction vector (or unit vector) of (L_i).

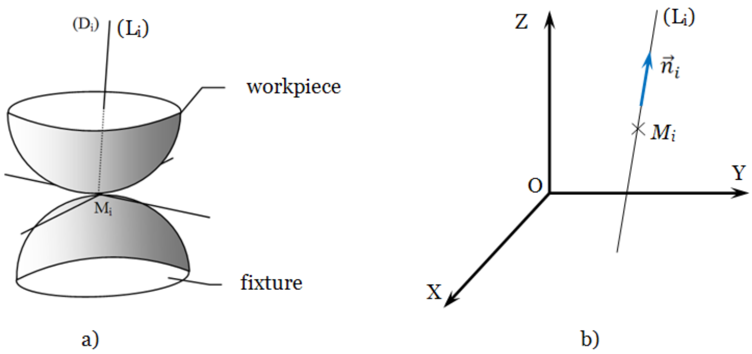


Fig. 3-7 Representation of a line in 3-space

(L_i) can be expressed by 6 scalars in the Plücker coordinates:

$$\mathrm{L_i} = \begin{Bmatrix} n_{X_i} & Z_i n_{Yi} - Y_i n_{Z_i} \\ n_{Y_i} & X_i n_{Zi} - Z_i n_{X_i} \\ n_{Z_i} & Y_i n_{Xi} - X_i n_{Y_i} \end{Bmatrix} \tag{3-16}$$

If $\vec{\hbar}_O = \vec{n}_i \times \overrightarrow{OM}_i$, (3-16) can be written as follows:

$$\mathrm{L_i} = \begin{Bmatrix} n_{X_i} & \hbar_{O,X_i} \\ n_{Y_i} & \hbar_{O,Y_i} \\ n_{Z_i} & \hbar_{O,Z_i} \end{Bmatrix} \tag{3-17}$$

(3-17) used to express one normal line in the Plücker coordinates. If we use a matrix to express 6 normal lines, they are shown as follows:

$$[\boldsymbol{B}] = \begin{bmatrix} n_{X_1} & n_{X_2} & n_{X_3} & n_{X_4} & n_{X_5} & n_{X_6} \\ n_{Y_1} & n_{Y_2} & n_{Y_3} & n_{Y_4} & n_{Y_5} & n_{Y_6} \\ n_{Z_1} & n_{Z_2} & n_{Z_3} & n_{Z_5} & n_{Z_5} & n_{Z_6} \\ \hbar_{O,X_1} & \hbar_{O,X_2} & \hbar_{O,X_3} & \hbar_{O,X_4} & \hbar_{O,X_5} & \hbar_{O,X_6} \\ \hbar_{O,Y_1} & \hbar_{O,Y_2} & \hbar_{O,Y_3} & \hbar_{O,Y_4} & \hbar_{O,Y_5} & \hbar_{O,Y_6} \\ \hbar_{O,Z_1} & \hbar g_{O,Z_2} & \hbar_{O,Z_3} & \hbar_{O,Z_4} & \hbar_{O,Z_5} & \hbar_{O,Z_6} \end{bmatrix} \tag{3-18}$$

In addition, the column-rank of matrix $[\boldsymbol{B}]$ can be calculated to check the independent of the six normal lines, notation r. The matrix is full rank if all of its columns are independent. A simple test for determining if the matrix is full rank is to calculate its determinant. If the determinant is zero, there are linearly dependent columns and the matrix is not full rank.

Thus, matrix $[\boldsymbol{B}]$ is used to present the six contact points (normal lines) of the fixture in 3-space. This will be used in a model of the reacting forces at the contact points in the next section.

3.3.2. Model of reacting forces

The workpiece fixed on the fixture is in static equilibrium when the reacting forces $\vec{F_i}$ at the contacts and the external force $\vec{P}$ (e.g. clamping force) are balanced (Fig. 3-8).

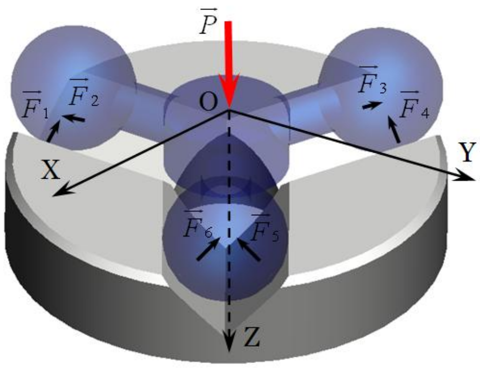


Fig. 3-8 Reacting forces and clamping force

The resultant on the workpiece can be shown as follows:

$$\sum_i F_i \cdot \vec{n}_i = -\vec{P} \tag{3-19}$$

$$\sum_i \overrightarrow{OM}_i \times F_i \cdot \vec{n}_i = -\overrightarrow{OM}_P \times \vec{P} \tag{3-20}$$

$$\Leftrightarrow \sum_i \left(\vec{n}_i \times \overrightarrow{OM}_i\right) \cdot F_i = \overrightarrow{OM}_P \times \vec{P} \tag{3-21}$$

where

- O is the origin of the workpiece-fixture coordinate system.
- M_i is the contact point i.
- M_P is the point of application of the clamping force $\vec{P}$.

Equations (3-19) (force-balance equation) and (3-21) (moment-balance equation) can be rewritten in the matrix form as follows:

$$\begin{bmatrix} \vec{n}_i \\ \vec{n}_i \times \overrightarrow{OM}_i \end{bmatrix} \cdot \begin{bmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \\ F_5 \\ F_6 \end{bmatrix} = \begin{bmatrix} P_X \\ P_Y \\ P_Z \\ Y_P.P_Z - Z_P.P_Y \\ Z_P.P_X - X_P.P_Z \\ X_P.P_Y - Y_P.P_X \end{bmatrix} \tag{3-22}$$

$$\Leftrightarrow [\boldsymbol{B}] \cdot \begin{bmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \\ F_5 \\ F_6 \end{bmatrix} = \begin{bmatrix} P_X \\ P_Y \\ P_Z \\ Y_P.P_Z - Z_P.P_Y \\ Z_P.P_X - X_P.P_Z \\ X_P.P_Y - Y_P.P_X \end{bmatrix} \tag{3-23}$$

so,

$$\begin{bmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \\ F_5 \\ F_6 \end{bmatrix} = [\boldsymbol{B}]^{-1} \cdot \begin{bmatrix} P_X \\ P_Y \\ P_Z \\ Y_P.P_Z - Z_P.P_Y \\ Z_P.P_X - X_P.P_Z \\ X_P.P_Y - Y_P.P_X \end{bmatrix} \tag{3-24}$$

In calculations of F_i, the angles of the V-Blocks are an important factor (corner effect). For instance, if we use the same clamping force $\vec{P}$ to fix the workpiece on the two fixtures with the two different angles of the V-blocks (90° and 120°) the reacting forces at the contact points will be different (Fig. 3-9).

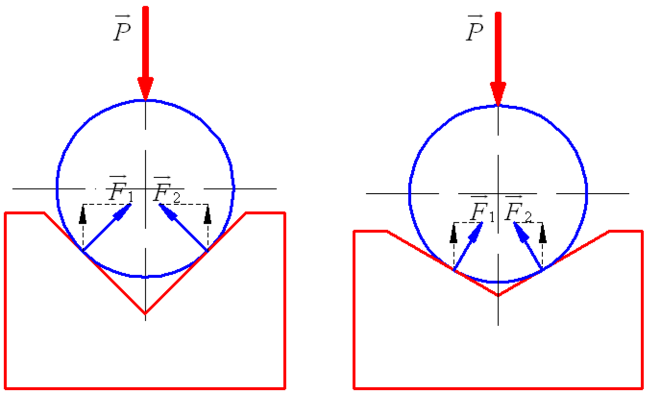


Fig. 3-9 Reacting forces of different angles of the V-block

From (3-24), matrix $[\boldsymbol{B}]^{-1}$ (or $[\boldsymbol{B}]$) presents the fixture configuration. It means each fixture configuration has a unique matrix $[\boldsymbol{B}]^{-1}$ (or $[\boldsymbol{B}]$). Based on this matrix, we propose the second indicator in which the norm of matrix $[\boldsymbol{B}]$ is used instead of the determinant of the matrix as in the first indicator.

In general, a norm of a matrix is not always calculated. For instance, we cannot calculate a norm of a non-homogenous matrix. In this case, we can calculate the pseudo-Euclidean norm instead of the norm of this matrix.

Consequently, we propose to calculate the pseudo condition of matrix $[\boldsymbol{B}]$ which is the product of pseudo-Euclidean norms of matrix $[\boldsymbol{B}]$ and $[\boldsymbol{B}]^{-1}$. It is named coefficient K and is calculated as follows:

$$\mathrm{K} = \|\boldsymbol{B}\|_E . \|\boldsymbol{B}^{-1}\|_E \tag{3-25}$$

where

- $\|\boldsymbol{B}\|_E, \|\boldsymbol{B}^{-1}\|_E$ are the pseudo-Euclidean norm of $[\boldsymbol{B}]$ and $[\boldsymbol{B}]^{-1}$, respectively.

$$\|\boldsymbol{B}\|_E = \sqrt{\sum_i \sum_j b_{ij}} \tag{3-26}$$

with b_{ij} is the value in the i^{th} row, j colum of $[\boldsymbol{B}]$.

From the matrix (3-24), the reacting forces at the contact points depend on the fixture configurations if the point of application of the clamping force does not change and the clamping force is constant. To see this, the Boys fixture is employed to calculate the coefficient K of different fixture configurations.

3.3.3. Application

Six fixture configurations of the Boys fixture (Tab. 3-1) are used to evaluate the fixture quality based on the coefficient K.

Parameters	The distances of the contact points R		
The angles of the V-blocks	V90R30	V90R40	V90R50
	V120R30	V120R40	V120R50

Tab. 3-1 Six fixture configurations

Fig. 3-10 shows the values of the coefficient K of different fixture configurations.

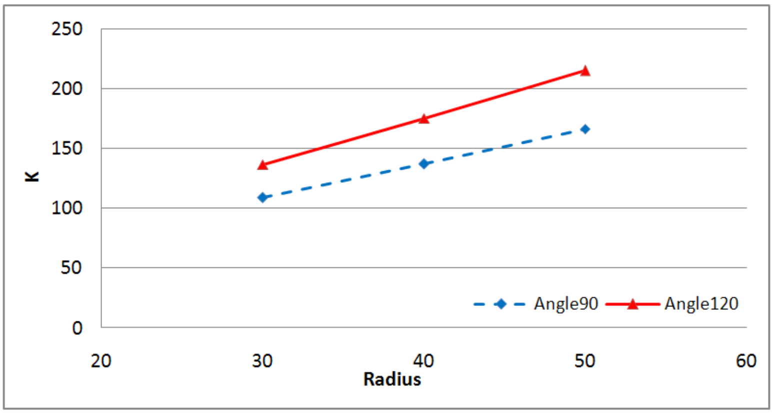


Fig. 3-10 The coefficient K of different fixture configurations

The results show that:

- "The larger the distances of the contact points, the greater the coefficient K". This can be explained by the stability of the fixture. It means that the workpiece fixed on the fixture is more stable if the locating points are larger.
- "The greater the angles of the V-blocks, the greater the coefficient K". This can be understood that the angles of the V-blocks influence the reacting forces at the contact points (Fig. 3-9). As mentioned earlier, a fixture is considered to be a good fixture if the influences of an external force (e.g. clamping force) are insignificant on the reacting force at the contacts.

Lastly, a great value of the coefficient K indicates that the influences of the external force (clamping force) on the reacting forces at the contact points between the workpiece and the fixture are insignificant and vice versa.

3.4. Conclusions

The two proposed indicators provide a method for evaluating the global quality of a fixture. This evaluation method can be used in fixture design for a preliminary estimation of fixture configurations.

To verify the proposed indicators, an experimental fixture is employed in the next section.

4. EXPERIMENTAL APPLICATION

4.1. Experimental fixture

An experimental fixture (Fig. 3-11) is measured. The measurement data are then analysed to evaluate the positioning defects of the workpiece on the fixture. This fixture is used to locate and hold a tri-axes workpiece. The experimental results are then used to compare the above theoretical results.

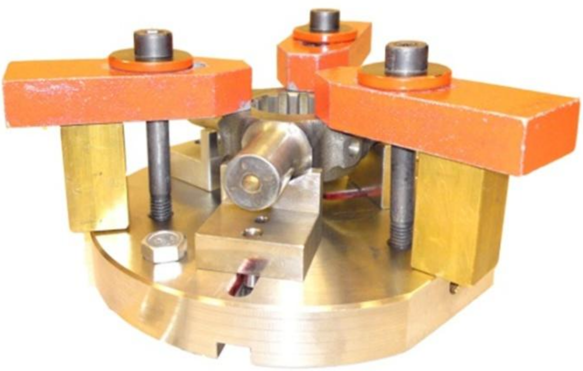


Fig. 3-11 Experimental fixture

The workpiece is fixed on the three short V-blocks that have two geometric parameters. They are the angles of V-blocks (V) and their positions (R). Their notations and values are shown in Tab. 3-2 and Fig. 3-12.

Angle of V-Blocks	Radius	
	33.5 *(mm)*	55 *(mm)*
90°	V90R1	V90R2
120°	V120R1	V120R2

Tab. 3-2 Geometric parameters of the fixture and their notations

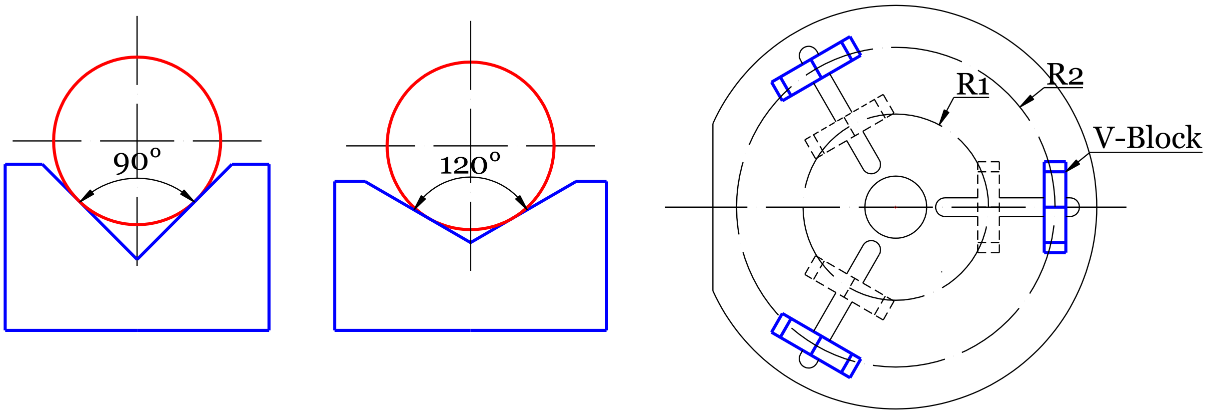


Fig. 3-12 Different fixture configurations

4.2. Measuring procedure

4.2.1. Measured points

In the first indicator, the displacements of the contact points are used to calculate the displacements of the workpiece on the fixture. However, the contact points cannot be measured in reality. Thus, we propose to measure two points on each trunnion of the workpiece for evaluating the workpiece displacements as in Fig. 3-13.

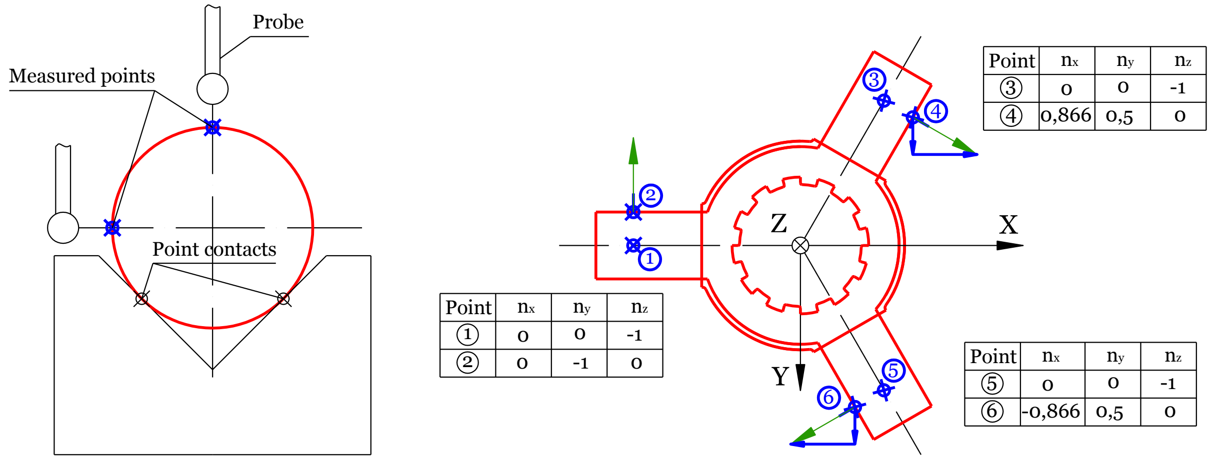


Fig. 3-13 Measured points on the workpiece

It can be noted that the normal vectors of the measured points will not change if the fixture configuration is changed (the angle of the V-blocks).

4.2.2. Measuring process

The workpiece is fixed on the fixture by three clamps in order to ensure the workpiece does not displace during the measurements. The clamping force is constant for all of the fixture configurations by a spring system as in Fig. 3-14.

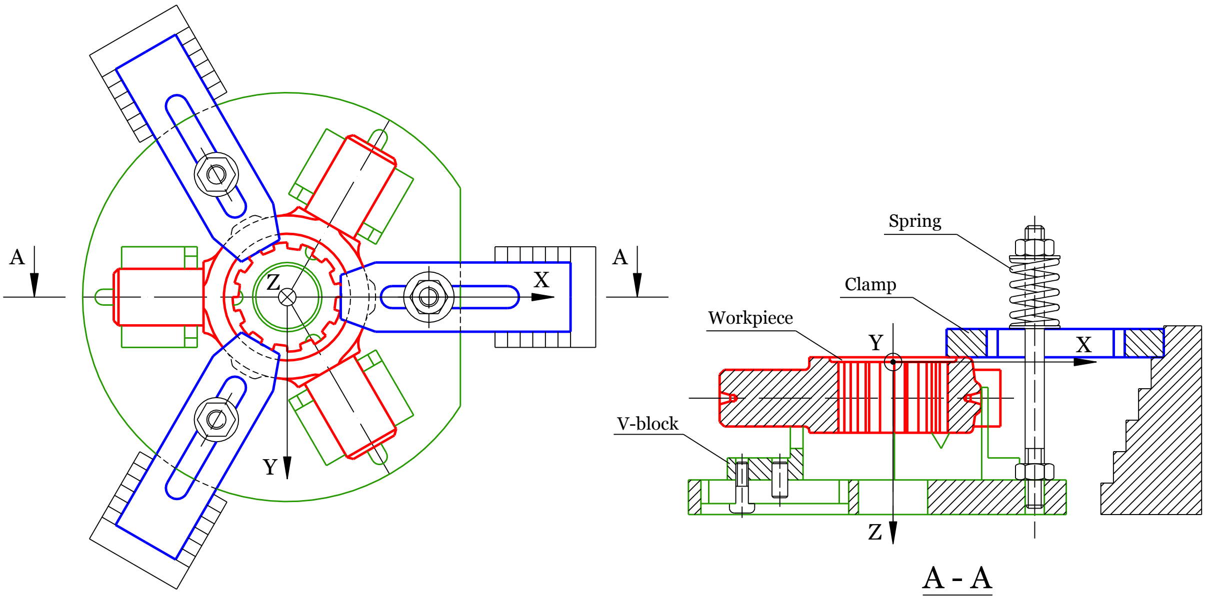


Fig. 3-14 Clamping the workpiece on the fixture

The coordinate system of the measuring program is created from the workpiece centre and the top surface of the workpiece (Fig. 3-14). The first installation of the workpiece is used to create this coordinate system.

The workpiece is then measured one hundred times for each fixture configuration. The workpiece is disassembled and reassembled for each new measurement. It is important to note that only one workpiece is used for the measurements.

4.2.3. Coordinate Measuring Machine

The CMM that is used in this study has the following technical data:

• Sip Orion

- *Technology:* *Touch probe*
- *Resolution:* *0.1 µm*
- *Precision:* *0.8 µm +L÷800*

4.3. Result analysis

As mentioned earlier, we consider that the displacements of the contact points correspond to the displacements of the measured points. Hence, from the relations of the equation (3-14) ($[\boldsymbol{\delta}] = [\boldsymbol{A}][\boldsymbol{\alpha}]$), we can obtain the displacements of the workpiece ($r_X, r_Y, r_Z, t_X, t_Y, t_Z$) on the fixture.

Fig. 3-15 and Fig. 3-16 show the standard deviations of the translations and the rotations of the workpiece fixed on the different fixture configurations that correspond to the matrix $[\boldsymbol{A}]$ in the equation (3-14). In this matrix, only the measured points on the workpiece change for each fixture configuration, meanwhile the normal vectors of the measured points do not change.

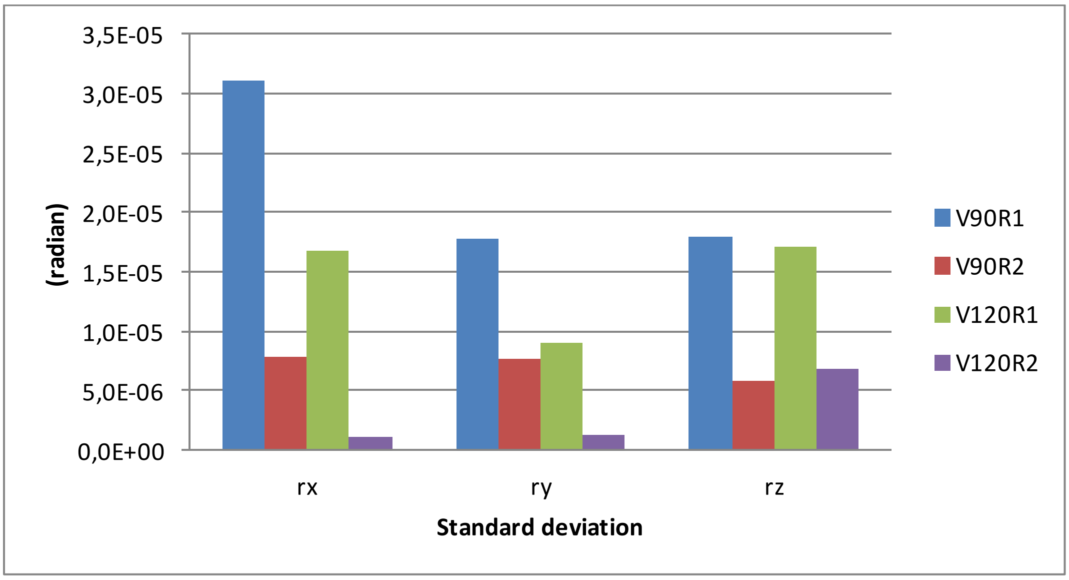


Fig. 3-15 Workpiece rotations on different fixture configurations

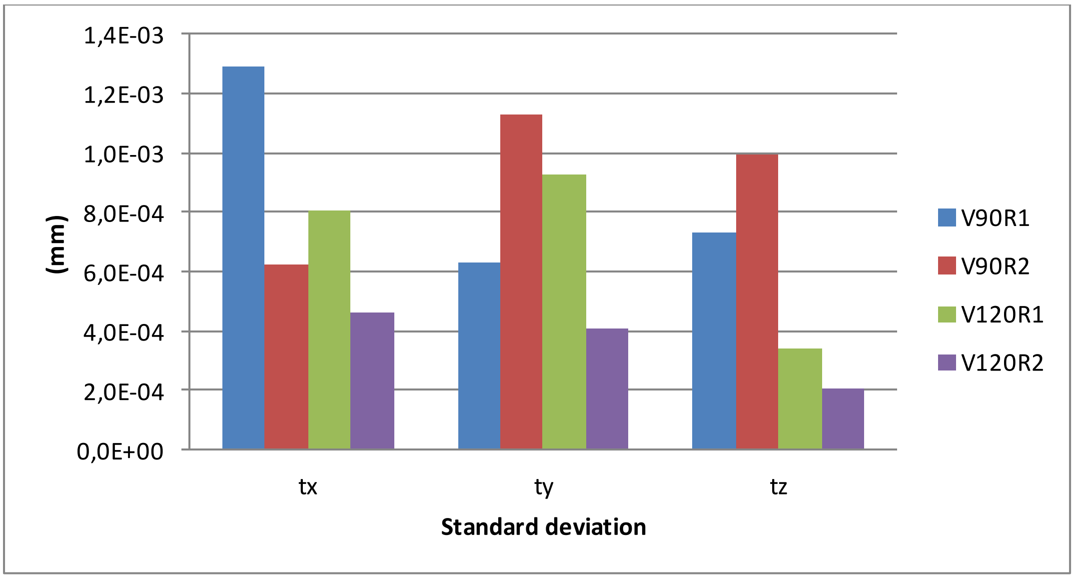


Fig. 3-16 Workpiece translations on different fixture configurations

The results show that:

- The greater the angle of the V-blocks, the smaller the displacements of the workpiece on the fixture (except t_Y and r_Z).
- The greater the distances between the contact points, the smaller the displacements of the workpiece on the fixture (except t_Y and t_Z).

In conclusion, for the parameter of the distance between the contact points (R), the experimental results appropriate the theoretical results for the two indicators. However, for the parameter of the angle of the V-blocks, the experimental results and the theoretical results are inappropriate. In this case, the second indicator "coefficient K" is proposed to use for evaluation of the quality of the fixture because the effects of the reacting forces at the contact points always exist in reality.

5. SUMMARY

In this chapter, we proposed two indicators for evaluating the quality of a fixture [DURET *et al.* 2010].

- In the first indicator "determinant", the quality of a fixture is evaluated based on the dispersions of the workpiece displacements on the fixture. This is represented by the absolute value of the determinant of the fixture configuration matrix.
- In the second indicator "coefficient K", the quality of a fixture is evaluated based on the sensibility of the reacting force at the contact points between the workpiece and the fixture. This is represented by the coefficient that is calculated from the norm of the fixture configuration matrix.

It is important to note that the fixture configuration matrices used in the calculation of the first indicator and the second one are not the same.

The two types of fixture are applied to show the results of the analyses from the two indicators.

An experimental fixture is then analysed to compare the experimental results with the results obtained from the two indicators. The comparison shows that the results obtained from the experimental fixture can be explained by the combination of the two indicators. Thus, a fixture is considered to be a good fixture if the displacements of the workpiece on the fixture are small and the influences of external force (clamping force) on the reacting forces at the contact points are insignificant (in other words, the angles of the V-blocks are great).

In addition, the simple indicators that are proposed can be integrated into software in order to choose quickly a fixture configuration from different geometric parameters.

CHAPTER 4

SIMULATION OF TOLERANCE ANALYSIS IN MANUFACTURING

1. INTRODUCTION

An essential step in manufacturing is to evaluate the quality of products in terms of functional tolerances. During manufacturing, a product can pass through one or several processes, machine tools, fixtures, cutting tools and the sequencing of operations that are called the process plan. Consequently, evaluation of a process plan in terms of functional tolerance is a direct step to control the quality of products, which is also known as tolerance analysis. In other words, controlling the quality of products is verifying whether the design tolerance requirements meet a given process plan with specified manufacturing deviations.

In the last two chapters, we presented the methods and the indicators for determining and evaluating manufacturing defects. The obtained results allow us to express machining defects and positioning defects as distributions with the associated statistics. This chapter presents a mathematical model for tolerance analysis based on Model of Manufactured Part (MMP). In this model, the experimental results obtained from chapter 2 are used as input variables for simulating manufacturing defects (machining and positioning defects) during design phases of a process plan. Nevertheless, to obtain the experimental results, we need to have appropriate equipment and time. In other words, they are very costly and time-consuming. To solve these difficulties, simulated defects presented in this chapter can also be used as input variables. The simulations allow to:

- verify the process plan in terms of functional tolerances
- or determine minimum tolerances of a batch of machined part based on an existing process plan and number of rejected parts per million (ppm).

In manufacturing, the process plan covers the selection of processes, machine tool, fixture, cutting tool and the sequencing of operations required to transform a workpiece into a finished product. During these processes, the workpiece can be located and fixed by one or several fixtures on one or more machine tools. Measuring instruments or test devices can be used for getting desired dimensional control and the required degree of surface finish on the workpieces.

In addition, to reduce cost and time, we present a model that is used to simulate positioning defects of the workpiece on the fixture in this chapter. The model is created by combining of a full factorial design and finite element models. The simulated results can be used in the above model for evaluating a process plan.

Consequently, the works carried out in this chapter can be summarized in the following diagram:

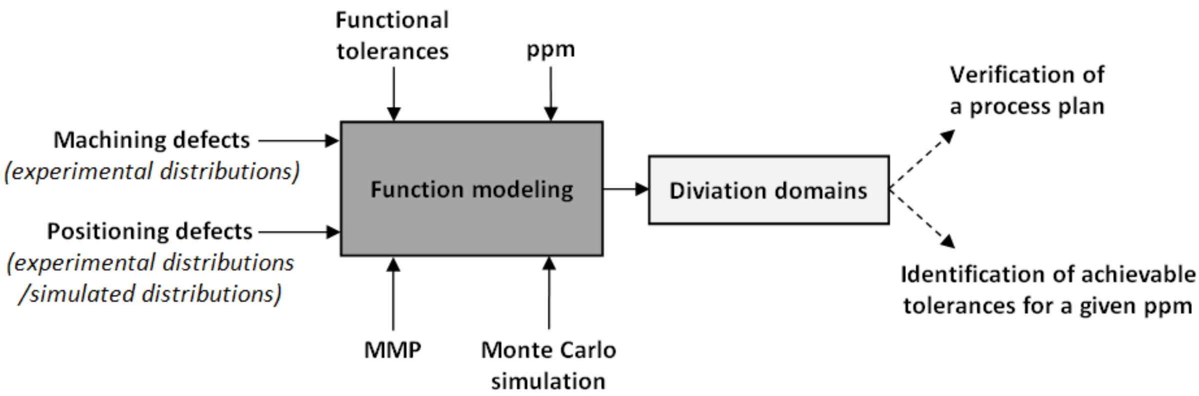


Fig. 4-1 Algorithms of tolerance analysis

Before going into the analyses, it is essential to remind the MMP and some definitions, such as positioning, machining errors, propagated variation, deviation zone, and tolerance zone in the next section.

2. SEVERAL DEFINITIONS

2.1. Errors on a machining operation

Because of the manufacturing imperfection, the error sources in machining operation can be classified as fixture errors, datum errors (workpiece errors), and machine tool errors. From another point of view, these errors can be classified as two types of errors: machining errors and positioning errors. The errors are illustrated clearly by the following example [LOOSE *et al.* 2007a]. The workpiece is represented as a metal cube, which passes through two setups in a machining operation. Geometric tolerancing of the workpiece is perpendicularity between the drilled hole and surface D; it is shown in Fig. 4-2.

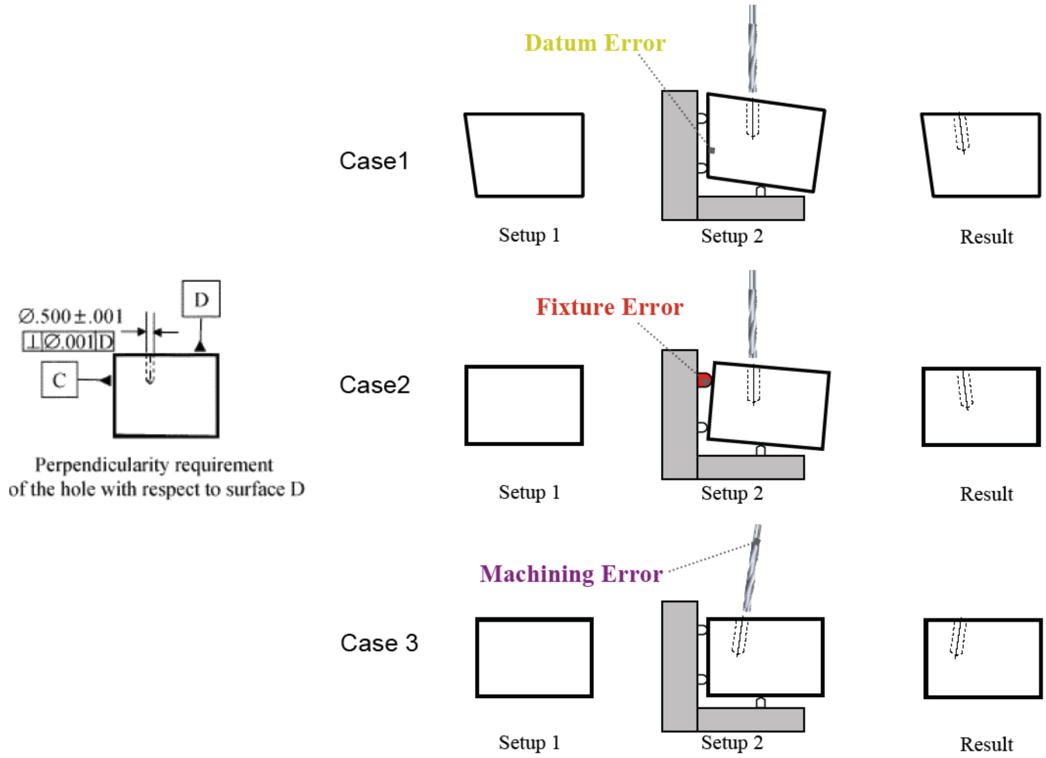


Fig. 4-2 Different errors can occur in a machining operation [LOOSE *et al.* 2007a]

2.1.1. *Positioning errors*

The positioning errors are illustrated in the two first cases (Fig. 4-2).

- In case 1, the positioning errors of the workpiece on the fixture are caused by datum errors, which are produced at the previous setup when the workpiece is prepared. Hence, the quality variation in this case is a propagated variation.
- In case 2, instead of datum errors the fixture locator errors are present. Because of the locator errors, surface D of the workpiece is not perpendicular to the tool path. As a result, the hole generated is not perpendicular to surface D.

From this example, it is clear that errors from the fixture or the workpiece are transferred into the newly generated surfaces.

2.1.2. *Machining errors*

In case 3 (Fig. 4-2), the workpiece position on the fixture is considered to be perfect. Nevertheless, errors of the machined surface can occur by errors of the machine tool. Many key factors can affect the accuracy of the machine tool, such as cutting tool (tool deflection, tool wear, and tool-path), cutting force, machining conditions, thermal errors, etc.

2.2. Propagated variation

Generally, a machined part passes through several machining operations, which are also known as a multistage machining process. The errors of each stage accumulate on the workpiece that will affect the machining accuracy at a subsequent stage. The machining errors of previous stages are known as propagated variation. Clearly, the propagated variation exists because part features produced at previous stages are used as the machining datum in the current operation.

Several models have been proposed to study the propagated variation in multistage machining processes. A state space model is proposed by [HUANG *et al.* 2000] to describe dimensional deviations in multistage machining processes. [ZHONG *et al.* 2002] proposed a model to study the variation propagation including workpiece deformation. Most of these models are used for orthogonal fixture layout, e.g. 3-2-1 fixture layout, and cannot be applied to a general fixture layout.

2.3. Model of Manufactured Part (MMP)

Villeneuve and Vignat [VIGNAT 2005, VIGNAT *et al.* 2007, VIGNAT *et al.* 2009, VILLENEUVE *et al.* 2004, 2007] proposed a method for modeling the geometrical and dimensional deviations produced in a multistage machining process, where propagated variations in the previous setup are also considered in further setups. A model obtained from this method is then used for simulation and evaluation of manufacturing processes, namely Model of Manufactured Part (MMP).

Deviations of surfaces on the MMP are described based on the SDT concept. For instance, deviations of a machined plane are expressed by two rotations and a translation that are measured in relations between an associated plane to the real surface and its nominal one. To better explain an example of the MMP, positioning deviations and machining deviations will be reminded as follows.

It is important to note that the defects generated by a machining process are considered the result of two independent phenomena in the MMP: the positioning and machining defects.

2.3.1. Positioning deviation

If a wokpiece is located and fixed on a fixture, positioning deviation is defined as differences between the real position of the workpiece and the nominal one on the fixture. The positioning deviations are due to geometric errors of the workpiece and fixture (Fig. 4-3).

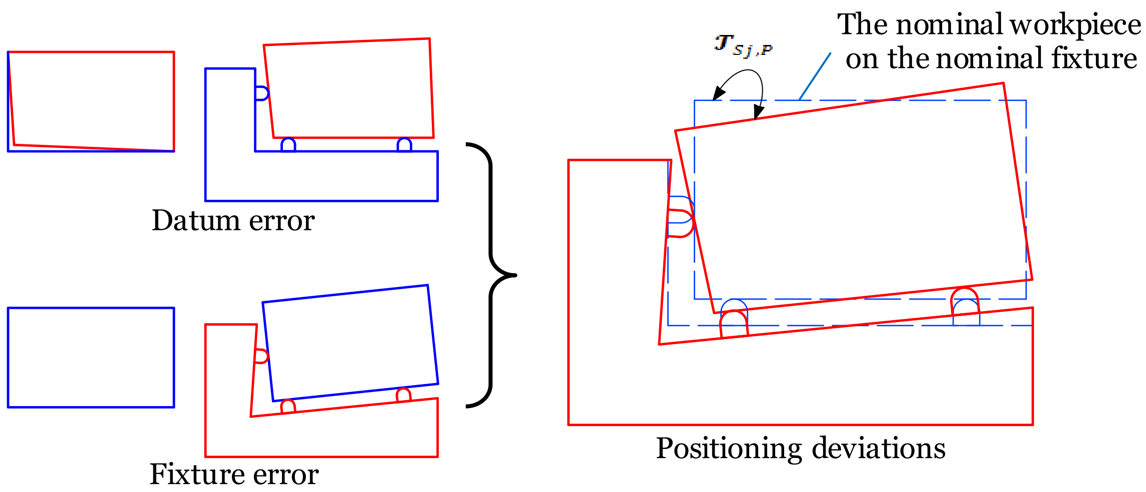


Fig. 4-3 Positioning deviation [KAMALI NEJAD 2009]

The positioning deviation of a workpiece in setup j is the summation of the datum errors, the fixture errors and the link errors of contacts between the workpiece and the fixture that are expressed by SDTs as equation (4-1).

$$\mathcal{T}_{Sj,P} = -\mathcal{T}_{P,Pi} + \mathcal{T}_{Sj,Hk} + \mathcal{T}_{Hk,Pi} \qquad (4\text{-}1)$$

where

- $\mathcal{T}_{Sj,P}$ is positioning deviation of setup j.
- $\mathcal{T}_{P,Pi}$ is datum error, which is also known as the MWP (Model of Workpiece from the previous setup).
- $\mathcal{T}_{Sj,Hk}$ is fixture errors of setup j.
- $\mathcal{T}_{Hk,Pi}$ is link errors of contacts between the workpiece and the fixture in setup j.

2.3.2. Machining deviation

As mentioned previously, many key factors can affect the accuracy of the machine tool. Thus, the machine tool can cause errors on the machine surface, but the fixture will not cause errors.

The machining deviation is expressed as deviations between the machined surface and the nominal surface that are created by the nominal machine tool.

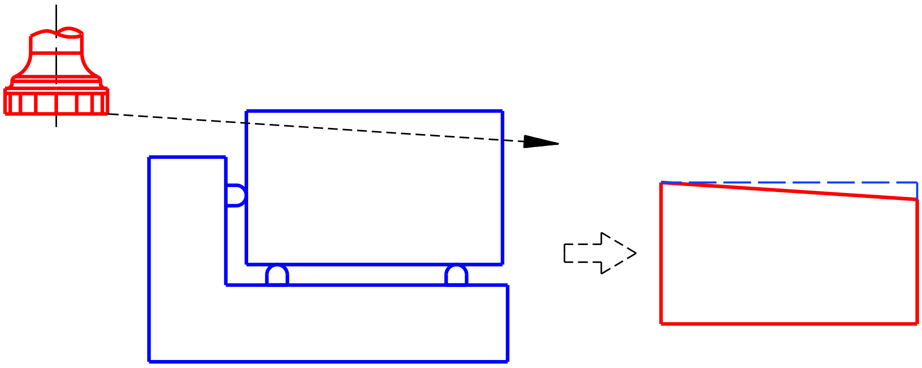


Fig. 4-4 Machining deviation [KAMALI NEJAD 2009]

Deviations of a machined surface in a setup, e.g. setup j, are expressed by a SDT, which is denoted $\mathcal{T}_{Sj,Pi}$.

Deviations of a machined surface compared to its nominal surface are denoted by a SDT $\mathcal{T}_{P,Pi}$. This can be expressed in relation to the positioning and machining deviation as follows:

$$\mathcal{T}_{P,Pi} = -\mathcal{T}_{Sj,P} + \mathcal{T}_{Sj,Pi} \qquad (4\text{-}2)$$

2.3.3. Positioning and machining defect

The above positioning and machining deviation are considered for a workpiece/machined part during manufacturing. In this study, if we calculate variances of positioning or machining deviation of a batch of parts, the values obtained are represented as positioning or machining defects.

2.4. Deviation domain and tolerance zone

It is difficult to manufacture a part to an exact size or geometry. Tolerances are therefore used to the allowable variability for certain geometrical dimensions or forms. They are also used to specify the shape, orientation, and location of features on a part. According to ISO: 1101, a geometrical tolerance specifies the zones in which a machined surface must be, namely tolerance zones.

There are different approaches to compare a machined part to its nominal part, such as virtual gauge approach [MAILHE *et al.* 2008], GapGP approach [KAMALI NEJAD 2009]. In this study, we use deviation domains and tolerance zones to do this.

Considering that a machined surface of a batch of parts must be inside a tolerance zone, the defects of these machined surfaces create a domain, namely deviation domain. The deviation domain is then compared with the tolerance zone to verify the machined parts. A SDT expresses deviations between a nominal surface and its machined surface. In other words, the deviation domain of a machined surface (of a batch of parts) is defined by all the SDTs that satisfy associated geometrical tolerance of this machined surface. The deviation domain of a surface therefore depends upon geometrical tolerances that are specified to it.

The following example is used to illustrate the deviation domains of different requirements, e.g. parallelism, location or perpendicularity.

A cylindrical part has two planes, namely P1 and P2. Let P1, P2 be two parallel planes that are expressed by two SDTs; Z_{CL} be a cylinder axis; L be a nominal distance between the two planes; and D be a diameter of this part.

2.4.1. Parallelism of two planes

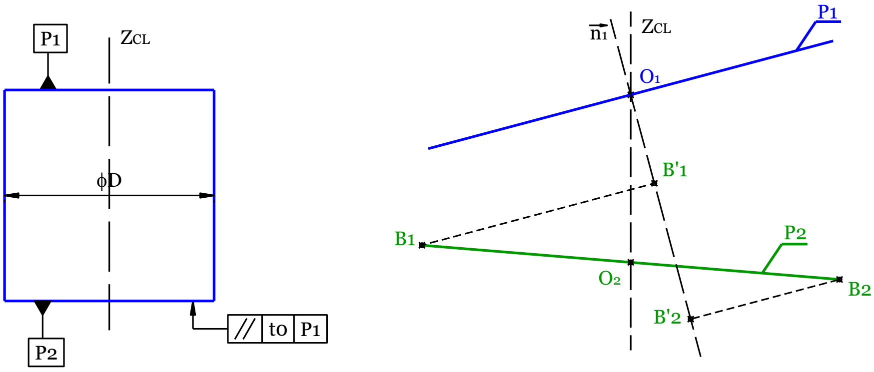


Fig. 4-5 Parallelism defect

If P1 is a referenced plane, parallelism defects of P2 are defined by the distance $\overline{B_1'B_2'}$ (Fig. 4-5), where *B'1* and *B'2* are projected points of contacting points delimiting the tolerance zone *B1* and *B2* on a normal vector $\overrightarrow{n_1}$ of P1. We can see that this defect does not depend on positions of these two planes. P2 satisfies its functional tolerance when the defect is inside the tolerance zone $(0, t_0)$. This is expressed by the inequalities as follows:

$$0 \leq \frac{\left|\overrightarrow{B_1B_2}.\vec{n}_1\right|}{\|\vec{n}_1\|} \leq t_0 \tag{4-3}$$

where

- *B1, B2* are defined based on two deviation rotations of P2.

2.4.2. Perpendicularity of planes

Let *A1, A2* and *B1, B2* be contacting points delimiting the tolerance zone of P1 and P2, respectively (Fig. 4-6). Perpendicularity defects of these planes compared to the axis Z_{CL} are defined by the distances $\overline{A_1'A_2'}$ and $\overline{B_1'B_2'}$, where *A'1, A'2* and *B'1, B'2* are projected points of the points *A1, A2* and *B1, B2*, respectively, on axis Z_{CL}. These defects do not depend on the position of these planes. The planes satisfy their functional tolerances (t_1 and t_2) when the defects are inside the tolerance zones $(0, t_1)$ and $(0, t_2)$, respectively. They can be expressed by the following inequalities:

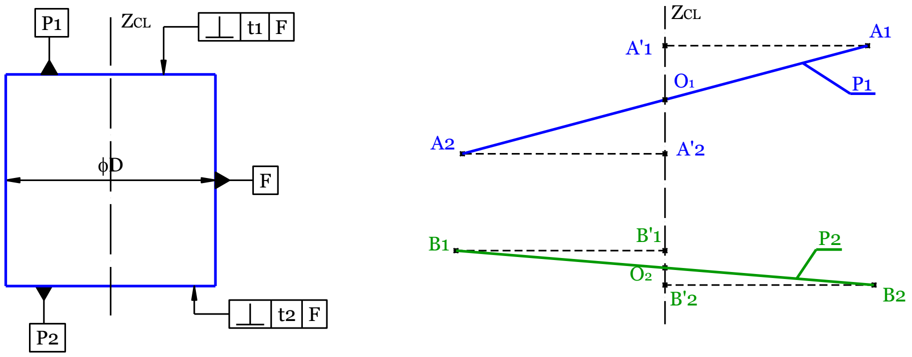


Fig. 4-6 Perpendicularity defects

$$0 \leq \frac{\left|\overrightarrow{A_1A_2}.\vec{n}_{CL}\right|}{\|\vec{n}_{CL}\|} \leq t_1$$
$$0 \leq \frac{\left|\overrightarrow{B_1B_2}.\vec{n}_{CL}\right|}{\|\vec{n}_{CL}\|} \leq t_2 \qquad (4\text{-}4)$$

where

- *A1, A2* and *B1, B2* are defined based on two deviation rotations of P1 and P2, respectively.

2.4.3. Location of two planes

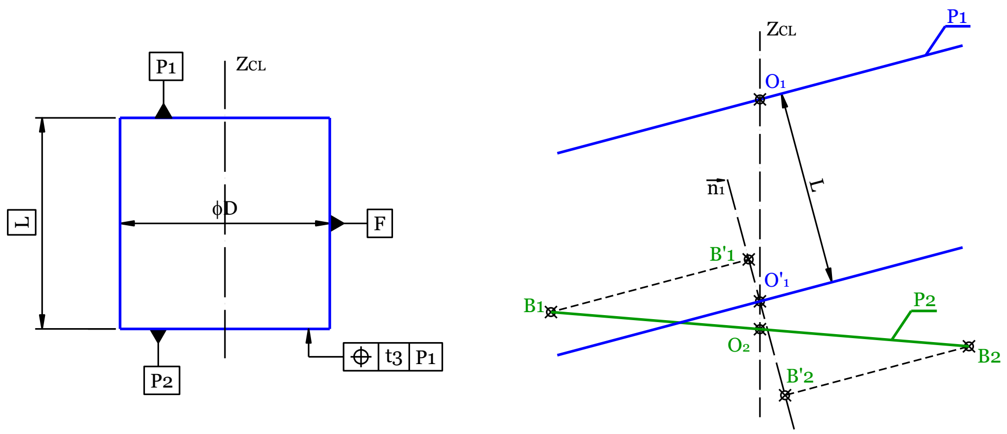


Fig. 4-7 Location defect

Let *B1* and *B2* be the two contacting points delimiting the tolerance zone (Fig. 4-7). Twice the maximum of the distances $\overline{O_1'B_1'}$ and $\overline{O_1'B_2'}$ gives us the location defect of P1 compared to referenced plane P2, where *B'1, B'2* are projected points of *B1, B2* on a normal vector $\overrightarrow{n_1}$ of P1 and $\overline{O_1O_1'}$ is the nominal distance from P1 to P2. This defect depends on the positions of these two planes. P2 satisfies its functional tolerance when the defect is inside the tolerance zone $(0, t_3)$. This is expressed by the inequalities as follows:

$$0 \leq 2.Max\left(\frac{\left|\overrightarrow{O_1'B_1}.\vec{n}_1\right|}{\|\vec{n}_1\|}, \frac{\left|\overrightarrow{O_1'B_2}.\vec{n}_1\right|}{\|\vec{n}_1\|}\right) \leq t_3 \qquad (4\text{-}5)$$

where *B1, B2* are defined based on the two deviation rotations and translation of P2.

We apply these models for each machined surface of the batch of machined parts. The inequalities that are used to express the relationships of the defects of machined surfaces and their tolerance are known as the mathematical expressions or mathematical models. These can then used in Monte Carlo simulation for validation of a manufacturing process or evaluation of percentage of waste products with an existing manufacturing process.

3. MANUFACTURING ERROR MODELS

In this section, a machined part is used as an example for simulating tolerance analysis in manufacturing. The objective is to verify a process plan of a batch of machined parts or determine tolerances of machined parts based on an existing process plan and number of rejected parts per million (ppm).

To do this, a batch of machined parts that passes through four setups with two machine tools (sawing machine and milling machine) is investigated. The machined part and its process plan are illustrated as follows.

3.1. Description of machined part and process plan

Workpieces in aluminum (2017 A) that have a diameter of D and a length of L_0 are cut from a long aluminum bar by a sawing machine. The workpieces are then located and fixed on a CNC milling machine by a three soft jaw-chucks (fixture) in order to machine different planes. The designed part and its requirement tolerances are shown in Fig. 4-8.

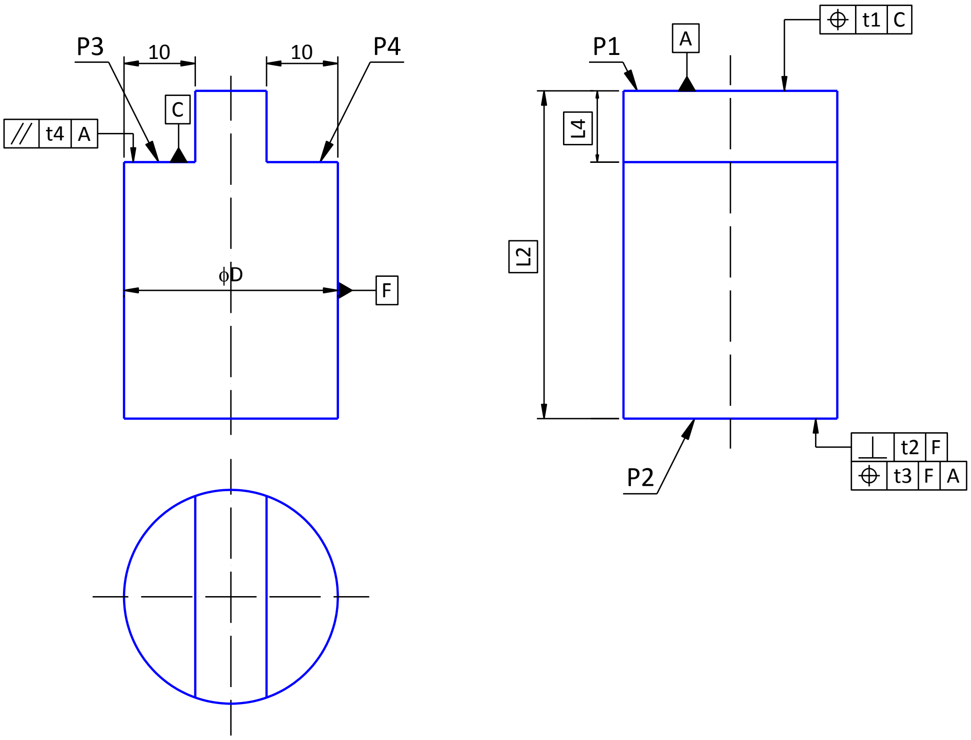


Fig. 4-8 Designed part

Four setups are illustrated in Fig. 4-9a. An end mill with a diameter of 20mm is used to machine surfaces of the workpieces on a CNC milling machine with two different tool-paths.

Setup 1

The length of each workpiece that is cut by a sawing machine is L_0. One of the cutting planes in this setup and the workpiece cylinder will be used to locate and fix the workpiece on a fixture in the next setup. One of the cutting planes is called workpiece locating plane (P0).

Setup 2

The workpieces are then located and fixed on the fixture in a CNC milling machine to machine a top surface of the workpiece, namely P1. A circle path is used to machine this plane with only one pass of the end mill (Fig. 4-9b). The depth of cut of this pass is 2mm.

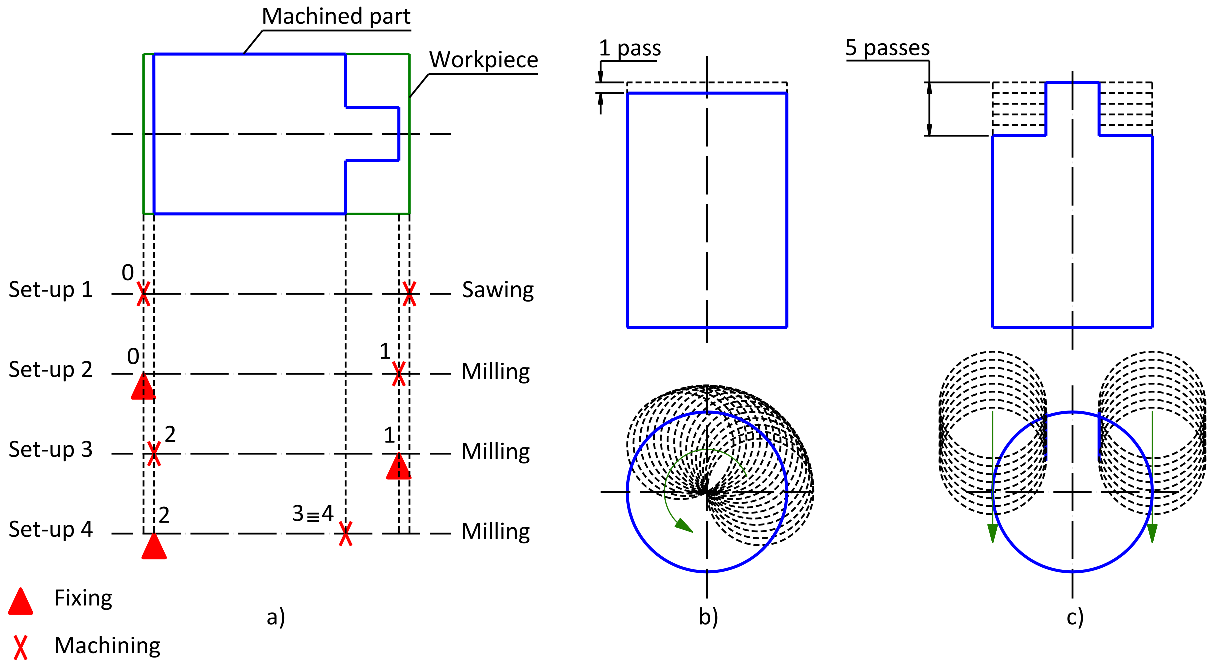


Fig. 4-9 Process plan (a) and tool paths on machined planes (b,c)

Setup 3

P1 machined in setup 2 and the workpiece cylinder are now used to locate and fix the workpiece in this setup. A plane is machined by a circle path with only one pass of the milling tool and 2mm of the depth of cut (Fig. 4-9b). The machined plane is called P2.

Setup 4

P2 and the workpiece cylinder are used to locate and fix the workpiece in this setup. P3 and P4 are then machined by a straight-line path with five passes of the end mill. Depth of cut of each pass is 2mm. The final passes on P3 and P4 are shown as Fig. 4-9c.

3.2. Model of Manufactured Part

The MMP is used to express the machining deviations of the machined surfaces or the positioning deviations of the workpiece on the fixture in each setup. Errors of each setup are accumulated on the workpiece that will affect the machining accuracy at subsequent setup. The quality of machined surfaces in each setup can be evaluated by verification of their defects compared to functional tolerances. For instance, parallelism, location or perpendicularity defects of the machined planes compare to the functional tolerances (Fig. 4-8). To do this, first, the SDT will be used to express defects of machined surfaces that may include machining defects and positioning defects. These defects are then used to determine deviation domains that are finally used for verification of the functional tolerances.

3.2.1. SDT of workpiece locating plane (P0)

Setup 1

One of the cutting planes that is used to locate the workpiece in setup 2 is measured for identifying the machining deviations of this plane. The machining deviations are expressed by a SDT. It is can be seen that only two rotations of this surface will influence on the machined surface in the next setup. Thus, the translation of this SDT is negligible.

$$\mathcal{T}_{S1,P0} = \begin{Bmatrix} r_{X_0} & 0 \\ r_{Y_0} & 0 \\ 0 & 0 \end{Bmatrix}_{O_0X_0Y_0Z_0} \tag{4-6}$$

where

- $\mathcal{T}_{S1,P0}$ is the SDT of the machined plane P0 in setup 1.
- $O_0X_0Y_0Z_0$ is the coordinate system of the nominal plane.

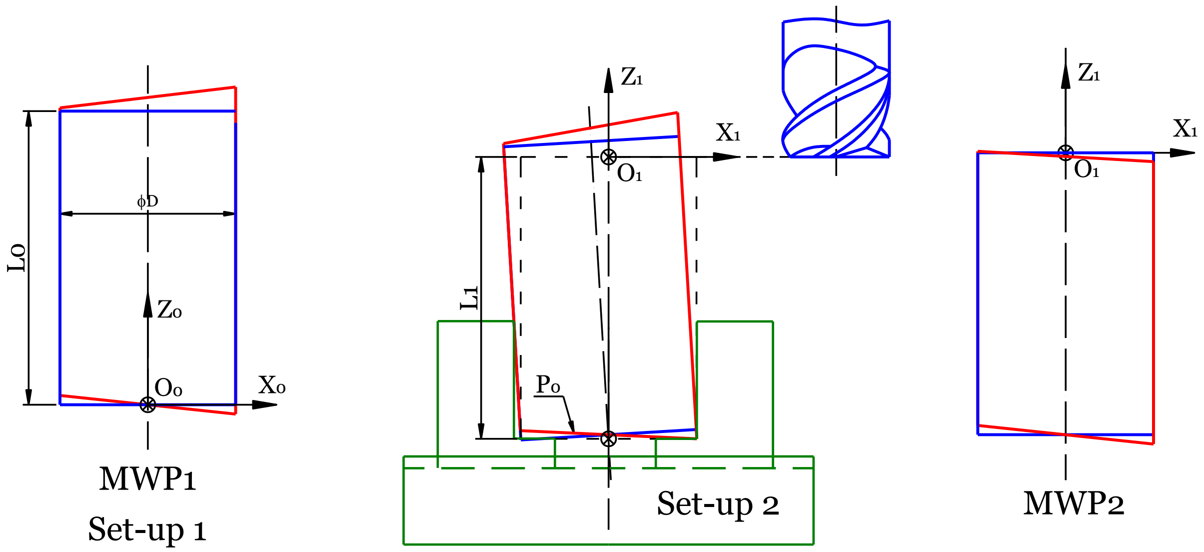


Fig. 4-10 Setup 1 and 2

3.2.2. SDT of machined plane P1

Setup 2

Defects of vertical machined surfaces are not considered in this study. Hence, translations along X and Y axis of the workpiece on the fixture are not necessary in this case. The positioning deviation of the workpiece on the fixture is so expressed by a SDT that includes two rotations of the workpiece cylinder around X and Y axis ($r_{X_{S2}}, r_{Y_{S2}}$) and the translation along Z axis ($t_{Z_{S2}}$).

$$\mathcal{T}_{S2,P} = \begin{Bmatrix} r_{X_{S2}} & 0 \\ r_{Y_{S2}} & 0 \\ 0 & t_{Z_{S2}} \end{Bmatrix}_{O_M X_M Y_M Z_M} \tag{4-7}$$

- $O_M X_M Y_M Z_M$ is the Machine Coordinate System (MCS).

Machined plane P1

Machining deviations of machined plane P1 compared to its nominal plane is expressed in the following equation:

$$\mathcal{T}_{P,P1} = -\mathcal{T}_{S2,P} + \mathcal{T}_{S2,P1} \tag{4-8}$$

$$\mathcal{T}_{P,P1} = -\begin{Bmatrix} r_{X_{S2}} & 0 \\ r_{Y_{S2}} & 0 \\ 0 & t_{Z_{S2}} \end{Bmatrix}_{O_M X_M Y_M Z_M} + \begin{Bmatrix} r_{X_1} & 0 \\ r_{Y_1} & 0 \\ 0 & t_{Z_1} \end{Bmatrix}_{O_1 X_1 Y_1 Z_1} \tag{4-9}$$

where

- $\mathcal{T}_{S2,P1}$ is the SDT of machining deviations of machined plane P1 in setup 2.
- $\mathcal{T}_{S2,P}$ is the SDT of positioning deviation of the workpiece on the fixture in setup 2.

The torsor $\mathcal{T}_{S2,P}$ can be rewritten in the $O_1 X_1 Y_1 Z_1$ by a changing of frame as follows:

$$\mathcal{T}_{S2,P} = \{\vec{R} \quad \vec{T}_{O_M}\}_{O_M X_M Y_M Z_M} = \{\vec{R} \quad \vec{T}_{O_M} + \vec{R} \wedge \overrightarrow{O_M O_1}\}_{O_1 X_1 Y_1 Z_1} \tag{4-10}$$

where

$$\vec{T}_{O_M} + \vec{R} \wedge \overrightarrow{O_M O_1} = \begin{Bmatrix} 0 \\ 0 \\ t_{Z_{S2}} \end{Bmatrix} + \begin{Bmatrix} r_{X_{S2}} \\ r_{Y_{S2}} \\ 0 \end{Bmatrix} \wedge \begin{Bmatrix} 0 \\ 0 \\ L_1 \end{Bmatrix} = \begin{Bmatrix} L_1 r_{X_{S2}} \\ -L_1 r_{Y_{S2}} \\ t_{Z_{S2}} \end{Bmatrix} \tag{4-11}$$

so

$$\mathcal{T}_{P,PA} = -\begin{Bmatrix} r_{X_{S2}} & L_1 r_{X_{S2}} \\ r_{Y_{S2}} & -L_1 r_{Y_{S2}} \\ 0 & t_{Z_{S2}} \end{Bmatrix}_{O_1 X_1 Y_1 Z_1} + \begin{Bmatrix} r_{X_1} & 0 \\ r_{Y_1} & 0 \\ 0 & t_{Z_A} \end{Bmatrix}_{O_1 X_1 Y_1 Z_1}$$

$$\Rightarrow \mathcal{T}_{P,P1} = \begin{Bmatrix} -r_{X_{S2}} + r_{X_1} & -L_1 r_{X_{S2}} \\ -r_{Y_{S2}} + r_{Y_1} & L_1 r_{Y_{S2}} \\ 0 & -t_{Z_{S2}} + t_{Z_1} \end{Bmatrix}_{O_1 X_1 Y_1 Z_1} \tag{4-12}$$

3.2.3. SDT of machined plane P2

Setup 3

Similarly, the positioning deviations and machining deviations are used to express defects of the machined planes in this setup.

Machined plane P2

The machining deviation of machined plane P2 compared to its nominal plane is expressed as in equation (4-13).

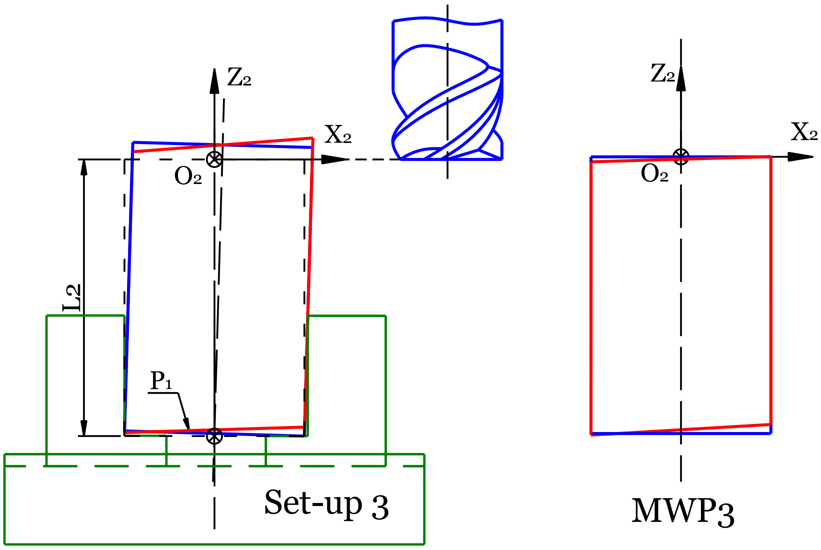


Fig. 4-11 Setup 3

$$\mathcal{T}_{P,P2} = \begin{Bmatrix} -r_{X_{S3}} + r_{X_2} & -L_2 r_{Y_{S3}} \\ -r_{Y_{S3}} + r_{Y_2} & L_2 r_{X_{S3}} \\ 0 & -t_{Z_{S3}} + t_{Z_2} \end{Bmatrix}_{O_2X_2Y_2Z_2} \quad (4\text{-}13)$$

or

$$\mathcal{T}_{P,P2} = \begin{Bmatrix} -r_{X_{S3}} - r_{X_2} & L_2 r_{X_{S3}} - L_2 r_{Y_{S3}} + L_2 r_{Y_2} \\ -r_{Y_{S3}} + r_{Y_2} & L_2 r_{Y_{S3}} - L_2 r_{X_{S3}} + L_2 r_{X_2} \\ 0 & t_{Z_{S3}} - t_{Z_2} \end{Bmatrix}_{O_1X_1Y_1Z_1} \quad (4\text{-}14)$$

3.2.4. SDTs of machined planes P3 and P4

Setup 4

Machined plane P3 and P4

The machining deviations of machined planes P3 and P4 compared to their nominal planes are expressed in equations (4-15) and (4-16).

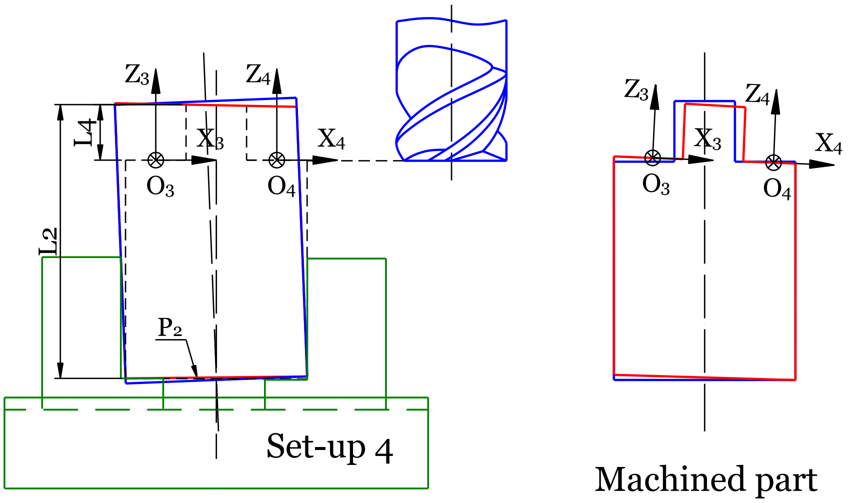


Fig. 4-12 Setup 4 and coordinate systems of the machined planes

$$\mathcal{T}_{P,P3} = \begin{Bmatrix} -r_{X_{S4}} + r_{X_3} & -L_2 r_{Y_{S4}} + L_4 r_{Y_3} \\ -r_{Y_{S4}} + r_{Y_3} & L_2 r_{X_{S4}} - L_4 r_{X_3} \\ 0 & -t_{Z_{S4}} - \dfrac{4R\sin^3\dfrac{\theta}{2}}{3(\theta-\sin\theta)} r_{Y_3} + t_{Z_3} \end{Bmatrix}_{O_1X_1Y_1Z_1} \quad (4\text{-}15)$$

$$\mathcal{T}_{P,P4} = \begin{Bmatrix} -r_{X_{S4}} + r_{X_4} & -L_2 r_{Y_{S4}} + L_4 r_{Y_4} \\ -r_{Y_{S4}} + r_{Y_4} & L_2 r_{X_{S4}} - L_4 r_{X_4} \\ 0 & -t_{Z_{S4}} + \dfrac{4R\sin^3\dfrac{\theta}{2}}{3(\theta-\sin\theta)} r_{Y_4} + t_{Z_4} \end{Bmatrix}_{O_1X_1Y_1Z_1} \quad (4\text{-}16)$$

where

- $\frac{4R\sin^3\frac{\theta}{2}}{3(\theta-\sin\theta)}$ is the X-coordinate of the centroid of circular segment (Fig. 4-13).

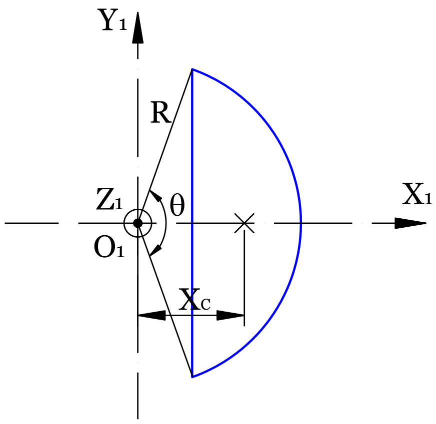


Fig. 4-13 Centroid of circular segment

To verify defects of the machined planes with their functional tolerances, deviation domains of these surfaces will be analyzed based on the SDTs obtained for comparison with the tolerance zones in the next section.

3.2.5. *Synthesis of SDTs of the machined planes*

The SDTs of the machined planes are summarized in the following table (Tab. 4-1).

Machined planes	SDTs
P1	$\mathcal{T}_{P,P1} = \begin{Bmatrix} -r_{X_{S2}} + r_{X_1} & -L_1 r_{X_{S2}} \\ -r_{Y_{S2}} + r_{Y_1} & L_1 r_{Y_{S2}} \\ 0 & -t_{Z_{S2}} + t_{Z_1} \end{Bmatrix}_{O_1X_1Y_1Z_1}$
P2	$\mathcal{T}_{P,P2} = \begin{Bmatrix} -r_{X_{S3}} - r_{X_2} & L_2 r_{X_{S3}} - L_2 r_{Y_{S3}} + L_2 r_{Y_2} \\ -r_{Y_{S3}} + r_{Y_2} & L_2 r_{Y_{S3}} - L_2 r_{X_{S3}} + L_2 r_{X_2} \\ 0 & t_{Z_{S3}} - t_{Z_2} \end{Bmatrix}_{O_1X_1Y_1Z_1}$
P3	$\mathcal{T}_{P,P3} = \begin{Bmatrix} -r_{X_{S4}} + r_{X_3} & -L_2 r_{Y_{S4}} + L_4 r_{Y_3} \\ -r_{Y_{S4}} + r_{Y_3} & L_2 r_{X_{S4}} - L_4 r_{X_3} \\ 0 & -t_{Z_{S4}} - \dfrac{4R\sin^3\dfrac{\theta}{2}}{3(\theta - \sin\theta)} r_{Y_3} + t_{Z_3} \end{Bmatrix}_{O_1X_1Y_1Z_1}$
P4	$\mathcal{T}_{P,P4} = \begin{Bmatrix} -r_{X_{S4}} + r_{X_4} & -L_2 r_{Y_{S4}} + L_4 r_{Y_4} \\ -r_{Y_{S4}} + r_{Y_4} & L_2 r_{X_{S4}} - L_4 r_{X_4} \\ 0 & -t_{Z_{S4}} + \dfrac{4R\sin^3\dfrac{\theta}{2}}{3(\theta - \sin\theta)} r_{Y_4} + t_{Z_4} \end{Bmatrix}_{O_1X_1Y_1Z_1}$

Tab. 4-1 The SDTs of the machined planes

3.3. Mathematical model based on the MMP

As mentioned previously, the MMP is based on the SDT that does not take into account form defects of machined surfaces. Thus, only orientation and positional tolerance are considered in the following models.

We apply these models for each machined surface of a batch of machined parts. The inequalities that are used to express the relationships of the defects of the machined surfaces and their tolerance are known as the mathematical expressions or mathematical models. These can then be used in Monte Carlo simulation for validation of a manufacturing process or evaluation of percentage of waste products with an existing manufacturing process.

3.3.1. *Model for verifying tolerances of machined plane A*

Location of plane A compared to referenced plane C

Plane A and C correspond to P1 and P3. Thus, the location of plane A compared to referenced plane C is determined based on the SDT as follows:

$$\mathcal{T}_{P3,P1} = -\mathcal{T}_{P,P3} + \mathcal{T}_{P,P1} \tag{4-17}$$

where

- $\mathcal{T}_{P,P1}$,$\mathcal{T}_{P,P3}$ are determined in Tab. 4-1.

$$\mathcal{T}_{P3,P1} = \begin{Bmatrix} r_{X_{S2}} - r_{X_1} + r_{X_{S4}} - r_{X_3} & -L_1 r_{X_{S2}} + L_2 r_{Y_{S4}} - L_4 r_{Y_3} \\ r_{Y_{S2}} - r_{Y_1} - r_{Y_{S4}} + r_{Y_3} & L_1 r_{Y_{S2}} - L_2 r_{X_{S4}} + L_4 r_{X_3} \\ 0 & -t_{Z_{S2}} + t_{Z_1} + t_{Z_{S4}} + \dfrac{4R\sin^3\frac{\theta}{2}}{3(\theta - \sin\theta)} r_{Y_3} - t_{Z_3} \end{Bmatrix}_{O_1X_1Y_1Z_1} \quad (4\text{-}18)$$

For location tolerance, which is a positional tolerance, three components (rotations and translation) are used to identify the contacting points delimiting the tolerance zone (A_1, A_2) as in the following equation:

$$(-t_{Z_{S2}} + t_{Z_1} + t_{Z_{S4}} + 9.1422 r_{Y_3} - t_{Z_3}) \pm a.(-r_{Y_{S2}} + r_{Y_1}) \mp b.(-r_{X_{S2}} + r_{X_1}) \quad (4\text{-}19)$$

The model, which will be used to verify the location tolerance of machined plane P1, is expressed as follows:

$$\begin{cases} 0 \le 2.Max\left(\dfrac{\left|\overrightarrow{O'_3A_1}.\vec{n}_{P3}\right|}{\|\vec{n}_{P3}\|}, \dfrac{\left|\overrightarrow{O'_3A_2}.\vec{n}_{P3}\right|}{\|\vec{n}_{P3}\|}\right) \le t_1 \\ a = R.\sin\left(\dfrac{\pi}{2} + \alpha\right) \\ b = R.\cos\left(\dfrac{\pi}{2} + \alpha\right) \\ \alpha = Arctan\left(\dfrac{-r_{Y_{S2}} + r_{Y_1}}{-r_{X_{S2}} + r_{X_1}}\right) \end{cases} \quad (4\text{-}20)$$

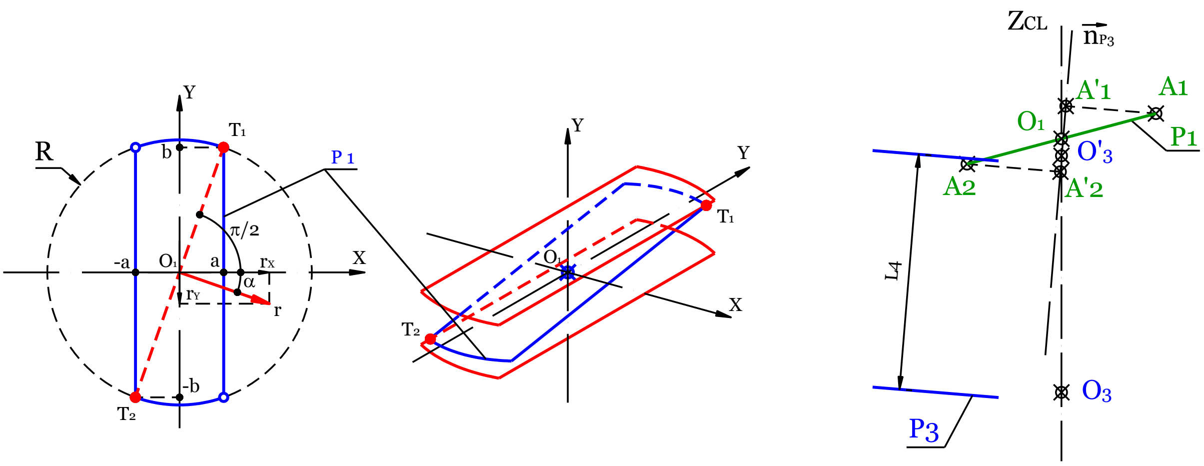


Fig. 4-14 Location defect of P1

3.3.2. Model for verifying tolerances of machined plane B

Perpendicularity of plane B compared to the cylinder axis

Plane B and the cylinder axis correspond to the bottom surface and cylinder F of the machined part (Fig. 4-8).

For perpendicularity tolerance, which is a tolerance of orientation, only components of rotations are necessary. Indeed, in this case, the position of the plane, which compares to its nominal coordinate system, is not important. Thus, we limit the component $\overrightarrow{\delta P}.\vec{Z} - t_Z$ must be inside the tolerance zone $(0, t)$. Two contacting points delimiting the tolerance zone (B_1, B_2) are defined by equations (4-21).

$$\pm a.\left(-r_{Y_{S3}} + r_{Y_2}\right) \mp b.\left(-r_{X_{S3}} - r_{X_2}\right) \tag{4-21}$$

The model, which will be used to verify the perpendicularity tolerance of machined plane A, is expressed in (4-22).

$$\begin{cases} 0 \leq \dfrac{\left|\overrightarrow{B_1B_2}.\vec{n}_{CL}\right|}{\|\vec{n}_{CL}\|} \leq t_2 \\ a = R.\sin\left(\dfrac{\pi}{2} + \alpha\right) \\ b = R.\cos\left(\dfrac{\pi}{2} + \alpha\right) \\ \alpha = Arctan\left(\dfrac{-r_{Y_{S3}} + r_{Y_2}}{-r_{X_{S3}} - r_{X_2}}\right) \end{cases} \tag{4-22}$$

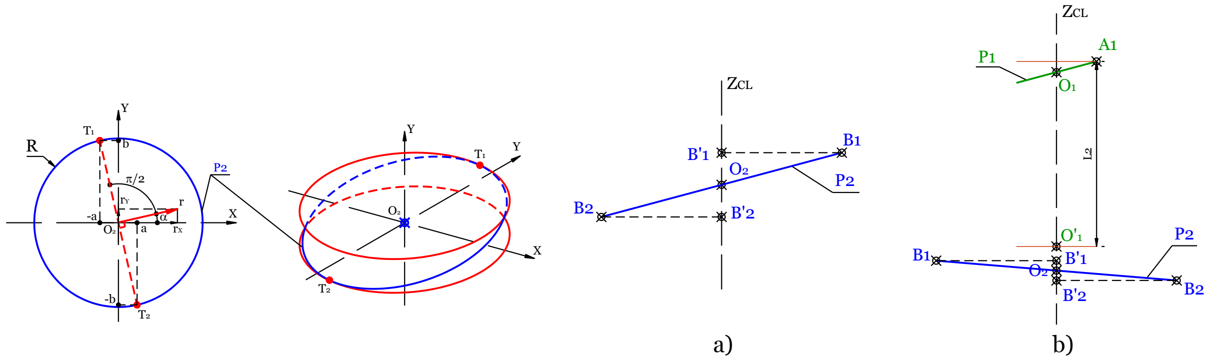


Fig. 4-15 Perpendicularity and Location defect of P2

Location of plane B compared to the cylinder axis and plane A

Location of plane B compared to referenced cylinder F and plane A by the following SDT:

$$\mathcal{T}_{P'1,P2} = -\mathcal{T}_{P,P'1} + \mathcal{T}_{P,P2} \tag{4-23}$$

where

- $\mathcal{T}_{P,P'1}$ is the SDT of the plane that passes through the maximum point of machined plane P1 (rotations equal zero).
- $\mathcal{T}_{P,P2}$ are determined in Tab. 4-1.

$$\mathcal{T}_{P'1,P2} = \begin{Bmatrix} -r_{X_{S3}} - r_{X_2} & L_2 r_{X_{S3}} - L_2 r_{Y_{S3}} + L_2 r_{Y_2} \\ -r_{Y_{S3}} + r_{Y_2} & L_2 r_{Y_{S3}} - L_2 r_{X_{S3}} + L_2 r_{X_2} \\ 0 & t_{Z_{S3}} - t_{Z_2} - t_{Z_{P'1}} \end{Bmatrix}_{O_1 X_1 Y_1 Z_1} \quad (4\text{-}24)$$

Relation of rotations and translation that are used to identify the contacting points delimiting the tolerance zone (B_1, B_2) is expressed in the following equation:

$$\left(t_{Z_{S3}} - t_{Z_2} - t_{Z_{P'1}}\right) \pm a.\left(-r_{Y_{S3}} + r_{Y_2}\right) \mp b.\left(-r_{X_{S3}} - r_{X_2}\right) \quad (4\text{-}25)$$

The model, which will be used to verify the location tolerance of the machined plane D, is expressed as follows:

$$\begin{cases} 0 \leq 2.Max\left(\dfrac{\left|\overrightarrow{O'_1B_1}.\vec{n}_{CL}\right|}{\|\vec{n}_{CL}\|}, \dfrac{\left|\overrightarrow{O'_1B_2}.\vec{n}_{CL}\right|}{\|\vec{n}_{CL}\|}\right) \leq t_3 \\ a = R.\sin\left(\dfrac{\pi}{2} + \alpha\right) \\ b = R.\cos\left(\dfrac{\pi}{2} + \alpha\right) \\ \alpha = Arctan\left(\dfrac{-r_{Y_{S3}} + r_{Y_2}}{-r_{X_{S3}} - r_{X_2}}\right) \end{cases} \quad (4\text{-}26)$$

3.3.3. Model for verifying tolerances of machined plane C

Parallelism of plane C compared to referenced plane A

For parallelism tolerance, which is an orientation tolerance, only components of rotation are necessary. Indeed, in this case, the position of the plane, which compares to its nominal coordinate system, is not important.

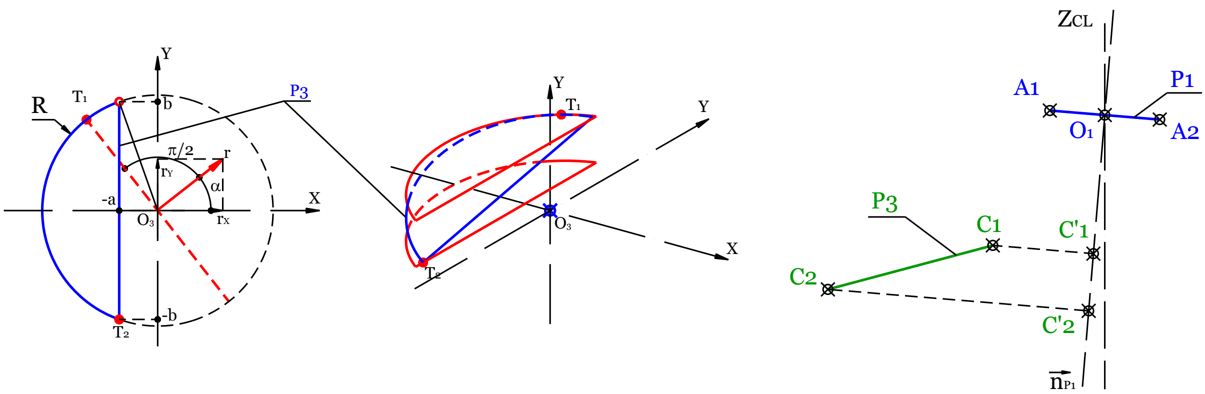


Fig. 4-16 Parallelism defect of P3

The model, which will be used to verify the parallelism tolerance of machined plane A, is expressed as follows:

$$\begin{cases} 0 \leq \dfrac{\left|\overrightarrow{C_1C_2}.\vec{n}_{P1}\right|}{\|\vec{n}_{P1}\|} \leq t_4 \\ a = R.\sin\left(\dfrac{\pi}{2} + \alpha\right) \\ b = R.\cos\left(\dfrac{\pi}{2} + \alpha\right) \\ \alpha = Arctan\left(\dfrac{-r_{Y_{S4}} + r_{Y_3}}{-r_{X_{S4}} + r_{X_3}}\right) \end{cases} \tag{4-27}$$

where C_1 and C_2 are determined by the following equations:

$$\pm a.\left(-r_{Y_{S4}} + r_{Y_3}\right) \mp b.\left(-r_{X_{S4}} + r_{X_3}\right) \tag{4-28}$$

The mathematical models that are obtained in this section will be used in Monte Carlo simulation in the next section for tolerance analysis.

4. TOLERANCE ANALYSIS

In this section, tolerance analysis in manufacturing is performed using Monte Carlo simulation. We first introduce Monte Carlo simulation and its advantages in simulation. We then provide the input variables that are used in Monte Carlo simulation. In this simulation, either we can use the results obtained from experiment in chapter 2, which include machining defects and positioning defects, or we can combine the machining defects obtained from experiment and positioning defects obtained from simulation as input variables.

4.1. Monte Carlo simulation

Monte Carlo simulation is a computerized mathematical technique based on the use of random numbers and probability statistics to investigate problems. As used here, models created based on the MMP are used to imitate real productions (outputs). The models are obtained in the previous section by mathematical expressions, where input variables are uncertainties of manufacture such as machining and positioning defects. The input variables are randomly generated from probability distributions to simulate the process of sampling from an actual population. Therefore, if the distributions of the input variables match real results, the outputs are closer to real productions. The data generated from the simulation can be represented as probability distributions or tolerance zones, etc (Fig. 4-17).

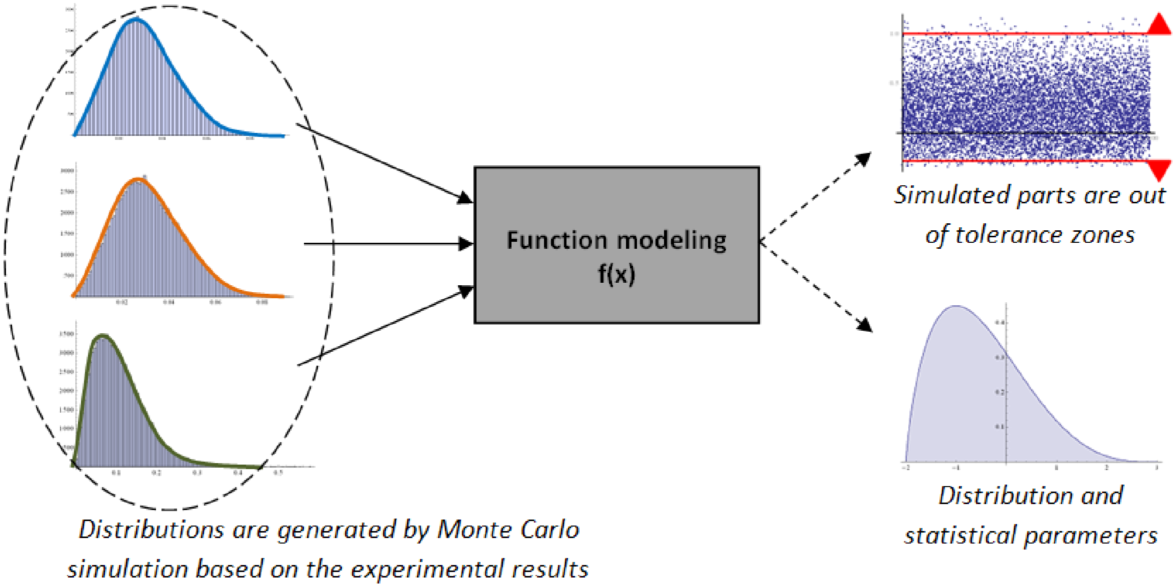


Fig. 4-17 Monte Carlo simulation in tolerance analysis

The algorithm of Monte Carlo simulation can be resumed by the following steps:

- Step 1: Create a parametric model, $y = f(x_1, x_2, ..., x_k)$.
- Step 2: Generate a set of random inputs, $x_{i1}, x_{i2}, ..., x_{ik}$.
- Step 3: Evaluate the model and store the results as y_i.
- Step 4: Repeat steps 2 and 3 for $i = 1$ to n.
- Step 5: Analyze the results using histograms, summary statistics, tolerance zones, etc.

Because of the advantages of Monte Carlo simulation, it is used to simulate defects of the machined part described in the previous section. This simulation is used for two objectives:

- ✓ *Verification of a process plan in terms of functional tolerances of a batch of machined parts;*
- ✓ *Determination of optimal tolerances of machined parts based on an existing process plan and a requirement of rejected parts per million.*

As mentioned previously, the input variables used in Monte Carlo simulation can be experimental results or combination of experimental results and simulated results. They are presented as follows:

4.2. Input variables obtained from experiment

The experimental results obtained in chapter 2 are now applied for tolerance analysis of the machined part presented in section 3.

4.2.1. Machining defects

Defects of a machined plane are expressed by the components of a SDT. Three different machining types of the machined surfaces are used in this simulation: sawing plane (P0), milling plane with two different tool-paths (P1, P2, P3 and P4). They are shown as in Tab. 4-2.

Surfaces	Components	Distributions	Parameters
Plane 0 *(WP locating plane)*	r_{X_0}	Normal	$m = 13.37 \times 10^{-3}$, $s = 2.91 \times 10^{-3}$
	r_{Y_0}	Normal	$m = -1.42 \times 10^{-3}$, $s = 2.85 \times 10^{-3}$
Plane 1 (or 2)	r_{X_1}	Normal	$m = -9.783 \times 10^{-5}$, $s = 2.812 \times 10^{-5}$
	r_{Y_1}	Normal	$m = -131 \times 10^{-5}$, $s = 6.459 \times 10^{-5}$
	t_{Z_1}	Pert	$min = -11.06 \times 10^{-3}$, $max = 1.62 \times 10^{-3}$, $c = -1.62 \times 10^{-3}$
Plane 3 (or 4)	r_{X_3}	Normal	$m = -4.79 \times 10^{-5}$, $s = 2.521 \times 10^{-5}$
	r_{Y_3}	Pert	$min = 1.41 \times 10^{-4}$, $max = 7.92 \times 10^{-4}$, $c = 6.688 \times 10^{-4}$
	t_{Z_3}	Pert	$min = -91.2 \times 10^{-4}$, $max = 17.4 \times 10^{-4}$, $c = 17.2 \times 10^{-4}$

Tab. 4-2 Distributions and their parameters of machining defects

4.2.2. Positioning defects

A workpiece is located and clamped on the fixture by its cylinder and one of its planes (workpiece locating surface). The workpiece locating plane is different in setup 2 and setup 3 or 4. Hence, positioning defects are considered as two cases as follows:

- Workpieces are fixed on the fixture by its cylinder and its sawing plane (setup 2). These are the same as the fixturing of workpieces in the experiment of chapter 2. Therefore, the experimental results are used to express components of positioning defects in this case.
- Workpieces are fixed on the fixture by its cylinder and its milling plane (setup 3 and 4). In this instance, rotation components of positioning defects are the same as setup 2; meanwhile, translation component is determined based on rotation deviations of the workpiece locating plane with the assumption that the workpiece locating plane and the fixture one always come into contact.

Setup	Components	Distributions	Parameters
2	$r_{X_{S2}}$	Normal	$m = -6.712 \times 10^{-5}$, $s = 56.043 \times 10^{-5}$
	$r_{Y_{S2}}$	Pert	$min = -1.15 \times 10^{-3}$, $max = 1.43 \times 10^{-3}$, $c = -0.606 \times 10^{-3}$
	$t_{Z_{S2}}$	Pert	$min = -29.68 \times 10^{-3}$, $max = 107.96 \times 10^{-3}$, $c = -16.93 \times 10^{-3}$
3 (or 4)	$r_{X_{S3}}$	Normal	$m = -6.712 \times 10^{-5}$, $s = 56.043 \times 10^{-5}$
	$r_{Y_{S3}}$	Pert	$min = -1.15 \times 10^{-3}$, $max = 1.43 \times 10^{-3}$, $c = -0.606 \times 10^{-3}$
	$t_{Z_{S3}}$	Calculate based on rotation deviation of the workpiece locating plane (R_1 *or* R_2) and its dimension	
	R_1 (R_2) (*vs. WP cylinder axis*)	Weibull	$\alpha = 51.8$, $\beta = 0.049$, $m = -0.037$

Tab. 4-3 Distributions and parameters of positioning defects

4.3. Input variables obtained from simulation

As mentioned before, the positioning defects can be determined by measuring experiments. Nevertheless, these are very costly and time-consuming; on the other hand, appropriate equipments are an essential requirement. For instance, a machined tool must be equipped with a measurement mean in order to measure a workpiece on a fixture inside this machine. Developing accurate models and methodologies for simulating the positioning defects and evaluating the factors that affect these defects can overstep these drawbacks.

The locating deviation of workpieces on fixtures can be the result of many causes that affect workpiece positions, orientation and static equilibrium. They could be clamping force, cutting force; deformations of a workpiece/fixture; friction coefficient of contacts between a workpiece and a fixture; geometrical errors of a workpiece/fixture.

In this section, an investigation of influences of different factors on positions of a workpiece fixed on a fixture is presented. Three treatment factors are proposed: geometrical error of workpiece locating plane, clamping force and coefficient of friction at contact surfaces between the workpiece and the fixture. For this purpose, a number of finite element simulations based on statistical two-level full factorial design of experiments method are conducted in order to determine mathematical models. The models are then used in Monte Carlo simulation for evaluating positions of the workpiece.

4.3.1. Method

To evaluate the positioning defects with the proposed factors, three methods have been used. Design of experiments method used to establish different models for finite element simulations. Responses obtained from the simulations are then used in Monte Carlo simulation in which the three factors and the responses of simulations are considered as input and output parameters. The combination of these methods is illustrated as in Fig. 4-18.

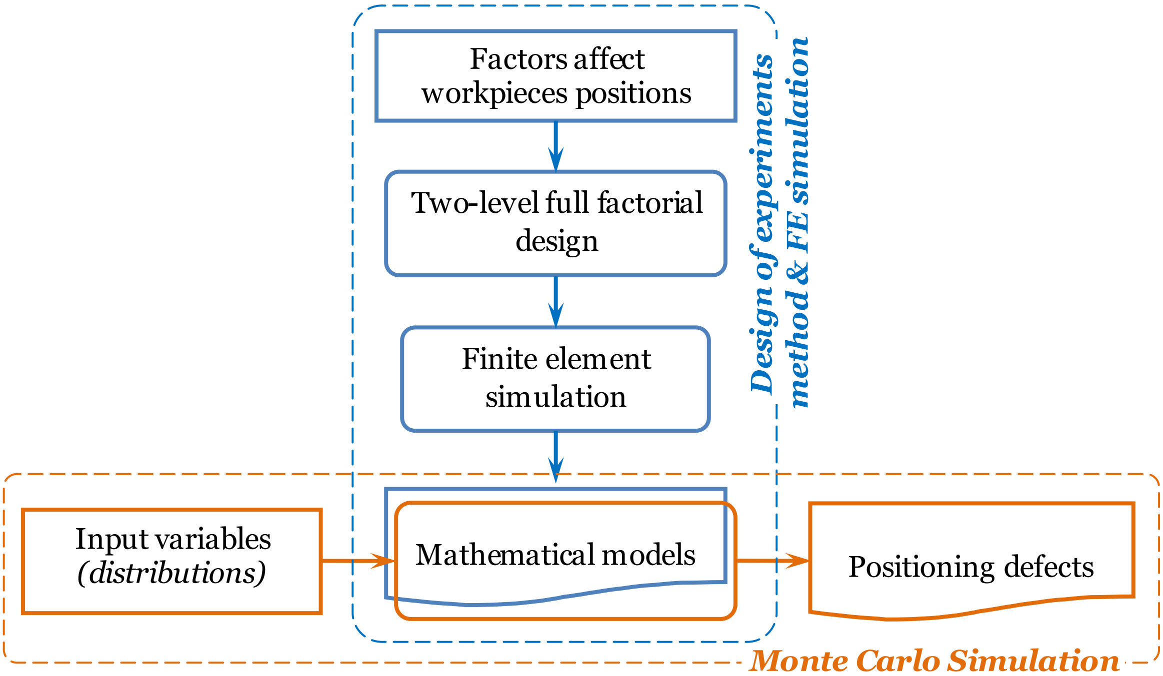


Fig. 4-18 Simulation diagram

4.3.1.1. *Design of experiments method*

"Design of experiments (DOE) is a systematic, rigorous approach to engineering problem-solving that applies principles and techniques at the data collection stage so as to ensure the generation of valid, defensible, and supportable engineering conclusions. In addition, all of this is carried out under the constraint of a minimal expenditure of engineering runs, time, and money." [NIST 2010]

This method can be used for 4 general engineering problem areas, such as comparative, screening/characterizing, modeling, and optimizing. The modeling is applied in our case, where the output from process modeling is a fitted mathematical function with estimated coefficients.

4.3.1.2. *Finite Element Analysis*

The Finite Element Analysis (FEA) is an applied technique in engineering that aims to evaluate the functionality of a certain product design before the prototypes are produced. Nowadays, it is being observed by different industries such as aeronautics, automotive, defense, and nuclear.

The FEA generally consists of a system of points drawn together to form a grid, namely a mesh. The model is then divided into several parts for mathematical analyses. Depending on the application of the product, several analyses must be made to give values to the variables such as stress, velocity, displacement, force etc. We have focused on the analyses of displacement in this research.

There is a lot of FEA software that can be used for analyzing 2D or 3D CAD models of the product. The Abaqus, which is a FEA program owned and supported by SIMULA, the Dassault Systèmes brand of Realistic Simulation, is chosen for finite element simulations.

4.3.2. Positioning defects

4.3.2.1. Finite element model and solution

Due to the ranges of parameters selected, it has been decided to use a two-level full factorial design. The notations, units and their levels chosen are summarized in Tab. 4-4. For the convenience of recording and processing the simulation data, the upper and lower levels of the parameters are coded as +1 and -1.

No.	Parameter	Notation	Unit	Levels			
				Original		Coded	
				Low	High	Low	High
1	Friction coefficient	μ		0.3	0.385	-1	1
2	Clamping force	P	N	7000	8000	-1	1
3	Rotation defect of workpiece locating plane	R_o	°	0.3	1.2	-1	1

Tab. 4-4 Parameters and their levels

The three factors (μ, P, R_o) and positioning defects (R, t_Z) can be illustrated in the model as Fig. 4-19.

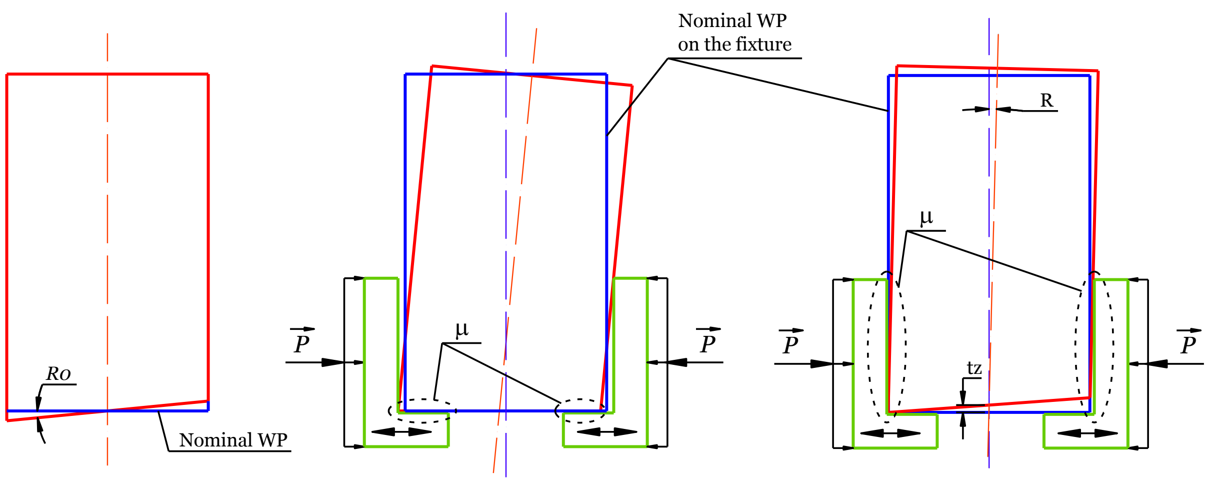


Fig. 4-19 The factors and positioning defects

The workpiece and fixture used in this simulation have been produced by aluminum (2017 A) and steel, respectively. Their material specification and mechanical properties are given in Tab. 4-5.

Material	Mass density ρ *(kg/m³)*	Young's modulus E *(N/m²)*	Poisson's ratio ν
Aluminum (2017 A)	2800	74×10^9	0.33
Steel	7800	200×10^9	0.3

Tab. 4-5 Mechanical properties of workpiece and fixture

The workpiece and fixture are modeled as a deformable body based on linear elasticity. Contact elements are used to simulate the contact phenomena between the workpiece and the fixturing elements. The analysis is accomplished using a non-linear finite element code Abaqus. The finite element is illustrated as Fig. 4-20.

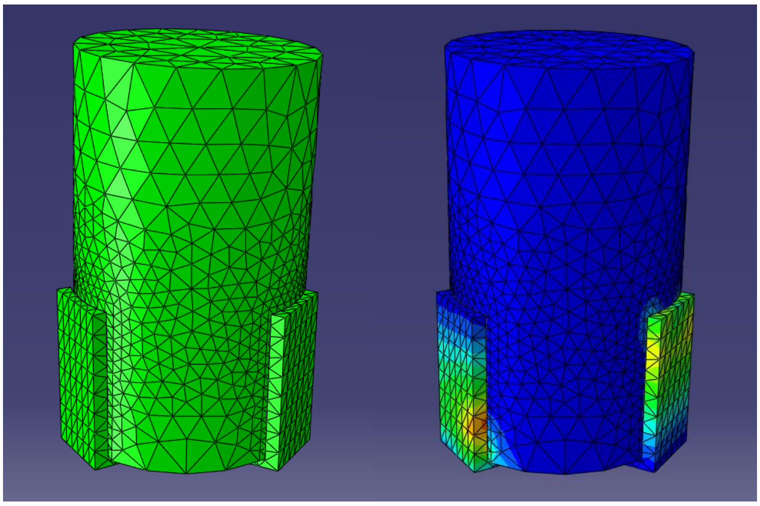


Fig. 4-20 Finite element model

The eight simulations were conducted on Abaqus with the condition given by a matrix of explanatory variables, namely a design matrix. The corresponding responses are expressed in Tab. 4-6.

Sim. No.	Coded value			Original value			Displacements of WP on the fixture	
	μ	P	R_o	μ	P	R_o	R *(radian)*	tZ *(mm)*
1	-1	-1	-1	0.3	7000	0.3	0.328×10^{-4}	0.077
2	-1	-1	+1	0.3	7000	1.2	0.699×10^{-4}	0.33
3	-1	+1	-1	0.3	8000	0.3	0.391×10^{-4}	0.075
4	-1	+1	+1	0.3	8000	1.2	0.682×10^{-4}	0.33
5	+1	-1	-1	0.385	7000	0.3	16.696×10^{-4}	0.044
6	+1	-1	+1	0.385	7000	1.2	24.831×10^{-4}	0.255
7	+1	+1	-1	0.385	8000	0.3	18.16×10^{-4}	0.041
8	+1	+1	+1	0.385	8000	1.2	24.94×10^{-4}	0.255

Tab. 4-6 Design matrix and corresponding responses

4.3.2.2. *Mathematical models*

Considering the displacements of the workpiece on the fixture (R, t_Z) as outputs, and the three parameters (μ, P, R_0) as inputs, we can express the relationship between the outputs and inputs by linear equations as follows:

$$R = \mu_R + E(\mu)_R.\mu + E(P)_R.P + E(R_0)_R.R + I(\mu,P)_R.\mu.P + I(\mu,R_0)_R.\mu.R_0 + I(P,R_0)_R.P.R_0 + I(\mu,P,R_0)_R.\mu.P.R_0 \tag{4-29}$$

$$t_Z = \mu_{t_Z} + E(\mu)_{t_Z}.\mu + E(P)_{t_Z}.P + E(R_0)_{t_Z}.R + I(\mu,P)_{t_Z}.\mu.P + I(\mu,R_0)_{t_Z}.\mu.R_0 + I(P,R_0)_{t_Z}.P.R_0 + I(\mu,P,R_0)_{t_Z}.\mu.P.R_0 \tag{4-30}$$

where

- μ_{-} average of an output response
- $E(-)_{-}$ effect of a factor of an output response
- $I(-,-)_{-}$ interaction of two factors of an output response
- $I(-,-,-)_{-}$ interaction of three factors of an output response

The effects and interactions obtained from the following matrix based on the design of experiments method.

$$\begin{bmatrix} Y_1 \\ Y_2 \\ Y_3 \\ Y_4 \\ Y_5 \\ Y_6 \\ Y_7 \\ Y_8 \end{bmatrix} = \begin{bmatrix} +1 & -1 & -1 & -1 & +1 & +1 & +1 & -1 \\ +1 & -1 & -1 & +1 & +1 & -1 & -1 & +1 \\ +1 & -1 & +1 & -1 & -1 & +1 & -1 & +1 \\ +1 & -1 & +1 & +1 & -1 & -1 & +1 & -1 \\ +1 & +1 & -1 & -1 & -1 & -1 & +1 & +1 \\ +1 & +1 & -1 & +1 & -1 & +1 & -1 & -1 \\ +1 & +1 & +1 & -1 & +1 & -1 & -1 & -1 \\ +1 & +1 & +1 & +1 & +1 & +1 & +1 & +1 \end{bmatrix} \begin{bmatrix} \mu_{-} \\ E(\mu)_{-} \\ E(P)_{-} \\ E(R_0)_{-} \\ I(\mu,P)_{-} \\ I(\mu,R_0)_{-} \\ I(P,R_0)_{-} \\ I(\mu,P,R_0)_{-} \end{bmatrix} \tag{4-31}$$

where,

- Y -vector of responses
- Matrix 8×8 is the design matrix

The averages, effects and interactions obtained by the matrix (4-31) are shown as Tab. 4-7.

Parameters	μ_	E(μ)	E(P)	E(R_O)	I(μ, P)	I(μ, R_O)	I(P, R_O)	I(μ, P, R_O)
R	1.084E-03	1.032E-03	2.02E-05	0.195E-03	1.91E-05	0.178E-03	-1.8E-05	-1.6E-05
tz	175.9E-03	27.21E-03	-0.69E-03	116.51E-03	-4.2E-05	-10.31E-03	0.361E-03	2.55E-05

Tab. 4-7 Averages, effects and interactions

The effects of the factors according to the simulations show that there is a difference between rotation and translation components of the positioning defects. For the rotation (R), the coefficient of friction has the greatest effect, and meanwhile the geometrical error of the workpiece locating surface plane has the greatest effect on the translation.

4.3.2.3. Positioning defects results

Using the mathematical models obtained from the previous section for Monte Carlo simulation, three input parameters (μ, P, R_0) and two output parameters (R, t_Z) are used in this simulation (Fig. 4-21), where the defects of workpiece locating plane R_0 were obtained and expressed by a distribution and its parameters in chapter 3. The coefficient of friction μ was recommended using lognormal distribution [STEELE 2008], and the clamping force is proposed using uniform distribution.

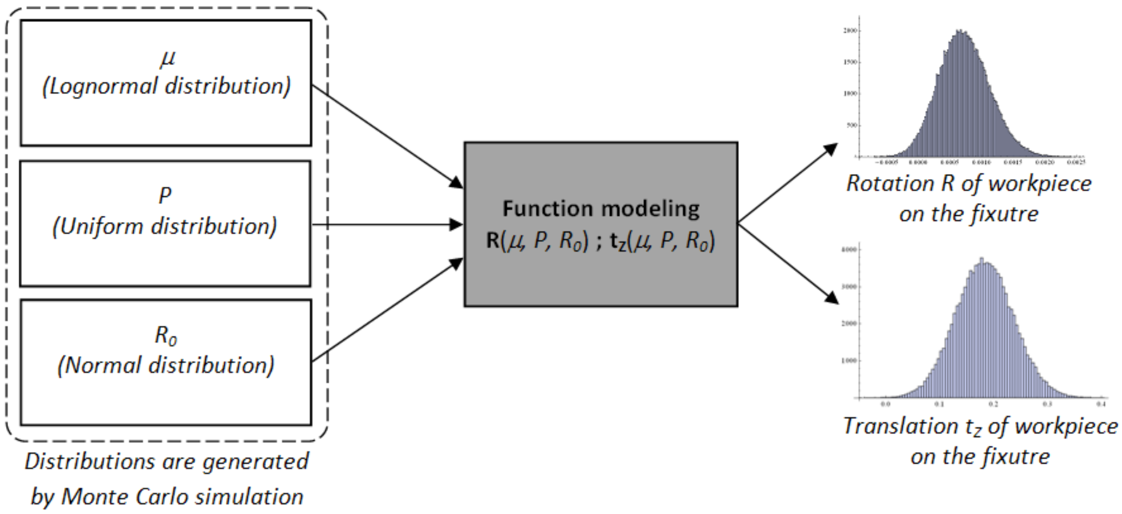
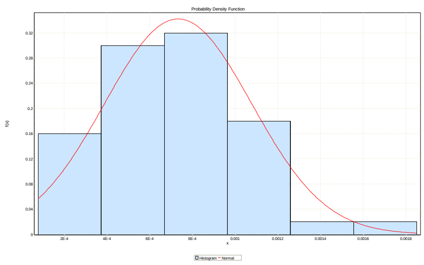
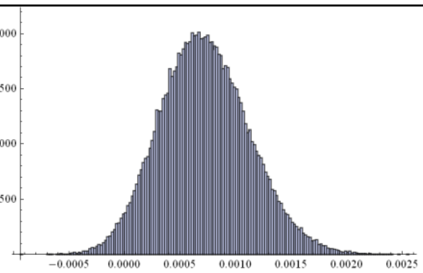
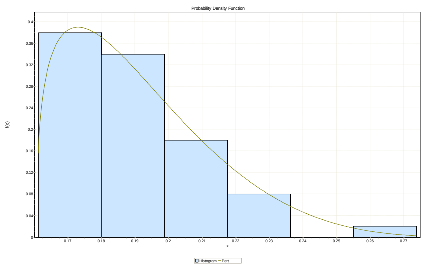
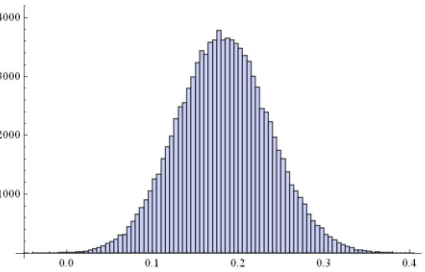


Fig. 4-21 Three input parameters are generated by Monte Carlo simulation

Number of iterations for simulation is a key factor in Monte Carlo simulation. [CVETKO *et al.* 1998] concluded that 10,000 iterations are large enough to obtain accurate results for Monte Carlo simulation.

Results of 100000 iterations of Monte Carlo simulation are shown in Tab. 4-8 for comparison with the experimental results.

Components	Histograms and parameters	
	Experiment	Simulation
R *(WP rotation)*	 $m = 7.335 \times 10^{-4}$ $s = 3.446 \times 10^{-4}$	 $m = 7.24 \times 10^{-4}$ $s = 4.01 \times 10^{-4}$
t_Z *(WP translation along Z-axis)*	 $m = 1.466 \times 10^{-1}$ $s = 2.064 \times 10^{-2}$	 $m = 1.83 \times 10^{-1}$ $s = 5.38 \times 10^{-2}$

Tab. 4-8 Experimental and simulated results

Using F-test for comparison of variances between the experimental and simulated results, this shows that there are no differences between the two rotation components. Nevertheless, differences between two translation components are significant. Thus, the proposed model has just satisfied the rotation component of the positioning defects. Some other factors should perhaps consider in the model, such as gaps between clamps and fixture base.

4.3.3. Conclusion

In this section, the mathematical models of a fixture are generated for the purpose of predicting positioning defects. Three factors that influence the workpiece positions are considered and two displacements of the workpiece are obtained. The factor influences on the components of the positioning defects are different. In addition, the results of Monte Carlo simulation have shown that only the rotation components of the positioning defects is satisfied for this simulation. The model therefore needs to be developed, for instance we need to consider another factor that affects these defects.

5. PRESENTATION OF RESULTS

As the two objectives are mentioned before, the results of the first simulation are used for verification of a process plan. It is important to note that Monte Carlo simulation with the experimental input variables obtains the results presented here.

5.1. Process plan verification

Monte Carlo simulation performs tolerance analysis using a random number generator, which selects values of each manufactured variable, based on the type of statistical distribution obtained from experimental measurements.

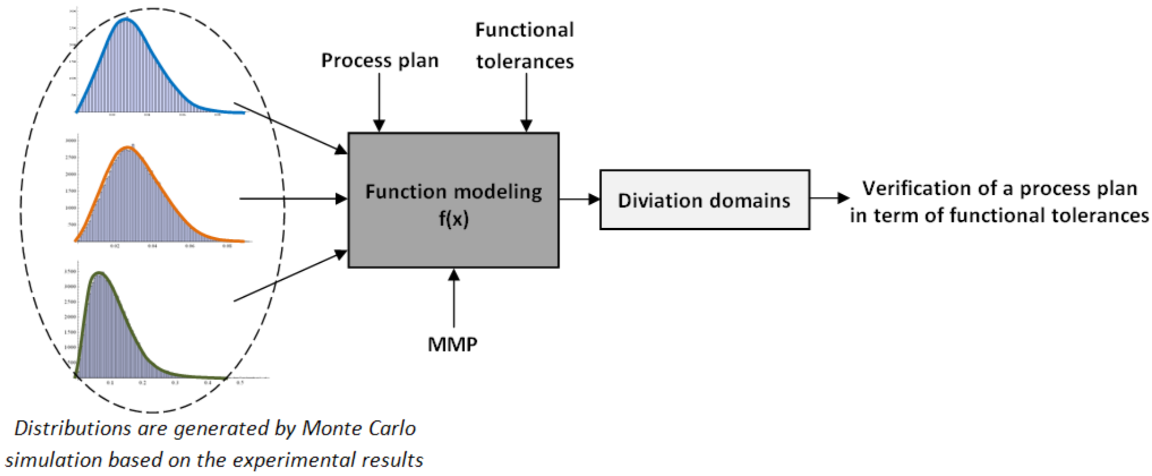


Fig. 4-22 Verification a process plan in term of functional tolerances

A process plan, tolerance requirements of the illustrative part are illustrated in Fig. 4-23.

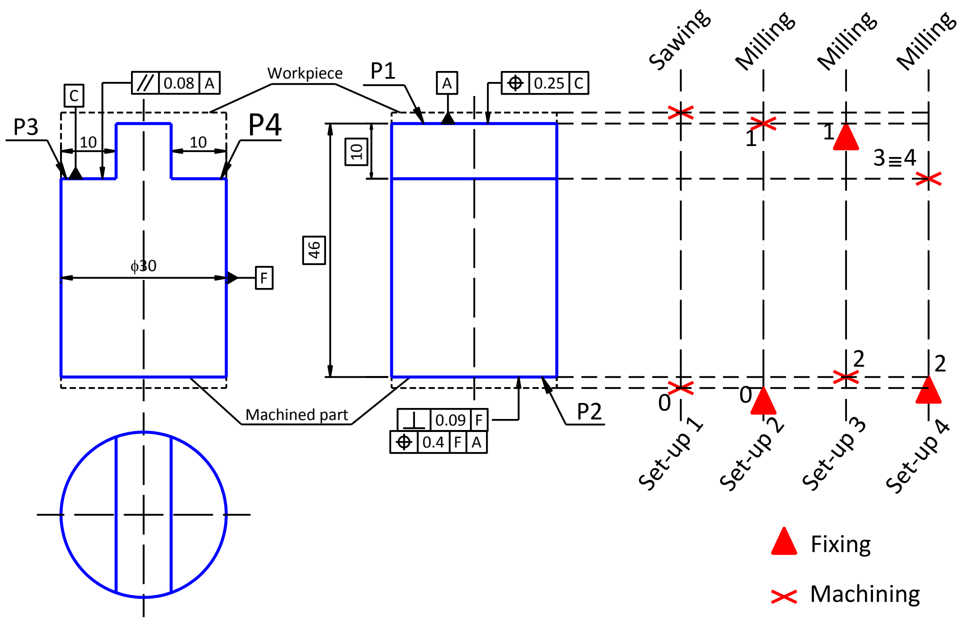
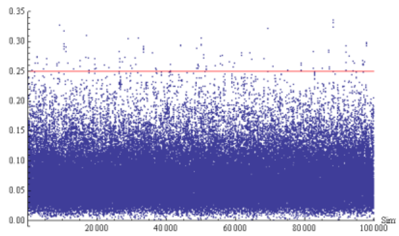
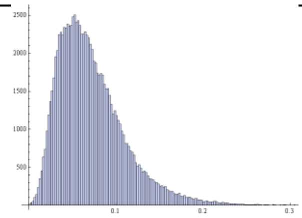
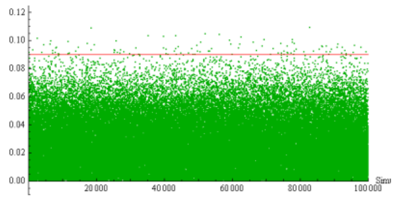
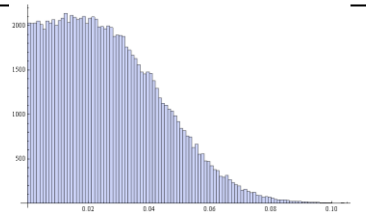
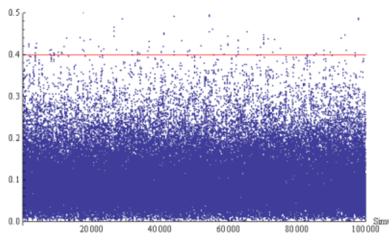
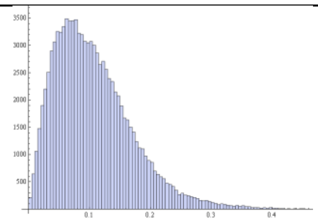
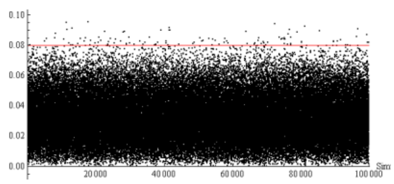
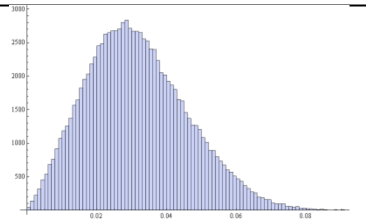


Fig. 4-23 Process plan and tolerance requirements of the illustrative part

The 100000 parts have been virtually manufactured with the above process plan. The defects of each machined surface are then verified to their functional tolerances. The following table shows results of the simulation such as the number of rejected parts, histograms with the associated statistics (estimates of the mean, variance, skewness, and kurtosis).

Machined surface planes	Tolerance requirements		Rejected parts *(part)*	Histograms
P_1	Location (vs. C)	0.25	 119/100000	 $m = 72.225 \times 10^{-3}$ $s = 1.485 \times 10^{-3}$ $skewness = 1.204$ $kurtosis = 5.186$
P_2	Perpendicularity (vs. F)	0.09	 97/100000	 $m = 26.603 \times 10^{-3}$ $s = 0.312 \times 10^{-3}$ $skewness = 0.633$ $kurtosis = 2.952$
	Location (vs. F and A)	0.4	 99/100000	 $m = 107.052 \times 10^{-3}$ $s = 4.157 \times 10^{-3}$ $skewness = 1.075$ $kurtosis = 4.663$
P_3	Parallelism (vs. A)	0.08	 108/100000	 $m = 31.598 \times 10^{-3}$ $s = 0.207 \times 10^{-3}$ $skewness = 0.436$ $kurtosis = 2.916$

Tab. 4-9 Simulation of validation of process plan

Tab. 4-9 shows the rejected parts per 100000 iterations and histograms that can be used to verify the process plan of the illustrative part. For instance, with the tolerance requirements as in Fig. 4-23 and number of rejected parts, e.g. 100 parts per million (or 10 parts per 100000), we can conclude that the above process plan is not satisfied.

Thanks to Monte Carlo simulation, we can also determine the functional tolerances of the illustrative parts to ensure the exiting process plan and number of rejected parts per million. These are shown as follows:

5.2. Determination of tolerances with an existing process plan

The objective is to determine achievable tolerances of a machined part based on a process plan and a ppm (Fig. 4-24).

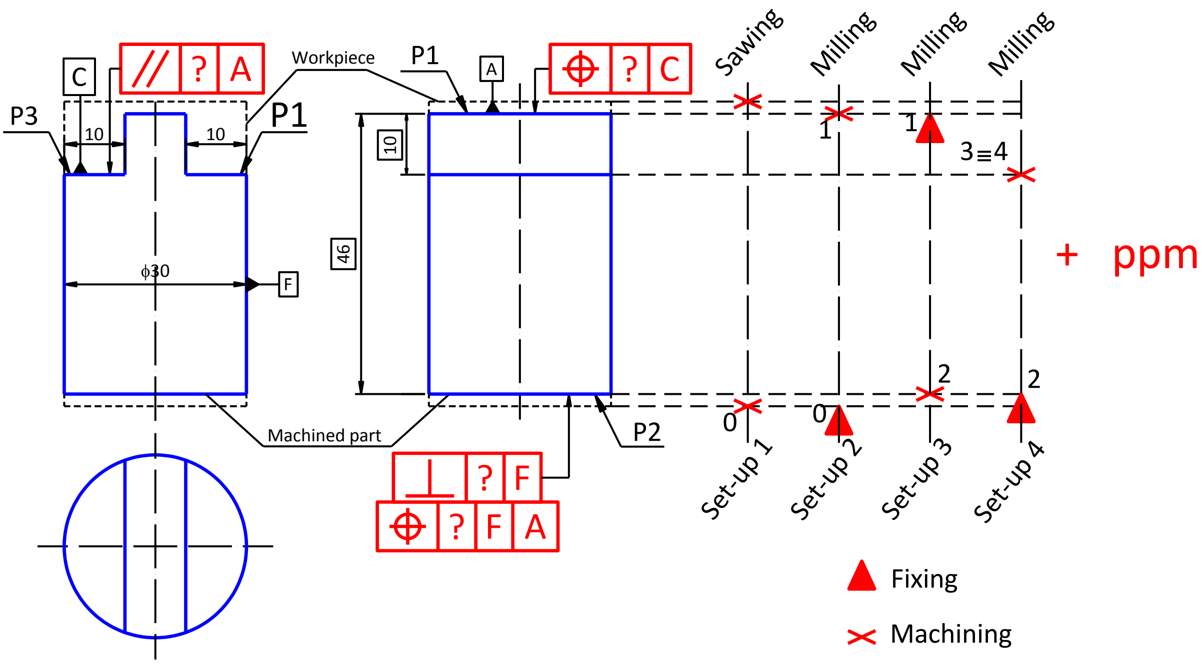


Fig. 4-24 A process plan and a ppm

Using Monte Carlo simulation with the existing process plan, the functional tolerances of the parts are determined to ensure number of rejected parts per million (ppm).

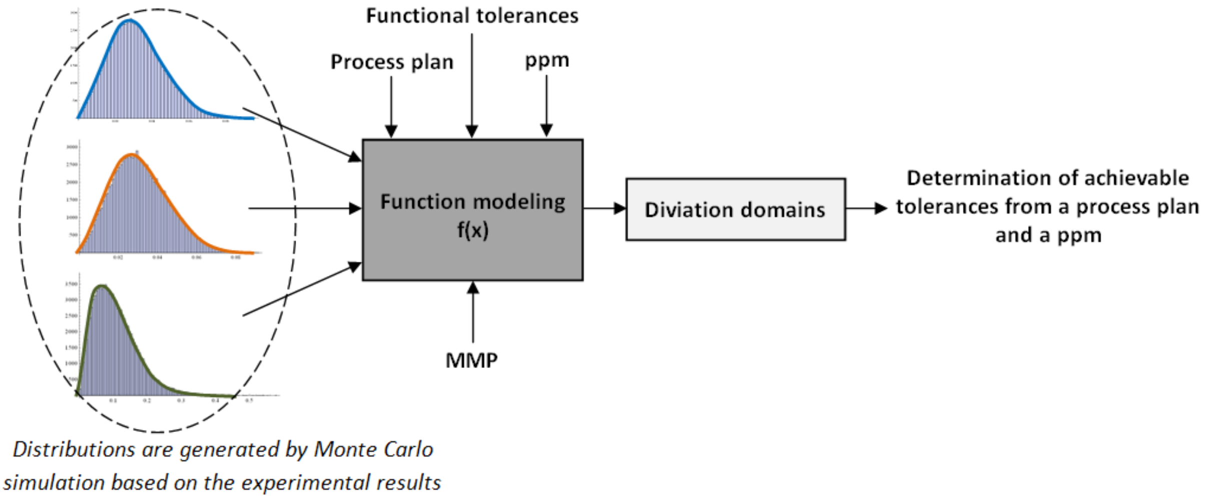


Fig. 4-25 Determination of achievable tolerances from a process plan and a ppm

Tab. 4-10 shows the predicted tolerances that ensure 100 ppm if the above process plan is used for manufacturing.

Machined surface planes	Predicted tolerances for 100 ppm *(mm)*		Initial tolerances *(mm)*
P_1	Location (vs. B)	0.305	0.25 *(1190 ppm)*
P_2	Perpendicularity (vs. F)	0.1	0.09 *(970 ppm)*
	Location (vs. F&A)	0.485	0.4 *(990 ppm)*
P_3	Parallelism (vs. A)	0.09	0.08 *(1080 ppm)*

Tab. 4-10 Predicted functional tolerances

The initial tolerances are also presented in Tab. 4-10 to clearly show the differences between the two results. For instance, if the initial tolerance of the location between machined plane A (P_1) and B is 0.25, the rejected parts per million (ppm) is 1190. Thus, in order to reduce the ppm in this case, the tolerance of the location must be expanded from 0.25 to 0.305.

6. SUMMARY

In this chapter, we introduced a model based on MMP that is used for tolerance analysis in manufacturing. The tolerance analysis is then performed using Monte Carlo simulation with the input variables obtained from experiments or simulations. The simulations can be used for:

- ✓ *verification of a process plan in terms of functional tolerances of a batch of machined parts;*
- ✓ *or determination of optimal tolerances of machined parts based on an existing process plan and requirement of rejected parts per million.*

In addition, we presented a model that combines design of experiment method, finite element model, and Monte Carlo simulation to determine positioning defects of workpieces fixed on a fixture. In this model, different factors, which affect the workpiece positioning, are considered and their effects are evaluated.

The simulation is performed in this chapter based on the results of a particular case that is carried out for obtaining the experimental distribution of manufacturing defects. The model based on MMP can be applied for different cases, but Monte Carlo simulation of this model need to use the input variables that match real results. As mentioned before, if the distributions of the input variables match real results, the outputs are closer to real productions. To have these input variables, we have several propositions as follows:

- Analyze the manufacturing defects of a sample batch of machined parts before carry out a large batch of machined parts;
- Create libraries of manufacturing defects of different types of fixture, different types of machined surfaces. However, this takes time and costly.

Conclusion & Perspectives

The purpose of the work in this thesis is to contribute for analyzing, measuring, and simulation of manufacturing defects in three-dimension. To achieve this we have presented the problems and limitations in identifications and simulations of manufacturing defects during a process. We have developed several methods based on the Small Displacement Torsor (SDT) concept. In addition, we have applied a method for analyzing form defects in order to complete analyses of machining defects.

The first proposed method based on the SDT concept is presented for identification of manufacturing defects. This method allows distinguishing the machining defect and positioning defects of a batch of parts during a process. Several propositions are used in the measurement method based on the study of [TICHADOU 2005], such as:

- Analysing the measurement results obtained from two different measurement means. The advantage of this analysis allows suppressing deviations of the data processing of two different measurement means.
- Evaluating the comparable capability of the measurement results obtained from two different measurement means.
- Determining a translation relationship between two machined planes that may not be parallel because of machining imperfection.
- Explaining the form defects of the machined planes based on the deflection errors of the milling tool and the workpiece.
- Calculating the form defect and orientation defect of the machined planes to complement the explanations of the form defects.
- Verifying whether there is a contact between the workpiece locating plane and the fixture's locating plane.

The results analyzed of the machining and positioning defect are expressed by different forms, e.g. SDT form; flatness, perpendicularity, and parallelism defects of machined surfaces; or distributions and statistical parameters. They can be used in different simulations, for example statistical parameters are usually used in Monte Carlo simulation, and components of SDTs are used in MMP simulations.

Moreover, we applied the parameterization method, which is usually used to analyze form defects of a part measured on a CMM with hundreds of measurement points, to complete the analysis of the form defects with a restricted number of measurement points (10 points on each machined surfaces). Even though, this number appears to be low, the modes of the form defects are almost obtained (comber, undulation, twist, etc).

Because of the important role of the positioning defect in the quality of a product during manufacturing, we proposed two simple indicators for evaluating the global quality of a fixture.

- In the first indicator "determinant", the quality of a fixture is evaluated based on the dispersions of the workpiece displacements on the fixture. This is represented by the absolute value of the determinant of the fixture configuration matrix. In this indicator, a fixture will be considered to be a good fixture if the dispersion of the displacements of the workpiece on the fixture is small.
- In the second indicator "coefficient K", the quality of a fixture is evaluated based on the sensibility of the reacting force at the contact points between the workpiece and the fixture. This is represented by a coefficient that is calculated from the norm of the fixture configuration matrix. In this case, a fixture will be considered to be a good fixture if the influences of an external force (e.g. clamping force), which is applied to the workpiece, are insignificant on the reacting force at the contacts.

The two indicators are then applied in a workpiece-fixture system that is measured on the CMM. The combination of the two indicators can explain the results of this experiment. The experimental results show that a fixture is considered to be a good fixture if the displacements of the workpiece on the fixture are small and the influences of external force (clamping force) on the reacting forces at the contact points are insignificant.

Furthermore, we developed a model for simulating positioning defects of a workpiece fixed on a three-jaw chuck. The model is a combination of three methods: design of experiments, finite element simulation, and Monte Carlo simulation. Three factors, which are assumed to be the most important in positioning defects, are used in this model. Based on the simulated results, the influences of these factors are estimated. The results obtained from simulations can be expressed by form of distributions or statistical parameters. These allow using simulation of tolerance analysis based on Monte Carlo simulation.

Last but not least, a model is developed based on Model of Manufactured Part (MMP) for tolerance analysis. This model is performed by the following steps:

Step 1. Defects of workpiece/machined part and fixture are expressed during a given process plan which may have functional tolerances of machined part or not. Here, the defects are defects of machined surfaces or defects of localization of a workpiece on a fixture in each setup of the process plan.

Step 2. MMP are created based on the SDTs for determining defects of the machined part at the end of manufacturing operations.

Step 3. Deviation domains are then created based on the defects in step 2. These deviation domains are mathematical expressions that will be used as models in Monte Carlo simulation.

Two objectives that can be obtained from Monte Carlo simulation with the mathematical expressions are:

- Verifying a given process plan with functional tolerances of the machined part by determination of the number of machined parts that are out of tolerance zones;
- Or determining functional tolerances of a batch of machined parts based on a given process plan (without functional tolerances) and a number of rejected parts per million.

The input parameters of Monte Carlo simulations can be the experimental results obtained in chapter 2 or a combination of the experimental results and results obtained from the simulation of positioning defects in chapter 4. The results of the analyses are expressed by the distributions and statistical parameters.

For any research of an applied nature, there are always possible extensions. The study was implemented for identification and simulation of defects in manufacturing. Further research is needed to achieve this goal faster and to become more accurate by:

- Developing a program with an interface for analyzing manufacturing defects from measuring data and expressing the defects in different form, e.g. distributions, statistical parameters, SDT form. The objective is easier for users;
- Measuring workpieces/machined parts by measurement means that are faster and more accurate, e.g. laser sensor, optical sensor, laser-scanner sensor;
- Investigating the simple indicators for different types of fixture, e.g. three-jaw chuck, and considering influences of change of external forces (e.g. clamping force) on the quality of workpiece localizations;

- Considering different important factors in the model of positioning defect simulation, e.g. gaps between clamps and fixture base in three-jaw chuck, and optimizing simulation time.

To have the input variables match real results that are used in Monte Carlo, we have several propositions:

- Analyze the manufacturing defects of a sample batch of machined parts before carry out a large batch of machined parts;
- Create libraries of manufacturing defects of different types of fixture, different types of machined surfaces. However, this takes time and costly.

Appendix

APPENDIX 1 – STATISTIC

All the measured results are treated by the statistical analysis, such as:

- Calculating the means, the standard deviations, and the variances of the manufacturing defects;
- Considering the relationship of two variables, e.g. a correlation;
- Comparing the variances of different variables;
- Expressing the forms of distributions and statistical parameters of the manufacturing defects.

We summarize some knowledge of statistics as follows:

Descriptive statistics of a variable

Descriptive statistics are used to describe the basic features of the data in a study. They can be the numbers, tables, chart, graphs, etc. In this thesis, the descriptive statistics of a variable are expressed by: a mean, a standard deviation/a variance, a type of a distribution, a graph. We also use skew index (symmetry) to evaluate how concentrated data are at the low or high end of the scale, and kurtosis index (peakedness) to evaluate how concentrated data are around a single value.

Evaluating the relationship of two variable

In addition, to evaluate the relationship of two variables, the correlation is one of the most common and most useful statistics. A correlation is a single number that describes the degree of relationship between two variables. Furthermore, we should use the bivariate plot during the evaluation.

There are different types of correlations for different circumstances. For instance, if we have two ordinal variables, we could use the Spearman rank Order Correlation (rho) or the Kendall rank order Correlation (tau). When both variables are measured at an interval level, the Pearson Product Moment Correlation can be used. This is a measure of linear relationship between two variables, which ranges from +1 to -1. If one variable tends to increases as the other decreases, the correlation coefficient is negative. Conversely, if the two variables tend to increase together the correlation coefficient is positive.

Probability distributions

Probability distributions are a fundamental concept in statistics. In practical uses, the probability can be used:

- To calculated confidence intervals for parameters and to calculate regions for hypothesis tests.
- For univariate data, it is often useful to determine a reasonable distributional model for the data.
- For simulation studies with random numbers generated from using a specific probability distribution are often needed.
- For verifying specific distributional assumptions that are used for computing statistical intervals and hypothesis tests.

There are a large number of distributions used in statistical applications. We provide several distributions that are used in this thesis.

Normal distribution

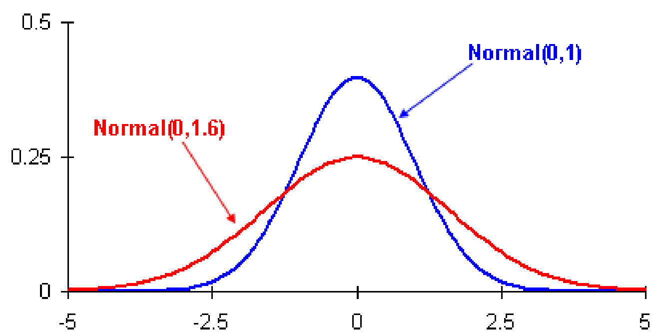
The general formula for probability density function of the normal distribution is:

$$f(x) = \frac{\exp\left(-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right)}{\sigma\sqrt{2\pi}}$$

where

- σ is the continuous scale parameter ($\sigma > 0$)
- μ is the continuous location parameter

The case where $\mu = 0$ and $\sigma = 1$ is called the standard normal distribution. The following is the plot of the standard normal probability density function, Normal(μ, σ).



Pert distribution

The Pert distribution is a special case of the Beta distribution specified by the parameters:

$$\alpha_1 = \frac{4m + b - 5a}{b - a}$$

$$\alpha_2 = \frac{5b - a - 4m}{b - a}$$

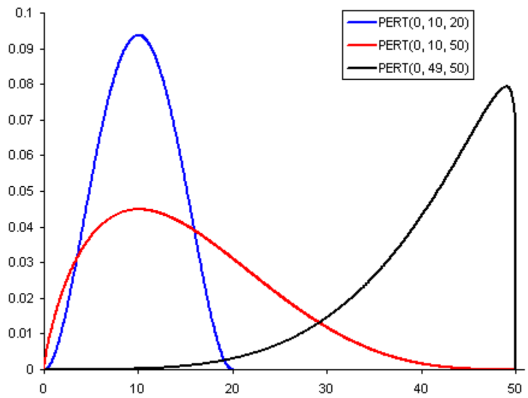
and the probability density function:

$$f(x) = \frac{1}{B(\alpha_1, \alpha_2)} \frac{(x - a)^{\alpha_1 - 1}(b - x)^{\alpha_2 - 1}}{(b - a)^{\alpha_1 + \alpha_2 - 1}}$$

where

- m is the continuous mode parameter ($a \leq m \leq b$)
- a, b is the continuous boundary parameter ($a < b$)
- B is the Beta function

The following is the plot of the standard Pert probability density function.



Test for equality of variances

This test is used to test if the variances of two populations are equal. There are different tests for different circumstances. For instance:

- Use Bartlett's test when the data come from normal distribution; Bartlett's test is not robust to departures from normality.
- Use Levene's test when the data come from continuous, but not necessarily normal distributions. This method considers the distances of the observation from their sample median rather than their sample mean, makes the test more robust for smaller samples.

If there are only two levels, an F-test is performed in place of Bartlett's test.

APPENDIX 2 – LEAST-SQUARES BEST-FIT METHOD

We provide here the least-squares best-fit method that is presented by [FORBES 1989]. This is one of the methods for reconstructing surfaces from measuring data. Different specific geometries can be considered, such as lines in a specified plane, lines in 3 dimensions, planes, circle in a specified plane, circles in 3 dimensions, spheres, cylinders, and cones. The following presents the plane and cylinder algorithms that are used in this thesis.

Cylinder

This method is allowed to reconstruct a cylinder from measuring points.

We assume that we wish to fit a cylinder to i points (x_i, y_i, z_i), where i is the measured points $(i = 13)$.

We specify a cylinder by:

- a point (x_0, y_0, z_0),
- a vector (a, b, c)pointing along the axis,
- and its radius.

To parameterize a cylinder properly we need a systematic way of deciding which point on the axis chooses, along with a constraint on(a, b, c).

Distance from a point to a cylinder

Given a cylinder specified by x_0, y_0, z_0, a, b, c and r as above, the distance from a point x_i, y_i, z_i to the cylinder is found from:

$$d_i = r_i - r$$

where

$$r_i = \frac{\sqrt{u_i^2 + v_i^2 + w_i^2}}{\sqrt{a^2 + b^2 + c^2}}$$

with

$$u_i = c(y_i - y_0) - b(z_i - z_0)$$

$$v_i = a(z_i - z_0) - b(x_i - x_0)$$

$$w_i = b(x_i - x_0) - b(y_i - y_0)$$

r_i is the distance of the i^{th} point to the cylinder axis; the reader may note that (u_i, v_i, w_i)can be expressed as a vector cross-product of $x[(x_i, y_i, z_i) - (x_0, y_0, z_0)]$ with (a, b, c).

From the point to the cylinder and we form the sum:

$$E = \sum_{i=1}^{n} d_i^2$$

$$v_i = a(z_i - z_0) - b(x_i - x_0)$$

$$w_i = b(x_i - x_0) - b(y_i - y_0)$$

This sum depend on the parameters $(x_0, y_0, z_0), (a, b, c)$, and r. Finding the best-fit cylinder amounts to find the values of the parameters which makes the sum of squares E take on its minimum value – hence the term least-squares fit. The Gauss-Newton algorithm is used to minimize the sum to squares of the distances. Therefore, the parameters $(x_0, y_0, z_0), (a, b, c)$, and r are obtained by the minimize process.

Plane

The procedure to fit a plane to n data points (x_i, y_i, z_i), where $i =$ from 1 to n, is given below.

A plane can be specified by:

- a point (x_0, y_0, z_0) on the plane,
- and the direction cosines (a, b, c) of the normal to the plane.

Any point (x, y, z) on the plane satisfies:

$$a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$$

It is known that the distance from a point (x_i, y_i, z_i) to a plane specified by x_0, y_0, z_0 and a, b, c is given by:

$$d_i = a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$$

The best-fit plane passes through the centroid (x_C, y_C, z_C) of the data and this specified a point on the P also the direction cosines have to be found out.

For this, (a, b, c) is the eigen vector associated with the smallest eigen value of:

$$m = M^T \cdot M$$

where

- M the matrix is formulated such that its first column is $x_i - \bar{x}$, second column $y_i - \bar{y}$ and third column $z_i - \bar{z}$.
- M^T is the transformation matrix of M.
- $\bar{x}, \bar{y}, \bar{z}$ is the average points.

$$\bar{x} = \sum_{i=1}^{n} \frac{x_i}{n}; \ \bar{y} = \sum_{i=1}^{n} \frac{y_i}{n}; \ \bar{z} = \sum_{i=1}^{n} \frac{z_i}{n}$$

Finally, we can obtain an associated plane that is represented by a point and a normal vector of this plane from the 3D measuring points.

APPENDIX 3 – RAW DATA

The following tables show the raw data that are obtained from measurements on the CNC machine and CMM.

N° part	N° point	Plan 1 (CNC)			Plan 2 (CNC)		
		X	Y	Z	X	Y	Z
1	1	-10	-8	48.1172	2.5	-8	38.1271
	2	-2.5	-8	48.1248	10	-8	38.1224
	3	-10	-4	48.1208	2.5	-4	38.1273
	4	-2.5	-4	48.1311	10	-4	38.1217
	5	-10	0	48.1207	2.5	0	38.1266
	6	-2.5	0	48.1369	10	0	38.1217
	7	-10	4	48.12	2.5	4	38.1262
	8	-2.5	4	48.1324	10	4	38.1224
	9	-10	8	48.1198	2.5	8	38.1268
	10	-2.5	8	48.1269	10	8	38.124
2	1	-10	-8	48.1238	2.5	-8	38.1287
	2	-2.5	-8	48.1308	10	-8	38.1238
	3	-10	-4	48.127	2.5	-4	38.1286
	4	-2.5	-4	48.1348	10	-4	38.1228
	5	-10	0	48.1276	2.5	0	38.128
	6	-2.5	0	48.1398	10	0	38.1237
	7	-10	4	48.1268	2.5	4	38.1274
	8	-2.5	4	48.1359	10	4	38.1241
	9	-10	8	48.1256	2.5	8	38.1281
	10	-2.5	8	48.1318	10	8	38.1255
3	1	-10	-8	48.124	2.5	-8	38.1291
	2	-2.5	-8	48.1305	10	-8	38.1266
	3	-10	-4	48.127	2.5	-4	38.1287
	4	-2.5	-4	48.1356	10	-4	38.1262
	5	-10	0	48.1268	2.5	0	38.1285
	6	-2.5	0	48.1406	10	0	38.1263
	7	-10	4	48.126	2.5	4	38.1289
	8	-2.5	4	48.1378	10	4	38.1268
	9	-10	8	48.1256	2.5	8	38.129
	10	-2.5	8	48.133	10	8	38.1283
4	1	-10	-8	48.1256	2.5	-8	38.1305
	2	-2.5	-8	48.1318	10	-8	38.1252
	3	-10	-4	48.1279	2.5	-4	38.1299
	4	-2.5	-4	48.1367	10	-4	38.124
	5	-10	0	48.1286	2.5	0	38.1288
	6	-2.5	0	48.1417	10	0	38.1239
	7	-10	4	48.1283	2.5	4	38.129
	8	-2.5	4	48.1383	10	4	38.1247
	9	-10	8	48.1271	2.5	8	38.1293
	10	-2.5	8	48.1337	10	8	38.1263
5	1	-10	-8	48.1237	2.5	-8	38.1309
	2	-2.5	-8	48.1309	10	-8	38.1269
	3	-10	-4	48.1273	2.5	-4	38.1307
	4	-2.5	-4	48.1358	10	-4	38.1262
	5	-10	0	48.1275	2.5	0	38.1304
	6	-2.5	0	48.1408	10	0	38.1265
	7	-10	4	48.1268	2.5	4	38.131
	8	-2.5	4	48.1372	10	4	38.1266
	9	-10	8	48.1253	2.5	8	38.1308
	10	-2.5	8	48.1324	10	8	38.1271
6	1	-10	-8	48.1251	2.5	-8	38.1305
	2	-2.5	-8	48.1323	10	-8	38.1279
	3	-10	-4	48.1288	2.5	-4	38.1305
	4	-2.5	-4	48.137	10	-4	38.1283
	5	-10	0	48.1288	2.5	0	38.1305
	6	-2.5	0	48.1418	10	0	38.1286
	7	-10	4	48.1281	2.5	4	38.13
	8	-2.5	4	48.1379	10	4	38.1283
	9	-10	8	48.1263	2.5	8	38.1308
	10	-2.5	8	48.133	10	8	38.1291

N° part	N° point	Plan 1 (CNC)			Plan 2 (CNC)		
		X	Y	Z	X	Y	Z
7	1	-10	-8	48.1258	2.5	-8	38.1311
	2	-2.5	-8	48.1339	10	-8	38.1279
	3	-10	-4	48.1292	2.5	-4	38.1312
	4	-2.5	-4	48.1389	10	-4	38.1279
	5	-10	0	48.1297	2.5	0	38.1309
	6	-2.5	0	48.1435	10	0	38.1284
	7	-10	4	48.129	2.5	4	38.1311
	8	-2.5	4	48.1405	10	4	38.1289
	9	-10	8	48.1279	2.5	8	38.1311
	10	-2.5	8	48.1359	10	8	38.1299
8	1	-10	-8	48.127	2.5	-8	38.1318
	2	-2.5	-8	48.1337	10	-8	38.1289
	3	-10	-4	48.1299	2.5	-4	38.1319
	4	-2.5	-4	48.1387	10	-4	38.1278
	5	-10	0	48.1302	2.5	0	38.1316
	6	-2.5	0	48.1436	10	0	38.1286
	7	-10	4	48.1297	2.5	4	38.1315
	8	-2.5	4	48.1401	10	4	38.1292
	9	-10	8	48.1285	2.5	8	38.1312
	10	-2.5	8	48.135	10	8	38.1304
9	1	-10	-8	48.1265	2.5	-8	38.1326
	2	-2.5	-8	48.1338	10	-8	38.1299
	3	-10	-4	48.1294	2.5	-4	38.1318
	4	-2.5	-4	48.1388	10	-4	38.129
	5	-10	0	48.1297	2.5	0	38.1318
	6	-2.5	0	48.1435	10	0	38.1291
	7	-10	4	48.1285	2.5	4	38.1318
	8	-2.5	4	48.1403	10	4	38.1292
	9	-10	8	48.1271	2.5	8	38.1321
	10	-2.5	8	48.1345	10	8	38.1308
10	1	-10	-8	48.1262	2.5	-8	38.1325
	2	-2.5	-8	48.1341	10	-8	38.1273
	3	-10	-4	48.1301	2.5	-4	38.1329
	4	-2.5	-4	48.1389	10	-4	38.1267
	5	-10	0	48.1305	2.5	0	38.1318
	6	-2.5	0	48.1432	10	0	38.1263
	7	-10	4	48.1295	2.5	4	38.1319
	8	-2.5	4	48.14	10	4	38.1276
	9	-10	8	48.1287	2.5	8	38.1324
	10	-2.5	8	48.1348	10	8	38.1292
11	1	-10	-8	48.1269	2.5	-8	38.1325
	2	-2.5	-8	48.134	10	-8	38.1276
	3	-10	-4	48.1293	2.5	-4	38.132
	4	-2.5	-4	48.1383	10	-4	38.1276
	5	-10	0	48.1291	2.5	0	38.1329
	6	-2.5	0	48.1432	10	0	38.1273
	7	-10	4	48.1288	2.5	4	38.1321
	8	-2.5	4	48.1402	10	4	38.1279
	9	-10	8	48.1272	2.5	8	38.1326
	10	-2.5	8	48.1349	10	8	38.1288
12	1	-10	-8	48.1256	2.5	-8	38.1324
	2	-2.5	-8	48.1341	10	-8	38.1276
	3	-10	-4	48.1294	2.5	-4	38.1322
	4	-2.5	-4	48.1391	10	-4	38.1264
	5	-10	0	48.1299	2.5	0	38.1316
	6	-2.5	0	48.1433	10	0	38.1264
	7	-10	4	48.1291	2.5	4	38.1317
	8	-2.5	4	48.1403	10	4	38.1275
	9	-10	8	48.1278	2.5	8	38.1319
	10	-2.5	8	48.1348	10	8	38.129

N° part	N° point	Plan 1 (CNC)			Plan 2 (CNC)		
		X	Y	Z	X	Y	Z
13	1	-10	-8	48.1265	2.5	-8	38.133
	2	-2.5	-8	48.1337	10	-8	38.1294
	3	-10	-4	48.1298	2.5	-4	38.1325
	4	-2.5	-4	48.1386	10	-4	38.1286
	5	-10	0	48.1301	2.5	0	38.1323
	6	-2.5	0	48.144	10	0	38.1289
	7	-10	4	48.1292	2.5	4	38.1322
	8	-2.5	4	48.1406	10	4	38.1296
	9	-10	8	48.1275	2.5	8	38.1323
	10	-2.5	8	48.1351	10	8	38.1309
14	1	-10	-8	48.127	2.5	-8	38.1325
	2	-2.5	-8	48.1348	10	-8	38.1286
	3	-10	-4	48.1295	2.5	-4	38.1321
	4	-2.5	-4	48.1394	10	-4	38.128
	5	-10	0	48.1297	2.5	0	38.1323
	6	-2.5	0	48.1445	10	0	38.1284
	7	-10	4	48.1292	2.5	4	38.1321
	8	-2.5	4	48.1406	10	4	38.1291
	9	-10	8	48.1283	2.5	8	38.1319
	10	-2.5	8	48.1353	10	8	38.1309
15	1	-10	-8	48.1264	2.5	-8	38.1326
	2	-2.5	-8	48.1339	10	-8	38.1291
	3	-10	-4	48.1295	2.5	-4	38.1326
	4	-2.5	-4	48.1394	10	-4	38.1281
	5	-10	0	48.1305	2.5	0	38.1327
	6	-2.5	0	48.1444	10	0	38.1285
	7	-10	4	48.1292	2.5	4	38.1325
	8	-2.5	4	48.1406	10	4	38.1294
	9	-10	8	48.1278	2.5	8	38.1324
	10	-2.5	8	48.1351	10	8	38.131
16	1	-10	-8	48.1272	2.5	-8	38.1327
	2	-2.5	-8	48.134	10	-8	38.1272
	3	-10	-4	48.1305	2.5	-4	38.1329
	4	-2.5	-4	48.1391	10	-4	38.1278
	5	-10	0	48.1309	2.5	0	38.1326
	6	-2.5	0	48.1443	10	0	38.1279
	7	-10	4	48.1306	2.5	4	38.1326
	8	-2.5	4	48.1409	10	4	38.1275
	9	-10	8	48.1294	2.5	8	38.1328
	10	-2.5	8	48.1346	10	8	38.1286
17	1	-10	-8	48.1273	2.5	-8	38.1334
	2	-2.5	-8	48.1344	10	-8	38.1295
	3	-10	-4	48.1307	2.5	-4	38.1331
	4	-2.5	-4	48.1397	10	-4	38.1283
	5	-10	0	48.1306	2.5	0	38.1323
	6	-2.5	0	48.1441	10	0	38.1283
	7	-10	4	48.1299	2.5	4	38.1328
	8	-2.5	4	48.1414	10	4	38.1296
	9	-10	8	48.1295	2.5	8	38.1331
	10	-2.5	8	48.1363	10	8	38.1308
18	1	-10	-8	48.1269	2.5	-8	38.1332
	2	-2.5	-8	48.1341	10	-8	38.1279
	3	-10	-4	48.1301	2.5	-4	38.1328
	4	-2.5	-4	48.1398	10	-4	38.1268
	5	-10	0	48.1309	2.5	0	38.1325
	6	-2.5	0	48.1448	10	0	38.1271
	7	-10	4	48.1301	2.5	4	38.1335
	8	-2.5	4	48.1417	10	4	38.1284
	9	-10	8	48.1287	2.5	8	38.1331
	10	-2.5	8	48.1359	10	8	38.1294
19	1	-10	-8	48.1267	2.5	-8	38.132
	2	-2.5	-8	48.1339	10	-8	38.1277
	3	-10	-4	48.1294	2.5	-4	38.1325
	4	-2.5	-4	48.1393	10	-4	38.1269
	5	-10	0	48.13	2.5	0	38.1323
	6	-2.5	0	48.1444	10	0	38.1271
	7	-10	4	48.1297	2.5	4	38.1325
	8	-2.5	4	48.1407	10	4	38.1285
	9	-10	8	48.1289	2.5	8	38.1331
	10	-2.5	8	48.135	10	8	38.1304

N° part	N° point	Plan 1 (CNC)			Plan 2 (CNC)		
		X	Y	Z	X	Y	Z
20	1	-10	-8	48.1267	2.5	-8	38.1346
	2	-2.5	-8	48.134	10	-8	38.1298
	3	-10	-4	48.1296	2.5	-4	38.134
	4	-2.5	-4	48.1396	10	-4	38.1292
	5	-10	0	48.1302	2.5	0	38.1341
	6	-2.5	0	48.1442	10	0	38.1295
	7	-10	4	48.1297	2.5	4	38.1339
	8	-2.5	4	48.1412	10	4	38.1295
	9	-10	8	48.1285	2.5	8	38.1351
	10	-2.5	8	48.1354	10	8	38.1305
21	1	-10	-8	48.127	2.5	-8	38.134
	2	-2.5	-8	48.1344	10	-8	38.1281
	3	-10	-4	48.1303	2.5	-4	38.1337
	4	-2.5	-4	48.1405	10	-4	38.1271
	5	-10	0	48.1308	2.5	0	38.1339
	6	-2.5	0	48.1445	10	0	38.1272
	7	-10	4	48.1299	2.5	4	38.134
	8	-2.5	4	48.1416	10	4	38.1279
	9	-10	8	48.1292	2.5	8	38.134
	10	-2.5	8	48.1358	10	8	38.1301
22	1	-10	-8	48.1278	2.5	-8	38.1341
	2	-2.5	-8	48.1346	10	-8	38.1295
	3	-10	-4	48.13	2.5	-4	38.1334
	4	-2.5	-4	48.1404	10	-4	38.1284
	5	-10	0	48.1309	2.5	0	38.1339
	6	-2.5	0	48.1449	10	0	38.1284
	7	-10	4	48.1298	2.5	4	38.1333
	8	-2.5	4	48.1415	10	4	38.1292
	9	-10	8	48.1298	2.5	8	38.1339
	10	-2.5	8	48.136	10	8	38.1309
23	1	-10	-8	48.1277	2.5	-8	38.1343
	2	-2.5	-8	48.1351	10	-8	38.1302
	3	-10	-4	48.13	2.5	-4	38.1342
	4	-2.5	-4	48.1407	10	-4	38.1292
	5	-10	0	48.1308	2.5	0	38.1341
	6	-2.5	0	48.1454	10	0	38.1293
	7	-10	4	48.1301	2.5	4	38.1337
	8	-2.5	4	48.1423	10	4	38.1292
	9	-10	8	48.1295	2.5	8	38.1342
	10	-2.5	8	48.1366	10	8	38.1313
24	1	-10	-8	48.1282	2.5	-8	38.1347
	2	-2.5	-8	48.1347	10	-8	38.129
	3	-10	-4	48.1303	2.5	-4	38.1345
	4	-2.5	-4	48.1403	10	-4	38.128
	5	-10	0	48.1313	2.5	0	38.1341
	6	-2.5	0	48.1451	10	0	38.1285
	7	-10	4	48.1306	2.5	4	38.1338
	8	-2.5	4	48.1419	10	4	38.1283
	9	-10	8	48.129	2.5	8	38.134
	10	-2.5	8	48.1364	10	8	38.1297
25	1	-10	-8	48.1278	2.5	-8	38.1346
	2	-2.5	-8	48.1356	10	-8	38.1294
	3	-10	-4	48.1305	2.5	-4	38.1341
	4	-2.5	-4	48.1417	10	-4	38.1294
	5	-10	0	48.1318	2.5	0	38.1342
	6	-2.5	0	48.1455	10	0	38.1293
	7	-10	4	48.131	2.5	4	38.1342
	8	-2.5	4	48.1426	10	4	38.13
	9	-10	8	48.13	2.5	8	38.1346
	10	-2.5	8	48.1366	10	8	38.132
26	1	-10	-8	48.1288	2.5	-8	38.1342
	2	-2.5	-8	48.1359	10	-8	38.1297
	3	-10	-4	48.131	2.5	-4	38.1343
	4	-2.5	-4	48.1419	10	-4	38.1289
	5	-10	0	48.1321	2.5	0	38.1344
	6	-2.5	0	48.1461	10	0	38.1294
	7	-10	4	48.1312	2.5	4	38.1345
	8	-2.5	4	48.1429	10	4	38.1299
	9	-10	8	48.1308	2.5	8	38.1347
	10	-2.5	8	48.1378	10	8	38.1316

N° part	N° point	Plan 1 (CNC)			Plan 2 (CNC)		
		X	Y	Z	X	Y	Z
27	1	-10	-8	48.1279	2.5	-8	38.1341
	2	-2.5	-8	48.1351	10	-8	38.1284
	3	-10	-4	48.1302	2.5	-4	38.1341
	4	-2.5	-4	48.1409	10	-4	38.1284
	5	-10	0	48.1306	2.5	0	38.1342
	6	-2.5	0	48.1455	10	0	38.1284
	7	-10	4	48.1302	2.5	4	38.1341
	8	-2.5	4	48.1415	10	4	38.1288
	9	-10	8	48.1282	2.5	8	38.1347
	10	-2.5	8	48.136	10	8	38.1308
28	1	-10	-8	48.1289	2.5	-8	38.1344
	2	-2.5	-8	48.1353	10	-8	38.1295
	3	-10	-4	48.1311	2.5	-4	38.1344
	4	-2.5	-4	48.1412	10	-4	38.1283
	5	-10	0	48.1313	2.5	0	38.1342
	6	-2.5	0	48.1453	10	0	38.1282
	7	-10	4	48.1305	2.5	4	38.1347
	8	-2.5	4	48.1426	10	4	38.1296
	9	-10	8	48.1304	2.5	8	38.1349
	10	-2.5	8	48.1374	10	8	38.1319
29	1	-10	-8	48.1284	2.5	-8	38.1347
	2	-2.5	-8	48.1351	10	-8	38.1297
	3	-10	-4	48.131	2.5	-4	38.1351
	4	-2.5	-4	48.1414	10	-4	38.1292
	5	-10	0	48.1316	2.5	0	38.135
	6	-2.5	0	48.1452	10	0	38.1292
	7	-10	4	48.1304	2.5	4	38.1346
	8	-2.5	4	48.1425	10	4	38.1295
	9	-10	8	48.1293	2.5	8	38.1349
	10	-2.5	8	48.1363	10	8	38.1309
30	1	-10	-8	48.1286	2.5	-8	38.1351
	2	-2.5	-8	48.1358	10	-8	38.1302
	3	-10	-4	48.1309	2.5	-4	38.1348
	4	-2.5	-4	48.1416	10	-4	38.1292
	5	-10	0	48.1316	2.5	0	38.1348
	6	-2.5	0	48.146	10	0	38.1297
	7	-10	4	48.1316	2.5	4	38.1347
	8	-2.5	4	48.143	10	4	38.1301
	9	-10	8	48.1306	2.5	8	38.1352
	10	-2.5	8	48.1372	10	8	38.1317
31	1	-10	-8	48.1287	2.5	-8	38.1347
	2	-2.5	-8	48.1351	10	-8	38.1297
	3	-10	-4	48.1305	2.5	-4	38.1348
	4	-2.5	-4	48.1411	10	-4	38.1291
	5	-10	0	48.1311	2.5	0	38.1347
	6	-2.5	0	48.1461	10	0	38.1291
	7	-10	4	48.1313	2.5	4	38.1344
	8	-2.5	4	48.1434	10	4	38.1293
	9	-10	8	48.1299	2.5	8	38.1347
	10	-2.5	8	48.1377	10	8	38.1304
32	1	-10	-8	48.1281	2.5	-8	38.1346
	2	-2.5	-8	48.1362	10	-8	38.1284
	3	-10	-4	48.1311	2.5	-4	38.1343
	4	-2.5	-4	48.1422	10	-4	38.1283
	5	-10	0	48.1318	2.5	0	38.1345
	6	-2.5	0	48.1464	10	0	38.1285
	7	-10	4	48.131	2.5	4	38.1344
	8	-2.5	4	48.1436	10	4	38.1294
	9	-10	8	48.1306	2.5	8	38.1349
	10	-2.5	8	48.1378	10	8	38.1313
33	1	-10	-8	48.1284	2.5	-8	38.1341
	2	-2.5	-8	48.1353	10	-8	38.1287
	3	-10	-4	48.1306	2.5	-4	38.1345
	4	-2.5	-4	48.1412	10	-4	38.1283
	5	-10	0	48.1312	2.5	0	38.1349
	6	-2.5	0	48.1449	10	0	38.1279
	7	-10	4	48.1302	2.5	4	38.1343
	8	-2.5	4	48.1413	10	4	38.1291
	9	-10	8	48.1291	2.5	8	38.1348
	10	-2.5	8	48.1358	10	8	38.1309

N° part	N° point	Plan 1 (CNC)			Plan 2 (CNC)		
		X	Y	Z	X	Y	Z
34	1	-10	-8	48.1285	2.5	-8	38.1352
	2	-2.5	-8	48.1346	10	-8	38.1298
	3	-10	-4	48.1304	2.5	-4	38.1351
	4	-2.5	-4	48.1405	10	-4	38.1293
	5	-10	0	48.1312	2.5	0	38.1351
	6	-2.5	0	48.1451	10	0	38.1295
	7	-10	4	48.1311	2.5	4	38.1346
	8	-2.5	4	48.1423	10	4	38.1303
	9	-10	8	48.1298	2.5	8	38.1353
	10	-2.5	8	48.1371	10	8	38.1315
35	1	-10	-8	48.1284	2.5	-8	38.1349
	2	-2.5	-8	48.1353	10	-8	38.129
	3	-10	-4	48.1307	2.5	-4	38.1345
	4	-2.5	-4	48.1418	10	-4	38.1285
	5	-10	0	48.1318	2.5	0	38.1344
	6	-2.5	0	48.1461	10	0	38.1284
	7	-10	4	48.1311	2.5	4	38.134
	8	-2.5	4	48.1428	10	4	38.1291
	9	-10	8	48.1302	2.5	8	38.1346
	10	-2.5	8	48.1366	10	8	38.1313
36	1	-10	-8	48.1288	2.5	-8	38.1347
	2	-2.5	-8	48.1355	10	-8	38.1286
	3	-10	-4	48.1304	2.5	-4	38.1347
	4	-2.5	-4	48.1411	10	-4	38.1281
	5	-10	0	48.1311	2.5	0	38.1346
	6	-2.5	0	48.146	10	0	38.1281
	7	-10	4	48.131	2.5	4	38.1343
	8	-2.5	4	48.1429	10	4	38.1288
	9	-10	8	48.1303	2.5	8	38.1344
	10	-2.5	8	48.1371	10	8	38.1308
37	1	-10	-8	48.1285	2.5	-8	38.1348
	2	-2.5	-8	48.1355	10	-8	38.1294
	3	-10	-4	48.1301	2.5	-4	38.1345
	4	-2.5	-4	48.1414	10	-4	38.1283
	5	-10	0	48.1314	2.5	0	38.1346
	6	-2.5	0	48.146	10	0	38.1294
	7	-10	4	48.131	2.5	4	38.1347
	8	-2.5	4	48.1432	10	4	38.1298
	9	-10	8	48.1303	2.5	8	38.135
	10	-2.5	8	48.1378	10	8	38.1312
38	1	-10	-8	48.1284	2.5	-8	38.1351
	2	-2.5	-8	48.1353	10	-8	38.1294
	3	-10	-4	48.1309	2.5	-4	38.1348
	4	-2.5	-4	48.1411	10	-4	38.1289
	5	-10	0	48.1326	2.5	0	38.135
	6	-2.5	0	48.1458	10	0	38.1292
	7	-10	4	48.132	2.5	4	38.1347
	8	-2.5	4	48.1429	10	4	38.1298
	9	-10	8	48.131	2.5	8	38.1354
	10	-2.5	8	48.137	10	8	38.1314
39	1	-10	-8	48.1282	2.5	-8	38.1349
	2	-2.5	-8	48.1353	10	-8	38.1294
	3	-10	-4	48.1303	2.5	-4	38.1347
	4	-2.5	-4	48.1409	10	-4	38.1284
	5	-10	0	48.1319	2.5	0	38.1348
	6	-2.5	0	48.1455	10	0	38.1288
	7	-10	4	48.1315	2.5	4	38.1345
	8	-2.5	4	48.1417	10	4	38.1297
	9	-10	8	48.1305	2.5	8	38.1348
	10	-2.5	8	48.1364	10	8	38.1312
40	1	-10	-8	48.1291	2.5	-8	38.135
	2	-2.5	-8	48.1362	10	-8	38.1297
	3	-10	-4	48.1314	2.5	-4	38.1349
	4	-2.5	-4	48.1419	10	-4	38.1291
	5	-10	0	48.1317	2.5	0	38.135
	6	-2.5	0	48.1457	10	0	38.1293
	7	-10	4	48.1316	2.5	4	38.1345
	8	-2.5	4	48.1428	10	4	38.1298
	9	-10	8	48.1304	2.5	8	38.1347
	10	-2.5	8	48.1374	10	8	38.1318

N° part	N° point	Plan 1 (CNC)			Plan 2 (CNC)		
		X	Y	Z	X	Y	Z
41	1	-10	-8	48.1283	2.5	-8	38.1347
	2	-2.5	-8	48.1353	10	-8	38.1292
	3	-10	-4	48.1303	2.5	-4	38.1349
	4	-2.5	-4	48.1408	10	-4	38.1286
	5	-10	0	48.1312	2.5	0	38.1351
	6	-2.5	0	48.1458	10	0	38.1291
	7	-10	4	48.1311	2.5	4	38.1349
	8	-2.5	4	48.1428	10	4	38.1293
	9	-10	8	48.1298	2.5	8	38.1349
	10	-2.5	8	48.1376	10	8	38.1312
42	1	-10	-8	48.1292	2.5	-8	38.1349
	2	-2.5	-8	48.1351	10	-8	38.1307
	3	-10	-4	48.1306	2.5	-4	38.1352
	4	-2.5	-4	48.1401	10	-4	38.1297
	5	-10	0	48.1312	2.5	0	38.1352
	6	-2.5	0	48.145	10	0	38.1296
	7	-10	4	48.1307	2.5	4	38.1344
	8	-2.5	4	48.142	10	4	38.1297
	9	-10	8	48.1296	2.5	8	38.135
	10	-2.5	8	48.1365	10	8	38.1317
43	1	-10	-8	48.1289	2.5	-8	38.1355
	2	-2.5	-8	48.1358	10	-8	38.1306
	3	-10	-4	48.1311	2.5	-4	38.1353
	4	-2.5	-4	48.1413	10	-4	38.1294
	5	-10	0	48.1317	2.5	0	38.135
	6	-2.5	0	48.146	10	0	38.1297
	7	-10	4	48.1314	2.5	4	38.1347
	8	-2.5	4	48.1432	10	4	38.1301
	9	-10	8	48.1309	2.5	8	38.1352
	10	-2.5	8	48.1375	10	8	38.132
44	1	-10	-8	48.1286	2.5	-8	38.1349
	2	-2.5	-8	48.1352	10	-8	38.1302
	3	-10	-4	48.1311	2.5	-4	38.1349
	4	-2.5	-4	48.141	10	-4	38.1295
	5	-10	0	48.1325	2.5	0	38.1348
	6	-2.5	0	48.1457	10	0	38.13
	7	-10	4	48.1317	2.5	4	38.1347
	8	-2.5	4	48.1424	10	4	38.1305
	9	-10	8	48.1309	2.5	8	38.1354
	10	-2.5	8	48.1375	10	8	38.1323
45	1	-10	-8	48.129	2.5	-8	38.1348
	2	-2.5	-8	48.1354	10	-8	38.1303
	3	-10	-4	48.1318	2.5	-4	38.135
	4	-2.5	-4	48.141	10	-4	38.1295
	5	-10	0	48.132	2.5	0	38.1346
	6	-2.5	0	48.1452	10	0	38.1294
	7	-10	4	48.1316	2.5	4	38.1345
	8	-2.5	4	48.1424	10	4	38.1301
	9	-10	8	48.1307	2.5	8	38.1355
	10	-2.5	8	48.1368	10	8	38.1316
46	1	-10	-8	48.1289	2.5	-8	38.1353
	2	-2.5	-8	48.1352	10	-8	38.1303
	3	-10	-4	48.1313	2.5	-4	38.1351

N° part	N° point	Plan 1 (CNC)			Plan 2 (CNC)		
		X	Y	Z	X	Y	Z
	4	-2.5	-4	48.1412	10	-4	38.1293
	5	-10	0	48.1324	2.5	0	38.1354
	6	-2.5	0	48.1454	10	0	38.1295
	7	-10	4	48.132	2.5	4	38.1351
	8	-2.5	4	48.1424	10	4	38.1302
	9	-10	8	48.131	2.5	8	38.1355
	10	-2.5	8	48.137	10	8	38.1318
47	1	-10	-8	48.1288	2.5	-8	38.135
	2	-2.5	-8	48.1353	10	-8	38.1298
	3	-10	-4	48.1312	2.5	-4	38.135
	4	-2.5	-4	48.1414	10	-4	38.1288
	5	-10	0	48.1324	2.5	0	38.1353
	6	-2.5	0	48.1456	10	0	38.1292
	7	-10	4	48.1317	2.5	4	38.1351
	8	-2.5	4	48.1422	10	4	38.1301
	9	-10	8	48.1305	2.5	8	38.1354
	10	-2.5	8	48.137	10	8	38.1317
48	1	-10	-8	48.129	2.5	-8	38.1354
	2	-2.5	-8	48.1354	10	-8	38.1299
	3	-10	-4	48.131	2.5	-4	38.1349
	4	-2.5	-4	48.1411	10	-4	38.1285
	5	-10	0	48.1321	2.5	0	38.1353
	6	-2.5	0	48.1455	10	0	38.1289
	7	-10	4	48.1318	2.5	4	38.1354
	8	-2.5	4	48.1428	10	4	38.1296
	9	-10	8	48.1303	2.5	8	38.1353
	10	-2.5	8	48.1371	10	8	38.1318
49	1	-10	-8	48.1286	2.5	-8	38.1356
	2	-2.5	-8	48.1352	10	-8	38.1302
	3	-10	-4	48.1309	2.5	-4	38.1349
	4	-2.5	-4	48.141	10	-4	38.1296
	5	-10	0	48.1319	2.5	0	38.1349
	6	-2.5	0	48.1459	10	0	38.1295
	7	-10	4	48.1313	2.5	4	38.1348
	8	-2.5	4	48.143	10	4	38.1298
	9	-10	8	48.1304	2.5	8	38.1352
	10	-2.5	8	48.1372	10	8	38.1315
50	1	-10	-8	48.1285	2.5	-8	38.1358
	2	-2.5	-8	48.1354	10	-8	38.1301
	3	-10	-4	48.1307	2.5	-4	38.1356
	4	-2.5	-4	48.1413	10	-4	38.1293
	5	-10	0	48.132	2.5	0	38.1353
	6	-2.5	0	48.146	10	0	38.1296
	7	-10	4	48.1318	2.5	4	38.135
	8	-2.5	4	48.1433	10	4	38.1304
	9	-10	8	48.1305	2.5	8	38.1352
	10	-2.5	8	48.1374	10	8	38.1324

N° part	N° point	Cylinder (CNC)		
		X	Y	Z
	1	16.9036	0.0029	26
	2	11.9598	-11.9561	26
	3	0.0030	-16.8953	26
	4	-11.9329	-11.9368	26
	5	-16.8711	-0.0030	26
	6	-16.8632	-0.0030	36
1	7	-11.9311	-11.9353	36
	8	0.0030	-16.8947	36
	9	11.9612	-11.9572	36
	10	16.9066	0.0029	36
	11	-16.8575	-0.0030	46
	12	-11.9276	-11.9319	46
	13	0.0030	-16.8932	46
	1	16.9050	0.0030	26
	2	11.9643	-11.9606	26
	3	0.0030	-16.9119	26
	4	-11.9437	-11.9478	26
	5	-16.8711	-0.0030	26
	6	-16.8707	-0.0030	36
2	7	-11.9456	-11.9502	36
	8	0.0030	-16.9178	36
	9	11.9675	-11.9639	36
	10	16.9072	0.0030	36
	11	-16.8694	-0.0030	46
	12	-11.9467	-11.9514	46
	13	0.0030	-16.9214	46
	1	16.8907	0.0030	26
	2	11.9449	-11.9410	26
	3	0.0030	-16.8888	26
	4	-11.9389	-11.9433	26
	5	-16.8852	-0.0030	26
	6	-16.8922	-0.0030	36
3	7	-11.9414	-11.9458	36
	8	0.0030	-16.8846	36
	9	11.9400	-11.9359	36
	10	16.8837	0.0030	36
	11	-16.8952	-0.0030	46
	12	-11.9408	-11.9450	46
	13	0.0030	-16.8785	46
	1	16.8914	0.0030	26
	2	11.9644	-11.9607	26
	3	0.0030	-16.9219	26
	4	-11.9569	-11.9612	26
	5	-16.8809	-0.0030	26
	6	-16.8845	-0.0030	36
4	7	-11.9610	-11.9653	36
	8	0.0030	-16.9289	36
	9	11.9677	-11.9642	36
	10	16.8899	0.0030	36
	11	-16.8860	-0.0030	46
	12	-11.9680	-11.9728	46
	13	0.0030	-16.9413	46
	1	16.9179	0.0030	26
	2	11.9686	-11.9648	26
	3	0.0030	-16.9107	26
	4	-11.9371	-11.9413	26
	5	-16.8576	-0.0030	26
	6	-16.8502	-0.0030	36
5	7	-11.9357	-11.9398	36
	8	0.0030	-16.9148	36
	9	11.9740	-11.9700	36
	10	16.9246	0.0030	36
	11	-16.8437	-0.0031	46
	12	-11.9359	-11.9403	46
	13	0.0030	-16.9210	46

N° part	N° point	Cylinder (CNC)		
		X	Y	Z
	1	16.9110	0.0030	26
	2	11.9604	-11.9566	26
	3	0.0030	-16.8968	26
	4	-11.9376	-11.9415	26
	5	-16.8572	-0.0030	26
	6	-16.8470	-0.0030	36
6	7	-11.9249	-11.9293	36
	8	0.0030	-16.8947	36
	9	11.9642	-11.9606	36
	10	16.9203	0.0029	36
	11	-16.8437	-0.0030	46
	12	-11.9223	-11.9267	46
	13	0.0030	-16.8902	46
	1	16.9106	0.0029	26
	2	11.9411	-11.9370	26
	3	0.0030	-16.8567	26
	4	-11.9109	-11.9149	26
	5	-16.8572	-0.0030	26
	6	-16.8531	-0.0030	36
7	7	-11.9005	-11.9041	36
	8	0.0030	-16.8388	36
	9	11.9364	-11.9324	36
	10	16.9174	0.0030	36
	11	-16.8446	-0.0030	46
	12	-11.8871	-11.8909	46
	13	0.0030	-16.8224	46
	1	16.8896	0.0029	26
	2	11.9584	-11.9544	26
	3	0.0030	-16.9183	26
	4	-11.9513	-11.9555	26
	5	-16.8791	-0.0030	26
	6	-16.8837	-0.0030	36
8	7	-11.9562	-11.9608	36
	8	0.0030	-16.9245	36
	9	11.9599	-11.9563	36
	10	16.8846	0.0030	36
	11	-16.8836	-0.0030	46
	12	-11.9587	-11.9632	46
	13	0.0030	-16.9273	46
	1	16.8990	0.0030	26
	2	11.9566	-11.9529	26
	3	0.0030	-16.8974	26
	4	-11.9346	-11.9390	26
	5	-16.8646	-0.0030	26
	6	-16.8634	-0.0030	36
9	7	-11.9340	-11.9383	36
	8	0.0030	-16.8976	36
	9	11.9583	-11.9549	36
	10	16.9025	0.0030	36
	11	-16.8622	-0.0030	46
	12	-11.9328	-11.9370	46
	13	0.0030	-16.8964	46
	1	16.9120	0.0030	26
	2	11.9587	-11.9551	26
	3	0.0030	-16.8926	26
	4	-11.9273	-11.9312	26
	5	-16.8523	-0.0030	26
	6	-16.8454	-0.0030	36
10	7	-11.9221	-11.9262	36
	8	0.0030	-16.8913	36
	9	11.9608	-11.9572	36
	10	16.9190	0.0029	36
	11	-16.8410	-0.0030	46
	12	-11.9177	-11.9224	46
	13	0.0030	-16.8876	46

N° part	N° point	Cylinder (CNC)		
		X	Y	Z
11	1	16.9016	0.0030	26
	2	11.9536	-11.9498	26
	3	0.0030	-16.8955	26
	4	-11.9343	-11.9389	26
	5	-16.8691	-0.0030	26
	6	-16.8637	-0.0030	36
	7	-11.9298	-11.9341	36
	8	0.0030	-16.8923	36
	9	11.9565	-11.9527	36
	10	16.9100	0.0030	36
	11	-16.8625	-0.0030	46
	12	-11.9290	-11.9337	46
	13	0.0030	-16.8906	46
12	1	16.9127	0.0030	26
	2	11.9643	-11.9608	26
	3	0.0030	-16.9057	26
	4	-11.9349	-11.9391	26
	5	-16.8593	-0.0030	26
	6	-16.8513	-0.0030	36
	7	-11.9317	-11.9359	36
	8	0.0030	-16.9040	36
	9	11.9665	-11.9627	36
	10	16.9179	0.0029	36
	11	-16.8419	-0.0030	46
	12	-11.9267	-11.9312	46
	13	0.0030	-16.9046	46
13	1	16.9016	0.0029	26
	2	11.9623	-11.9584	26
	3	0.0030	-16.9155	26
	4	-11.9580	-11.9629	26
	5	-16.8892	-0.0030	26
	6	-16.8683	-0.0030	36
	7	-11.9458	-11.9504	36
	8	0.0030	-16.9208	36
	9	11.9658	-11.9622	36
	10	16.9052	0.0029	36
	11	-16.8674	-0.0030	46
	12	-11.9491	-11.9540	46
	13	0.0030	-16.9287	46
14	1	16.8823	0.0029	26
	2	11.9600	-11.9562	26
	3	0.0030	-16.9211	26
	4	-11.9571	-11.9618	26
	5	-16.8865	-0.0030	26
	6	-16.8891	-0.0030	36
	7	-11.9638	-11.9685	36
	8	0.0030	-16.9332	36
	9	11.9642	-11.9607	36
	10	16.8789	0.0029	36
	11	-16.8925	-0.0030	46
	12	-11.9712	-11.9761	46
	13	0.0030	-16.9452	46
15	1	16.8833	0.0030	26
	2	11.9549	-11.9509	26
	3	0.0030	-16.9183	26
	4	-11.9546	-11.9595	26
	5	-16.8875	-0.0030	26
	6	-16.8901	-0.0030	36
	7	-11.9609	-11.9654	36
	8	0.0030	-16.9270	36
	9	11.9567	-11.9530	36
	10	16.8787	0.0030	36
	11	-16.8942	-0.0030	46
	12	-11.9674	-11.9721	46
	13	0.0030	-16.9364	46

N° part	N° point	Cylinder (CNC)		
		X	Y	Z
16	1	16.8848	0.0030	26
	2	11.9391	-11.9349	26
	3	0.0030	-16.8817	26
	4	-11.9373	-11.9418	26
	5	-16.8844	-0.0030	26
	6	-16.8880	-0.0030	36
	7	-11.9359	-11.9404	36
	8	0.0030	-16.8761	36
	9	11.9347	-11.9308	36
	10	16.8831	0.0030	36
	11	-16.8885	-0.0031	46
	12	-11.9326	-11.9371	46
	13	0.0030	-16.8692	46
17	1	16.8996	0.0030	26
	2	11.9427	-11.9390	26
	3	0.0030	-16.8758	26
	4	-11.9279	-11.9324	26
	5	-16.8699	-0.0030	26
	6	-16.8683	-0.0030	36
	7	-11.9237	-11.9282	36
	8	0.0030	-16.8701	36
	9	11.9400	-11.9361	36
	10	16.9030	0.0030	36
	11	-16.8658	-0.0030	46
	12	-11.9199	-11.9243	46
	13	0.0030	-16.8651	46
18	1	16.8929	0.0030	26
	2	11.9527	-11.9490	26
	3	0.0030	-16.9037	26
	4	-11.9420	-11.9466	26
	5	-16.8749	-0.0030	26
	6	-16.8761	-0.0030	36
	7	-11.9411	-11.9454	36
	8	0.0030	-16.9019	36
	9	11.9530	-11.9493	36
	10	16.8936	0.0030	36
	11	-16.8737	-0.0030	46
	12	-11.9404	-11.9451	46
	13	0.0030	-16.9033	46
19	1	16.9092	0.0030	26
	2	11.9598	-11.9561	26
	3	0.0030	-16.8998	26
	4	-11.9316	-11.9361	26
	5	-16.8608	-0.0030	26
	6	-16.8561	-0.0030	36
	7	-11.9274	-11.9317	36
	8	0.0030	-16.8978	36
	9	11.9662	-11.9625	36
	10	16.9167	0.0030	36
	11	-16.8480	-0.0030	46
	12	-11.9218	-11.9263	46
	13	0.0030	-16.8960	46
20	1	16.9042	0.0030	26
	2	11.9585	-11.9550	26
	3	0.0030	-16.9010	26
	4	-11.9365	-11.9410	26
	5	-16.8661	-0.0030	26
	6	-16.8642	-0.0030	36
	7	-11.9357	-11.9401	36
	8	0.0030	-16.9032	36
	9	11.9603	-11.9565	36
	10	16.9065	0.0030	36
	11	-16.8606	-0.0030	46
	12	-11.9325	-11.9372	46
	13	0.0030	-16.9013	46

N° part	N° point	Cylinder (CNC)		
		X	Y	Z
	1	16.8835	0.0030	26
	2	11.9458	-11.9419	26
	3	0.0030	-16.9007	26
	4	-11.9465	-11.9513	26
	5	-16.8870	-0.0030	26
	6	-16.8888	-0.0030	36
21	7	-11.9473	-11.9516	36
	8	0.0030	-16.9013	36
	9	11.9461	-11.9419	36
	10	16.8820	0.0030	36
	11	-16.8944	-0.0030	46
	12	-11.9507	-11.9554	46
	13	0.0030	-16.9014	46
	1	16.9129	0.0030	26
	2	11.9585	-11.9545	26
	3	0.0030	-16.8927	26
	4	-11.9253	-11.9296	26
	5	-16.8551	-0.0030	26
	6	-16.8453	-0.0030	36
22	7	-11.9183	-11.9224	36
	8	0.0030	-16.8889	36
	9	11.9617	-11.9580	36
	10	16.9219	0.0030	36
	11	-16.8395	-0.0030	46
	12	-11.9141	-11.9182	46
	13	0.0030	-16.8866	46
	1	16.9081	0.0030	26
	2	11.9531	-11.9494	26
	3	0.0030	-16.8855	26
	4	-11.9258	-11.9303	26
	5	-16.8610	-0.0030	26
	6	-16.8535	-0.0030	36
23	7	-11.9199	-11.9239	36
	8	0.0030	-16.8791	36
	9	11.9537	-11.9500	36
	10	16.9165	0.0030	36
	11	-16.8495	-0.0030	46
	12	-11.9148	-11.9191	46
	13	0.0030	-16.8752	46
	1	16.9180	0.0030	26
	2	11.9629	-11.9593	26
	3	0.0030	-16.8992	26
	4	-11.9291	-11.9336	26
	5	-16.8534	-0.0030	26
	6	-16.8425	-0.0030	36
24	7	-11.9237	-11.9279	36
	8	0.0030	-16.9010	36
	9	11.9694	-11.9657	36
	10	16.9297	0.0030	36
	11	-16.8350	-0.0030	46
	12	-11.9211	-11.9256	46
	13	0.0030	-16.9020	46
	1	16.9186	0.0030	26
	2	11.9568	-11.9531	26
	3	0.0030	-16.8838	26
	4	-11.9208	-11.9254	26
	5	-16.8594	-0.0030	26
	6	-16.8455	-0.0030	36
25	7	-11.9128	-11.9167	36
	8	0.0030	-16.8791	36
	9	11.9581	-11.9547	36
	10	16.9266	0.0030	36
	11	-16.8358	-0.0030	46
	12	-11.9039	-11.9080	46
	13	0.0030	-16.8730	46

N° part	N° point	Cylinder (CNC)		
		X	Y	Z
	1	16.9110	0.0030	26
	2	11.9514	-11.9477	26
	3	0.0030	-16.8775	26
	4	-11.9200	-11.9241	26
	5	-16.8573	-0.0030	26
	6	-16.8527	-0.0030	36
26	7	-11.9121	-11.9162	36
	8	0.0030	-16.8676	36
	9	11.9490	-11.9453	36
	10	16.9168	0.0030	36
	11	-16.8453	-0.0030	46
	12	-11.9039	-11.9082	46
	13	0.0030	-16.8590	46
	1	16.9132	0.0029	26
	2	11.9621	-11.9584	26
	3	0.0030	-16.9037	26
	4	-11.9325	-11.9372	26
	5	-16.8602	-0.0030	26
	6	-16.8508	-0.0030	36
27	7	-11.9313	-11.9355	36
	8	0.0030	-16.9077	36
	9	11.9668	-11.9630	36
	10	16.9228	0.0029	36
	11	-16.8444	-0.0030	46
	12	-11.9306	-11.9350	46
	13	0.0030	-16.9123	46
	1	16.8935	0.0030	26
	2	11.9483	-11.9445	26
	3	0.0030	-16.8917	26
	4	-11.9377	-11.9421	26
	5	-16.8794	-0.0030	26
	6	-16.8749	-0.0030	36
28	7	-11.9315	-11.9357	36
	8	0.0030	-16.8834	36
	9	11.9455	-11.9415	36
	10	16.8973	0.0029	36
	11	-16.8743	-0.0030	46
	12	-11.9293	-11.9337	46
	13	0.0030	-16.8805	46
	1	16.9135	0.0030	26
	2	11.9578	-11.9545	26
	3	0.0030	-16.8893	26
	4	-11.9245	-11.9288	26
	5	-16.8548	-0.0030	26
	6	-16.8408	-0.0030	36
29	7	-11.9162	-11.9204	36
	8	0.0030	-16.8836	36
	9	11.9628	-11.9594	36
	10	16.9256	0.0030	36
	11	-16.8343	-0.0030	46
	12	-11.9143	-11.9186	46
	13	0.0030	-16.8814	46
	1	16.9160	0.0030	26
	2	11.9625	-11.9587	26
	3	0.0030	-16.8942	26
	4	-11.9256	-11.9301	26
	5	-16.8533	-0.0030	26
	6	-16.8460	-0.0031	36
30	7	-11.9218	-11.9262	36
	8	0.0030	-16.8935	36
	9	11.9653	-11.9613	36
	10	16.9231	0.0030	36
	11	-16.8389	-0.0030	46
	12	-11.9165	-11.9210	46
	13	0.0030	-16.9013	46

N° part	N° point	Cylinder (CNC)		
		X	Y	Z
31	1	16.9011	0.0030	26
	2	11.9489	-11.9449	26
	3	0.0030	-16.8859	26
	4	-11.9357	-11.9400	26
	5	-16.8690	-0.0030	26
	6	-16.8628	-0.0030	36
	7	-11.9229	-11.9271	36
	8	0.0030	-16.8782	36
	9	11.9487	-11.9449	36
	10	16.9093	0.0030	36
	11	-16.8596	-0.0030	46
	12	-11.9204	-11.9248	46
	13	0.0030	-16.8741	46
32	1	16.9104	0.0030	26
	2	11.9497	-11.9458	26
	3	0.0030	-16.8709	26
	4	-11.9174	-11.9214	26
	5	-16.8562	-0.0030	26
	6	-16.8486	-0.0030	36
	7	-11.9075	-11.9114	36
	8	0.0030	-16.8592	36
	9	11.9494	-11.9455	36
	10	16.9230	0.0030	36
	11	-16.8383	-0.0030	46
	12	-11.8962	-11.9004	46
	13	0.0030	-16.8519	46
33	1	16.8804	0.0029	26
	2	11.9490	-11.9451	26
	3	0.0030	-16.9124	26
	4	-11.9518	-11.9565	26
	5	-16.8901	-0.0030	26
	6	-16.8961	-0.0031	36
	7	-11.9544	-11.9591	36
	8	0.0030	-16.9138	36
	9	11.9478	-11.9441	36
	10	16.8764	0.0030	36
	11	-16.9020	-0.0030	46
	12	-11.9609	-11.9658	46
	13	0.0030	-16.9187	46
34	1	16.9067	0.0030	26
	2	11.9600	-11.9560	26
	3	0.0030	-16.8991	26
	4	-11.9353	-11.9398	26
	5	-16.8610	-0.0030	26
	6	-16.8538	-0.0030	36
	7	-11.9317	-11.9359	36
	8	0.0030	-16.8984	36
	9	11.9638	-11.9602	36
	10	16.9138	0.0030	36
	11	-16.8518	-0.0030	46
	12	-11.9315	-11.9358	46
	13	0.0030	-16.9004	46
35	1	16.9103	0.0030	26
	2	11.9626	-11.9590	26
	3	0.0030	-16.9063	26
	4	-11.9370	-11.9413	26
	5	-16.8609	-0.0030	26
	6	-16.8575	-0.0030	36
	7	-11.9367	-11.9412	36
	8	0.0030	-16.9112	36
	9	11.9673	-11.9637	36
	10	16.9143	0.0030	36
	11	-16.8506	-0.0030	46
	12	-11.9357	-11.9403	46
	13	0.0030	-16.9152	46

N° part	N° point	Cylinder (CNC)		
		X	Y	Z
36	1	16.8930	0.0029	26
	2	11.9597	-11.9560	26
	3	0.0030	-16.9108	26
	4	-11.9488	-11.9532	26
	5	-16.8808	-0.0030	26
	6	-16.8801	-0.0030	36
	7	-11.9510	-11.9557	36
	8	0.0030	-16.9161	36
	9	11.9624	-11.9586	36
	10	16.8929	0.0030	36
	11	-16.8829	-0.0030	46
	12	-11.9572	-11.9618	46
	13	0.0030	-16.9250	46
37	1	16.9027	0.0030	26
	2	11.9465	-11.9425	26
	3	0.0030	-16.8826	26
	4	-11.9309	-11.9354	26
	5	-16.8718	-0.0030	26
	6	-16.8707	-0.0030	36
	7	-11.9262	-11.9306	36
	8	0.0030	-16.8739	36
	9	11.9441	-11.9402	36
	10	16.9039	0.0030	36
	11	-16.8679	-0.0030	46
	12	-11.9223	-11.9269	46
	13	0.0030	-16.8700	46
38	1	16.9065	0.0030	26
	2	11.9629	-11.9592	26
	3	0.0030	-16.9028	26
	4	-11.9365	-11.9408	26
	5	-16.8634	-0.0030	26
	6	-16.8576	-0.0030	36
	7	-11.9333	-11.9378	36
	8	0.0030	-16.9028	36
	9	11.9652	-11.9616	36
	10	16.9120	0.0030	36
	11	-16.8531	-0.0030	46
	12	-11.9337	-11.9383	46
	13	0.0030	-16.9055	46
39	1	16.9111	0.0030	26
	2	11.9568	-11.9529	26
	3	0.0030	-16.8909	26
	4	-11.9287	-11.9331	26
	5	-16.8622	-0.0030	26
	6	-16.8550	-0.0030	36
	7	-11.9229	-11.9270	36
	8	0.0030	-16.8883	36
	9	11.9591	-11.9554	36
	10	16.9184	0.0030	36
	11	-16.8493	-0.0030	46
	12	-11.9188	-11.9231	46
	13	0.0030	-16.8867	46
40	1	16.9078	0.0030	26
	2	11.9490	-11.9454	26
	3	0.0030	-16.8766	26
	4	-11.9199	-11.9243	26
	5	-16.8639	-0.0030	26
	6	-16.8556	-0.0027	36
	7	-11.9108	-11.9150	36
	8	0.0030	-16.8688	36
	9	11.9492	-11.9457	36
	10	16.9150	0.0030	36
	11	-16.8538	-0.0030	46
	12	-11.9049	-11.9091	46
	13	0.0030	-16.8611	46

N° part	N° point	Cylinder (CNC)		
		X	Y	Z
41	1	16.9038	0.0030	26
	2	11.9497	-11.9463	26
	3	0.0030	-16.8817	26
	4	-11.9270	-11.9317	26
	5	-16.8671	-0.0030	26
	6	-16.8596	-0.0030	36
	7	-11.9200	-11.9238	36
	8	0.0030	-16.8732	36
	9	11.9486	-11.9450	36
	10	16.9077	0.0030	36
	11	-16.8561	-0.0030	46
	12	-11.9186	-11.9230	46
	13	0.0030	-16.8735	46
42	1	16.8898	0.0030	26
	2	11.9477	-11.9439	26
	3	0.0030	-16.8950	26
	4	-11.9404	-11.9447	26
	5	-16.8843	-0.0030	26
	6	-16.8844	-0.0030	36
	7	-11.9394	-11.9437	36
	8	0.0030	-16.8929	36
	9	11.9462	-11.9423	36
	10	16.8910	0.0030	36
	11	-16.8870	-0.0030	46
	12	-11.9403	-11.9449	46
	13	0.0030	-16.8916	46
43	1	16.9159	0.0030	26
	2	11.9536	-11.9501	26
	3	0.0030	-16.8819	26
	4	-11.9213	-11.9257	26
	5	-16.8594	-0.0030	26
	6	-16.8512	-0.0030	36
	7	-11.9144	-11.9184	36
	8	0.0030	-16.8769	36
	9	11.9553	-11.9516	36
	10	16.9227	0.0030	36
	11	-16.8449	-0.0030	46
	12	-11.9079	-11.9122	46
	13	0.0030	-16.8690	46
44	1	16.9128	0.0030	26
	2	11.9586	-11.9550	26
	3	0.0030	-16.8948	26
	4	-11.9303	-11.9345	26
	5	-16.8596	-0.0030	26
	6	-16.8521	-0.0030	36
	7	-11.9249	-11.9293	36
	8	0.0030	-16.8903	36
	9	11.9605	-11.9564	36
	10	16.9212	0.0030	36
	11	-16.8469	-0.0031	46
	12	-11.9214	-11.9260	46
	13	0.0030	-16.8904	46
45	1	16.8820	0.0030	26
	2	11.9429	-11.9389	26
	3	0.0030	-16.8919	26
	4	-11.9439	-11.9483	26
	5	-16.8862	-0.0030	26
	6	-16.8908	-0.0030	36
	7	-11.9447	-11.9492	36
	8	0.0030	-16.8904	36
	9	11.9392	-11.9350	36
	10	16.8776	0.0030	36
	11	-16.8922	-0.0030	46
	12	-11.9444	-11.9490	46
	13	0.0030	-16.8904	46
46	1	16.8882	0.0030	26
	2	11.9504	-11.9465	26
	3	0.0030	-16.8991	26
	4	-11.9454	-11.9500	26
	5	-16.8866	-0.0030	26
	6	-16.8873	-0.0030	36
	7	-11.9460	-11.9504	36
	8	0.0030	-16.8999	36
	9	11.9506	-11.9469	36
	10	16.8875	0.0030	36
	11	-16.8921	-0.0030	46
	12	-11.9497	-11.9547	46
	13	0.0030	-16.9034	46
47	1	16.8747	0.0030	26
	2	11.9506	-11.9467	26
	3	0.0030	-16.9156	26
	4	-11.9573	-11.9621	26
	5	-16.8981	-0.0030	26
	6	-16.9029	-0.0031	36
	7	-11.9637	-11.9682	36
	8	0.0030	-16.9251	36
	9	11.9527	-11.9487	36
	10	16.8706	0.0030	36
	11	-16.9117	-0.0030	46
	12	-11.9723	-11.9770	46
	13	0.0030	-16.9323	46
48	1	16.8877	0.0030	26
	2	11.9518	-11.9477	26
	3	0.0030	-16.9029	26
	4	-11.9439	-11.9487	26
	5	-16.8809	-0.0030	26
	6	-16.8823	-0.0030	36
	7	-11.9427	-11.9473	36
	8	0.0030	-16.9028	36
	9	11.9529	-11.9489	36
	10	16.8893	0.0030	36
	11	-16.8820	-0.0030	46
	12	-11.9446	-11.9492	46
	13	0.0030	-16.9043	46
49	1	16.9142	0.0029	26
	2	11.9594	-11.9558	26
	3	0.0030	-16.9036	26
	4	-11.9355	-11.9397	26
	5	-16.8635	-0.0030	26
	6	-16.8562	-0.0030	36
	7	-11.9323	-11.9364	36
	8	0.0030	-16.9080	36
	9	11.9642	-11.9607	36
	10	16.9197	0.0029	36
	11	-16.8542	-0.0030	46
	12	-11.9302	-11.9348	46
	13	0.0030	-16.9038	46
50	1	16.9093	0.0030	26
	2	11.9600	-11.9564	26
	3	0.0030	-16.9093	26
	4	-11.9399	-11.9443	26
	5	-16.8662	-0.0030	26
	6	-16.8585	-0.0030	36
	7	-11.9376	-11.9422	36
	8	0.0030	-16.9101	36
	9	11.9643	-11.9609	36
	10	16.9154	0.0030	36
	11	-16.8581	-0.0030	46
	12	-11.9374	-11.9420	46
	13	0.0030	-16.9132	46

N° part	N° point	Plan 0 (CMM)			Plan 1 (CMM)			Plan 2 (CMM)		
		X	Y	Z	X	Y	Z	X	Y	Z
1	1	10.001751	-7.998732	-0.265418	-9.998136	-7.998799	47.730584	2.501892	-7.998864	37.714725
	2	-9.998088	-7.998928	-0.137611	-2.498752	-7.998737	47.722968	10.001610	-7.998308	37.699666
	3	10.001285	-2.998446	-0.286810	-9.998095	-3.998147	47.700895	2.501932	-3.998646	37.684014
	4	-9.998046	-2.998643	-0.199498	-2.498712	-3.997654	47.696277	10.002076	-3.998589	37.667954
	5	10.003361	3.001457	-0.352779	-9.998053	0.001190	47.670706	2.501973	0.002052	37.653303
	6	-9.998494	3.001261	-0.257463	-2.498244	0.001252	47.672586	10.002107	0.001605	37.638242
	7	10.001361	8.001185	-0.434669	-9.998011	4.001858	47.638016	2.502013	4.001373	37.623591
	8	-9.998451	8.001489	-0.327851	-2.498210	4.002343	47.638895	10.001640	4.001847	37.609030
	9	0.002951	5.001951	-0.297810	-9.997969	8.002037	47.610827	2.502054	8.001972	37.593880
	10	0.002957	-4.999381	-0.172531	-2.498176	8.001597	47.605205	10.001672	8.002438	37.580818
2	1	10.003127	-7.998732	-0.277418	-9.998135	-7.998800	47.699586	2.501892	-7.998864	37.692227
	2	-9.998087	-7.998930	-0.266608	-2.498751	-7.998738	47.698969	10.001611	-7.999238	37.682167
	3	10.001285	-2.998447	-0.307809	-9.999022	-3.998153	47.672897	2.501006	-3.998645	37.663016
	4	-9.998044	-2.999145	-0.281496	-2.498282	-3.998591	47.674278	10.002073	-3.998589	37.653955
	5	10.001326	3.001457	-0.354779	-9.998052	0.001189	47.645707	2.501973	0.001126	37.634804
	6	-9.998493	3.001259	-0.326462	-2.498247	0.001251	47.650587	10.002106	0.001603	37.626243
	7	10.001361	8.001185	-0.430169	-9.998010	4.001854	47.616018	2.502014	4.001373	37.607593
	8	-9.998451	8.001488	-0.371850	-2.498213	4.001416	47.620896	10.001638	4.001845	37.599531
	9	0.001037	5.001949	-0.370308	-9.997968	8.001118	47.590328	2.502054	8.001057	37.580881
	10	0.002958	-4.998181	-0.275028	-2.498178	8.001180	47.590705	10.001671	8.001525	37.574318
3	1	10.001751	-7.998732	-0.267918	-9.998135	-7.998800	47.700586	2.501893	-7.998864	37.687728
	2	-9.998088	-7.998929	-0.192109	-2.498751	-7.998305	47.693969	10.001611	-7.998807	37.678168
	3	10.001285	-2.998447	-0.318809	-9.999022	-3.998153	47.674897	2.501006	-3.998645	37.662016
	4	-9.998046	-2.999142	-0.177998	-2.498283	-3.998591	47.673778	10.002073	-3.998164	37.650456
	5	10.001326	3.001457	-0.346279	-9.998052	0.001189	47.652707	2.501051	0.001126	37.637304
	6	-9.998495	3.001262	-0.194464	-2.498247	0.001677	47.655587	10.002106	0.001603	37.625743
	7	10.001360	8.001186	-0.377170	-9.998010	4.001856	47.627017	2.501096	4.001373	37.613592
	8	-9.998453	8.001492	-0.205853	-2.498212	4.001418	47.628896	10.001639	4.001846	37.602031
	9	0.001035	5.001951	-0.293309	-9.997969	8.001119	47.604327	2.502054	8.001971	37.591380
	10	0.002958	-4.999570	-0.284029	-2.498177	8.001596	47.602205	10.001672	8.001526	37.580818
4	1	10.003014	-7.998731	-0.212920	-9.999076	-7.998799	47.731584	2.501892	-7.998864	37.719225
	2	-9.998087	-7.998929	-0.197109	-2.498752	-7.998298	47.727467	10.001610	-7.998808	37.707165
	3	10.001284	-2.998445	-0.254810	-9.999029	-3.998147	47.702395	2.501932	-3.998646	37.689514
	4	-9.998045	-2.999143	-0.229997	-2.498713	-3.998586	47.701277	10.002077	-3.998589	37.676454
	5	10.001326	3.001457	-0.336779	-9.998053	0.001190	47.674705	2.501046	0.001126	37.659802
	6	-9.998494	3.001259	-0.313962	-2.498244	0.001681	47.677086	10.002109	0.001183	37.647241
	7	10.001361	8.001185	-0.421170	-9.998011	4.001859	47.643016	2.502013	4.001373	37.630091
	8	-9.998451	8.001488	-0.362350	-2.498210	4.001421	47.645395	10.001641	4.001849	37.619529
	9	0.001036	5.001950	-0.357808	-9.997969	8.001119	47.615826	2.502054	8.001057	37.602379
	10	0.002957	-4.998180	-0.214030	-2.498175	8.001599	47.613204	10.001674	8.001528	37.591817
5	1	10.001249	-7.997605	-0.114921	-9.999080	-7.998798	47.749083	2.501890	-7.998864	37.750723
	2	-9.998087	-7.999694	-0.214109	-2.498754	-7.998293	47.752466	10.001609	-7.998808	37.745162
	3	10.001284	-2.998444	-0.194311	-9.999033	-3.998143	47.721394	2.500993	-3.998646	37.720511
	4	-9.998045	-2.999143	-0.229497	-2.498714	-3.999019	47.727275	10.002083	-3.998152	37.715951
	5	10.001324	3.001460	-0.224281	-9.998054	0.001190	47.695204	2.501039	0.001126	37.692300
	6	-9.998494	3.000760	-0.279463	-2.498240	0.001687	47.704084	10.002115	0.001614	37.687239
	7	10.001359	8.001188	-0.299671	-9.998012	4.001864	47.665515	2.501085	4.001373	37.664088
	8	-9.998452	8.000990	-0.289851	-2.498206	4.001427	47.674893	10.001647	4.001856	37.661026
	9	0.002922	5.001951	-0.283310	-9.998893	8.001120	47.638325	2.502052	8.001057	37.637377
	10	0.002457	-4.998179	-0.193530	-2.498172	8.001605	47.643203	10.001680	8.001535	37.634814
6	1	10.001250	-7.997541	-0.163421	-9.999067	-7.998800	47.693587	2.501892	-7.998864	37.694727
	2	-9.998087	-7.998430	-0.243108	-2.498751	-7.998738	47.696469	10.002042	-7.998807	37.689667
	3	10.001284	-2.998445	-0.237810	-9.999021	-3.998154	47.667897	2.501005	-3.998645	37.668015
	4	-9.999899	-2.999144	-0.265997	-2.498711	-3.998591	47.674778	10.002075	-3.999089	37.663955
	5	10.001325	3.001459	-0.272280	-9.998975	0.001189	47.643707	2.501973	0.001126	37.643304
	6	-9.998493	3.000759	-0.352961	-2.498247	0.001676	47.654087	10.002108	0.001605	37.638742
	7	10.001359	8.001187	-0.347171	-9.998929	4.001854	47.618518	2.501095	4.001373	37.617592
	8	-9.998451	8.000989	-0.352350	-2.498212	4.001418	47.627396	10.001640	4.000929	37.614030
	9	0.001036	5.001950	-0.351308	-9.998883	8.001118	47.593328	2.502054	8.001057	37.593380
	10	0.002958	-4.998181	-0.264529	-2.498177	8.001596	47.598705	10.001673	8.001527	37.589817

N° part	N° point	Plan 0 (CMM)			Plan 1 (CMM)			Plan 2 (CMM)		
		X	Y	Z	X	Y	Z	X	Y	Z
7	1	10.001252	-7.997295	-0.309418	-9.999056	-7.998378	47.643590	2.501894	-7.998441	37.640231
	2	-9.998086	-7.998432	-0.328106	-2.498748	-7.999239	47.645973	10.001612	-7.999229	37.634671
	3	10.001285	-2.998447	-0.310809	-9.999010	-3.998164	47.621900	2.501015	-3.998645	37.619019
	4	-9.998043	-2.999146	-0.339995	-2.498709	-3.998601	47.627781	10.002067	-3.998171	37.610959
	5	10.001325	3.001458	-0.308779	-9.998965	0.001188	47.601210	2.501975	0.001126	37.598307
	6	-9.998493	3.000759	-0.332962	-2.498252	0.001668	47.613089	10.001687	0.001597	37.592746
	7	10.001359	8.001188	-0.321671	-9.998008	4.001847	47.583020	2.502015	4.001373	37.578095
	8	-9.998451	8.000988	-0.355350	-2.498217	4.000910	47.590398	10.001635	4.000929	37.572533
	9	0.001036	5.001950	-0.325309	-9.997967	8.001118	47.562829	2.502055	8.001057	37.559382
	10	0.002959	-4.998182	-0.319527	-2.498181	8.001590	47.567707	10.001669	8.001522	37.555320
8	1	10.003136	-7.997347	-0.282419	-9.999075	-7.998799	47.725585	2.501892	-7.998864	37.712226
	2	-9.998088	-7.998429	-0.193610	-2.498752	-7.998300	47.720468	10.002044	-7.998808	37.697666
	3	10.001285	-2.998446	-0.279310	-9.999028	-3.998148	47.698395	2.501002	-3.998646	37.682514
	4	-9.998045	-2.999144	-0.243497	-2.498279	-3.998587	47.696277	10.002076	-3.999089	37.668454
	5	10.001326	3.001458	-0.327279	-9.998981	0.001190	47.671706	2.501047	0.001126	37.654803
	6	-9.998494	3.000760	-0.262963	-2.498244	0.001680	47.672586	10.002108	0.001606	37.640742
	7	10.001359	8.001187	-0.351171	-9.998934	4.000936	47.640016	2.501093	4.001373	37.626591
	8	-9.998451	8.000989	-0.332351	-2.498210	4.001421	47.642395	10.001640	4.000929	37.613030
	9	0.001036	5.001950	-0.323809	-9.998888	8.001119	47.613826	2.501138	8.001057	37.599879
	10	0.002958	-4.998181	-0.259029	-2.498176	8.001598	47.610704	10.001673	8.001527	37.587317
9	1	10.001251	-7.998731	-0.234919	-9.999070	-7.998365	47.704086	2.501892	-7.998430	37.698727
	2	-9.999916	-7.998930	-0.253109	-2.498751	-7.999238	47.703969	10.001611	-7.998807	37.690167
	3	10.001285	-2.997097	-0.262810	-9.999023	-3.999082	47.677897	2.501004	-3.998217	37.672015
	4	-9.999937	-2.998145	-0.286497	-2.498282	-3.998590	47.680778	10.002075	-3.998589	37.662955
	5	10.001325	3.001458	-0.317279	-9.998977	0.001615	47.653707	2.501049	0.001126	37.646803
	6	-9.998493	3.000759	-0.326962	-2.498246	0.001251	47.660587	10.002107	0.001605	37.637742
	7	10.001360	8.001186	-0.369170	-9.998931	4.000935	47.626017	2.501094	4.001373	37.620592
	8	-10.000562	8.000988	-0.383850	-2.498211	4.001419	47.632896	10.001640	4.000929	37.613030
	9	0.001036	5.001950	-0.353308	-9.998885	8.001119	47.601327	2.502054	8.001057	37.596380
	10	0.002958	-5.000055	-0.276529	-2.498177	8.001597	47.603205	10.001673	8.001527	37.589317
10	1	10.001252	-7.998733	-0.295917	-9.999066	-7.998369	47.687587	2.501893	-7.998433	37.686228
	2	-9.999938	-7.998930	-0.264608	-2.498751	-7.999670	47.690470	10.002041	-7.998807	37.680167
	3	10.001285	-2.998446	-0.281810	-9.999019	-3.998656	47.661398	2.501006	-3.998645	37.661016
	4	-9.999958	-2.999145	-0.297496	-2.498283	-3.999020	47.668779	10.002073	-3.999089	37.653455
	5	10.001325	3.001458	-0.318279	-9.998974	0.001189	47.639708	2.501051	0.001126	37.635804
	6	-9.998494	3.001260	-0.308462	-2.498247	0.001675	47.649087	10.002106	0.001604	37.629243
	7	10.001359	8.001187	-0.325671	-9.998928	4.001353	47.614518	2.501096	4.001791	37.611092
	8	-9.998450	8.000988	-0.387350	-2.498213	4.001417	47.622396	10.001639	4.001429	37.605030
	9	0.001036	5.001950	-0.354808	-9.998882	8.001532	47.589328	2.501141	8.001470	37.586380
	10	0.002959	-4.998182	-0.300528	-2.498178	8.000765	47.593705	10.001672	8.001526	37.581818
11	1	10.001251	-7.997397	-0.255419	-9.999064	-7.998370	47.679588	2.501893	-7.998435	37.675729
	2	-9.999968	-7.998431	-0.281108	-2.498750	-7.999669	47.679970	10.002039	-7.998807	37.669168
	3	10.001285	-2.998446	-0.296309	-9.999018	-3.999083	47.653898	2.501009	-3.998645	37.650017
	4	-9.999931	-2.999145	-0.283497	-2.498285	-3.998595	47.658779	10.002072	-3.999089	37.641956
	5	10.001326	3.001457	-0.327779	-9.998972	0.001189	47.630208	2.501054	0.001126	37.624805
	6	-9.998493	3.000759	-0.321962	-2.498249	0.001673	47.639088	10.002104	0.001602	37.617244
	7	10.001360	8.001187	-0.366670	-9.998926	4.001351	47.604018	2.501098	4.001373	37.600593
	8	-10.000482	8.000989	-0.351351	-2.498214	4.001415	47.612397	10.001638	4.000929	37.593531
	9	0.001036	4.999943	-0.340809	-9.998880	8.001118	47.580328	2.502055	8.001057	37.576381
	10	0.002959	-5.000133	-0.315528	-2.498179	8.001593	47.583206	10.001671	8.001524	37.569819
12	1	10.001251	-7.998732	-0.274918	-9.999069	-7.998365	47.703586	2.501892	-7.999364	37.703226
	2	-9.999932	-7.998930	-0.261608	-2.498316	-8.000108	47.707469	10.002043	-7.998808	37.696166
	3	10.001284	-2.997110	-0.255811	-9.999023	-3.998652	47.677897	2.501003	-3.998216	37.676015
	4	-9.999975	-2.998645	-0.305496	-2.498281	-3.999020	47.684278	10.002076	-3.999945	37.668454
	5	10.001325	3.001459	-0.268280	-9.998977	0.001614	47.652207	2.501049	0.001126	37.649303
	6	-9.998494	3.000760	-0.308462	-2.498246	0.000751	47.662587	10.002108	0.001606	37.641742
	7	10.002761	8.001188	-0.291672	-9.998930	4.001355	47.624517	2.501094	4.001373	37.622591
	8	-10.000503	8.000988	-0.359851	-2.498211	4.001419	47.634896	10.001641	4.000929	37.616030
	9	0.001036	4.999975	-0.326309	-9.998884	8.001534	47.599327	2.501139	8.001057	37.597380
	10	0.002959	-4.998182	-0.306028	-2.498177	8.000764	47.603205	10.001674	8.001528	37.591317

N° part	N° point	Plan 0 (CMM)			Plan 1 (CMM)			Plan 2 (CMM)		
		X	Y	Z	X	Y	Z	X	Y	Z
13	1	10.001249	-7.997599	-0.119421	-9.999071	-7.998363	47.712086	2.501892	-7.999364	37.706726
	2	-9.999829	-7.998429	-0.199110	-2.498752	-7.999674	47.711468	10.002043	-7.998808	37.696166
	3	10.001284	-2.998444	-0.198811	-9.999025	-3.998651	47.684396	2.501003	-3.998646	37.677515
	4	-9.999881	-2.999144	-0.256497	-2.498281	-3.999020	47.687277	10.002075	-3.999089	37.666954
	5	10.001325	3.001459	-0.275280	-9.998978	0.001189	47.657707	2.501049	0.001551	37.649303
	6	-9.998493	3.000759	-0.334962	-2.498245	0.000751	47.664086	10.002108	0.001183	37.638742
	7	10.001360	8.002735	-0.358671	-9.998931	4.000935	47.628517	2.501094	4.001373	37.620592
	8	-10.000490	8.001488	-0.354351	-2.498211	4.000997	47.633896	10.001640	4.000929	37.611530
	9	0.001036	4.999932	-0.345309	-9.998884	8.001534	47.600327	2.502054	8.001472	37.594380
	10	0.001195	-4.999942	-0.212530	-2.498177	8.000264	47.601705	10.001673	8.000700	37.587317
14	1	10.001251	-7.997444	-0.227419	-9.999074	-7.998361	47.724085	2.501892	-7.999364	37.710226
	2	-9.999753	-7.998428	-0.146111	-2.498752	-8.000112	47.719468	10.002043	-7.999240	37.694666
	3	10.001285	-2.997052	-0.286810	-9.999027	-3.998649	47.694896	2.501003	-3.998646	37.678515
	4	-9.999780	-2.999877	-0.194499	-2.498280	-3.998587	47.693277	10.002075	-3.999089	37.663455
	5	10.001327	3.001456	-0.392278	-9.998980	0.001190	47.666206	2.501048	0.001126	37.649803
	6	-10.000410	3.000760	-0.298463	-2.498245	0.000751	47.668586	10.002107	0.001604	37.634243
	7	10.001361	8.002952	-0.438670	-9.998932	4.001357	47.634517	2.501094	4.001373	37.619592
	8	-10.000583	8.001488	-0.391850	-2.498211	4.001419	47.636396	10.001639	4.000929	37.605030
	9	0.001036	4.999905	-0.356809	-9.998886	8.001535	47.604827	2.501140	8.001057	37.590380
	10	0.001193	-4.999944	-0.213530	-2.498177	8.000764	47.601705	10.001672	8.001525	37.577818
15	1	10.001251	-7.997365	-0.272918	-9.999074	-7.998361	47.721085	2.501892	-7.999364	37.710226
	2	-9.999683	-7.998427	-0.091112	-2.498752	-8.000112	47.718468	10.001610	-7.998808	37.695666
	3	10.001285	-2.997039	-0.293310	-9.999027	-3.998649	47.693896	2.501002	-3.998215	37.680015
	4	-9.999726	-2.998642	-0.156999	-2.498280	-3.999520	47.693277	10.002075	-3.999944	37.665455
	5	10.002815	3.002947	-0.332280	-9.998980	0.001617	47.666706	2.501048	0.001551	37.651303
	6	-10.000234	3.001762	-0.196965	-2.498245	0.000751	47.669086	10.002107	0.000683	37.637242
	7	10.001360	8.002740	-0.360171	-9.998933	4.001358	47.636517	2.501094	4.001793	37.622092
	8	-10.000319	8.001490	-0.271852	-2.498210	4.001420	47.638895	10.001640	4.000095	37.608530
	9	0.001035	5.001451	-0.301809	-9.998886	8.001536	47.605827	2.501139	8.001472	37.595880
	10	0.001152	-4.999986	-0.238530	-2.498176	8.000764	47.604205	10.001673	8.000700	37.583318
16	1	10.001251	-7.997342	-0.285418	-9.999068	-7.998367	47.697087	2.501893	-7.999364	37.685228
	2	-9.999899	-7.998930	-0.242109	-2.498318	-8.000103	47.691970	10.002040	-7.998807	37.670668
	3	10.001285	-2.996996	-0.314810	-9.999022	-3.998653	47.673897	2.501006	-3.998219	37.661016
	4	-9.999854	-2.998644	-0.240498	-2.498283	-3.999520	47.673278	10.002072	-3.999937	37.645456
	5	10.001326	3.002928	-0.324280	-9.998977	0.001614	47.651707	2.501051	0.001549	37.636804
	6	-10.000391	3.001760	-0.288963	-2.498247	0.000326	47.654587	10.002105	0.000763	37.622743
	7	10.001360	8.000686	-0.391170	-9.998930	4.001355	47.625517	2.501096	4.001791	37.613092
	8	-10.000329	7.999613	-0.278352	-2.498212	4.000497	47.629396	10.001638	4.000098	37.599031
	9	0.001036	4.999963	-0.332309	-9.998885	8.001535	47.602827	2.501140	8.001057	37.590380
	10	0.001063	-5.000077	-0.288029	-2.498177	8.000265	47.599705	10.001672	8.000613	37.576818
17	1	10.001251	-7.997350	-0.280918	-9.998634	-7.998368	47.691087	2.501893	-7.999364	37.687228
	2	-9.999955	-7.998431	-0.274108	-2.498751	-7.999670	47.690970	10.002041	-7.998807	37.677668
	3	10.001285	-2.996988	-0.318810	-9.999021	-3.998654	47.667397	2.501006	-3.999146	37.662516
	4	-9.999923	-2.998644	-0.278997	-2.498283	-3.999520	47.672778	10.002073	-3.999939	37.652456
	5	10.001326	3.003009	-0.359779	-9.998975	0.001613	47.646207	2.501051	0.001549	37.638804
	6	-10.000430	3.001260	-0.308463	-2.498247	0.000826	47.653587	10.002106	0.000683	37.628743
	7	10.001360	8.001186	-0.381670	-9.998930	4.001355	47.622017	2.501095	4.001791	37.614592
	8	-10.000360	8.000990	-0.294352	-2.498212	4.000497	47.628396	10.001639	4.000096	37.605530
	9	0.001036	5.001449	-0.364308	-9.998884	8.001534	47.598827	2.501140	8.001057	37.592380
	10	0.001020	-5.000121	-0.309529	-2.498177	8.000265	47.600205	10.001673	8.000613	37.585317
18	1	10.001252	-7.999233	-0.311417	-9.999070	-7.998364	47.707586	2.501892	-7.999364	37.698727
	2	-9.999836	-7.998929	-0.203110	-2.498751	-8.000107	47.705469	10.002042	-7.998807	37.686167
	3	10.001285	-2.998947	-0.321809	-9.999024	-3.998651	47.681896	2.501004	-3.998217	37.671515
	4	-9.999884	-2.999144	-0.257497	-2.498281	-3.999020	47.684278	10.002074	-3.999942	37.658955
	5	10.001326	3.002961	-0.339279	-9.998978	0.001616	47.658707	2.501050	0.001550	37.644804
	6	-10.000391	3.001260	-0.288963	-2.498246	0.000751	47.663586	10.002107	0.000761	37.633243
	7	10.001359	8.002725	-0.354671	-9.998931	4.001356	47.629517	2.501094	4.001792	37.620092
	8	-10.000457	8.001489	-0.340351	-2.498211	4.000497	47.635896	10.001640	4.000095	37.608030
	9	0.001036	4.999942	-0.341309	-9.998886	8.001119	47.604827	2.501139	8.001472	37.595380
	10	0.001054	-5.000086	-0.292529	-2.498176	8.000680	47.605205	10.001672	8.000701	37.583318

N° part	N° point	Plan 0 (CMM)			Plan 1 (CMM)			Plan 2 (CMM)		
		X	Y	Z	X	Y	Z	X	Y	Z
19	1	10.001250	-7.997501	-0.191420	-9.998635	-7.998366	47.699086	2.501892	-7.999364	37.698227
	2	-9.999798	-7.998929	-0.178110	-2.498317	-8.000106	47.701469	10.002042	-7.999239	37.689167
	3	10.001284	-2.997191	-0.206311	-9.999022	-3.998653	47.673897	2.501004	-3.998217	37.671515
	4	-9.999759	-2.998642	-0.180499	-2.498282	-3.999520	47.680278	10.002074	-3.999516	37.661455
	5	10.001324	3.002710	-0.204282	-9.998976	0.001614	47.649707	2.501049	0.001550	37.645804
	6	-10.000270	2.999986	-0.220465	-2.498246	0.000325	47.659587	10.002107	0.000761	37.636242
	7	10.001357	8.002453	-0.213173	-9.998930	4.001355	47.623517	2.501094	4.000873	37.620092
	8	-10.000278	8.001491	-0.249852	-2.498211	4.000576	47.632396	10.001640	4.000094	37.611530
	9	0.001035	5.001452	-0.246810	-9.998884	8.001534	47.598827	2.501139	8.001472	37.596880
	10	0.001230	-4.998679	-0.189530	-2.498177	8.000264	47.602205	10.001673	8.000700	37.588317
20	1	10.001251	-7.999231	-0.223419	-9.999069	-7.999300	47.702086	2.501892	-7.999364	37.698227
	2	-9.999946	-7.999431	-0.268608	-2.498751	-7.999672	47.702469	10.002042	-7.999239	37.689667
	3	10.001285	-2.998946	-0.286810	-9.999023	-3.998653	47.675397	2.501004	-3.999146	37.671015
	4	-9.999956	-2.999145	-0.295996	-2.498282	-3.999520	47.679778	10.002075	-3.999516	37.661955
	5	10.001326	3.002931	-0.325780	-9.998976	0.001614	47.650707	2.501049	0.001550	37.645304
	6	-10.000505	3.001259	-0.342962	-2.498246	0.000325	47.659087	10.002107	0.000761	37.636242
	7	10.001360	8.002788	-0.380171	-9.998930	4.001355	47.623017	2.501095	4.001792	37.618592
	8	-10.000513	8.001488	-0.364351	-2.498212	4.000576	47.630896	10.001640	4.000095	37.609530
	9	0.001036	5.001449	-0.363808	-9.998884	8.001534	47.597827	2.501139	8.001472	37.593880
	10	0.001055	-5.000085	-0.292029	-2.498177	8.000264	47.600705	10.001673	8.000200	37.585317
21	1	10.001252	-7.999233	-0.296417	-9.998635	-7.999299	47.710086	2.501892	-7.999364	37.700227
	2	-9.999717	-7.998927	-0.118612	-2.498751	-7.999673	47.705969	10.002041	-7.999238	37.683667
	3	10.001285	-2.998446	-0.294309	-9.999025	-3.998651	47.684896	2.501004	-3.998645	37.672515
	4	-9.999755	-2.999142	-0.177499	-2.498281	-3.999520	47.684778	10.002074	-3.999089	37.656455
	5	10.001326	3.000957	-0.336279	-9.998979	0.001616	47.660206	2.501049	0.001550	37.646803
	6	-10.000262	3.000761	-0.215464	-2.498245	-0.000103	47.664087	10.002106	0.000262	37.631243
	7	10.001360	8.002731	-0.357171	-9.998932	4.001357	47.633017	2.501094	4.001792	37.621092
	8	-10.000324	8.000990	-0.275352	-2.498211	4.000575	47.635896	10.001639	4.000096	37.605530
	9	0.001036	4.999970	-0.328809	-9.998886	8.001536	47.607327	2.501139	8.001472	37.595880
	10	0.001162	-4.999976	-0.233030	-2.498176	8.000264	47.605205	10.001672	8.000701	37.582318
22	1	10.001251	-7.999232	-0.272418	-9.998634	-7.999300	47.692587	2.501892	-7.999364	37.696227
	2	-9.999951	-7.999431	-0.271608	-2.498318	-7.999671	47.697469	10.002043	-7.999240	37.691167
	3	10.002668	-2.997063	-0.281311	-9.999021	-3.998655	47.666897	2.501005	-3.999146	37.669515
	4	-9.999973	-2.998645	-0.304496	-2.498282	-3.999520	47.676778	10.002075	-3.999516	37.664455
	5	10.001325	3.000958	-0.313779	-9.998975	0.001612	47.643207	2.501050	0.001126	37.644304
	6	-10.000460	3.000759	-0.322462	-2.498246	0.000325	47.657087	10.002108	0.000683	37.639742
	7	10.001359	8.002643	-0.317672	-9.998929	4.001354	47.618518	2.501094	4.000873	37.620592
	8	-10.000551	8.000988	-0.379350	-2.498212	4.000576	47.629896	10.001641	4.000511	37.615530
	9	0.001036	5.001450	-0.338309	-9.998883	8.001533	47.592828	2.501139	8.001472	37.596380
	10	0.001017	-4.998682	-0.311028	-2.498177	8.000264	47.600705	10.001674	8.000199	37.592817
23	1	10.001251	-7.999232	-0.268918	-9.999065	-7.999300	47.685587	2.501893	-7.999364	37.686728
	2	-9.999874	-7.999430	-0.227109	-2.498319	-7.999670	47.689970	10.002041	-7.999238	37.680667
	3	10.001285	-2.998946	-0.280310	-9.999020	-3.998655	47.663398	2.501006	-3.999146	37.662516
	4	-9.999968	-2.999145	-0.301496	-2.498283	-3.999520	47.670779	10.002074	-3.999089	37.655955
	5	10.002832	3.000957	-0.339779	-9.998974	0.001612	47.639708	2.501051	0.001549	37.638804
	6	-10.000508	3.000759	-0.344462	-2.498247	0.000326	47.652587	10.002107	0.000261	37.632243
	7	10.001360	8.000686	-0.373670	-9.998928	4.001354	47.616018	2.501095	4.000873	37.614592
	8	-10.000619	7.999319	-0.404851	-2.499132	4.000577	47.625896	10.001640	4.000512	37.609030
	9	0.001037	5.001449	-0.370808	-9.998883	8.001533	47.592328	2.501140	8.001057	37.592380
	10	0.000999	-5.000142	-0.319528	-2.498677	8.000680	47.597205	10.001673	8.000613	37.587317
24	1	10.001250	-7.999230	-0.156420	-9.999068	-7.998366	47.699087	2.501892	-7.999364	37.703726
	2	-9.999918	-7.998430	-0.253609	-2.498317	-7.999673	47.704969	10.002044	-7.999241	37.698166
	3	10.001284	-2.998945	-0.214811	-9.999022	-3.998653	47.672897	2.501003	-3.999146	37.676515
	4	-9.999939	-2.999145	-0.287497	-2.498281	-3.999520	47.683278	10.002076	-3.999518	37.671454
	5	10.001325	3.000959	-0.271780	-9.998976	0.000689	47.648207	2.501048	0.001551	37.649803
	6	-10.000562	3.000758	-0.366962	-2.498246	0.000325	47.662087	10.002108	0.000259	37.644742
	7	10.001359	8.000687	-0.346671	-9.998929	4.001355	47.621017	2.501094	4.000873	37.623591
	8	-10.000544	7.999394	-0.376851	-2.499133	4.000075	47.633896	10.001641	4.000511	37.618029
	9	0.001036	5.001450	-0.358308	-9.998883	8.001533	47.594827	2.501138	8.001472	37.598379
	10	0.001141	-4.999997	-0.245030	-2.499093	8.000264	47.602705	10.001674	8.000199	37.594317

N° part	N° point	Plan 0 (CMM)			Plan 1 (CMM)			Plan 2 (CMM)		
		X	Y	Z	X	Y	Z	X	Y	Z
25	1	10.001251	-7.999232	-0.278918	-9.999061	-7.999301	47.665589	2.501893	-7.999364	37.668729
	2	-9.999993	-7.999431	-0.293108	-2.498750	-7.999667	47.671471	10.002038	-7.999234	37.663169
	3	10.002649	-2.998946	-0.271310	-9.999015	-3.998660	47.642399	2.501010	-3.999146	37.644517
	4	-10.000008	-2.999145	-0.321496	-2.498285	-3.999446	47.652780	10.002071	-3.999512	37.638457
	5	10.001325	3.000958	-0.301779	-9.998970	0.000688	47.620209	2.501055	0.001546	37.621305
	6	-10.000551	3.000758	-0.362462	-2.498249	0.000329	47.633588	10.002104	0.000265	37.613744
	7	10.001359	8.000688	-0.299671	-9.998924	4.001350	47.596519	2.501099	4.000873	37.596093
	8	-10.000569	8.000988	-0.386350	-2.498214	4.000079	47.607897	10.001637	4.000515	37.592031
	9	0.001036	5.001450	-0.335809	-9.998878	8.001529	47.572329	2.501143	8.001468	37.574881
	10	0.001040	-5.000100	-0.299529	-2.498180	8.000268	47.579206	10.001671	8.000203	37.570318
26	1	10.001252	-7.999233	-0.319417	-9.999060	-7.999301	47.664589	2.501893	-7.999364	37.664229
	2	-10.000025	-7.999431	-0.309607	-2.498749	-7.999667	47.667471	10.002037	-7.999233	37.656669
	3	10.001285	-2.998947	-0.300309	-9.999015	-3.998660	47.642399	2.501011	-3.999146	37.640518
	4	-10.000022	-2.999146	-0.327996	-2.498286	-3.999946	47.650780	10.002070	-3.999511	37.632457
	5	10.001326	3.000957	-0.338779	-9.998970	0.000689	47.623709	2.501055	0.001545	37.618806
	6	-9.998493	3.000759	-0.340461	-2.498249	0.000329	47.635088	10.002104	0.000265	37.610744
	7	10.001359	8.000687	-0.350171	-9.998925	4.001351	47.601019	2.501099	4.000873	37.596093
	8	-10.000550	8.000988	-0.378850	-2.498214	4.000079	47.608897	10.001637	4.000516	37.588532
	9	0.001036	5.001450	-0.350808	-9.998880	8.001530	47.577828	2.501143	8.001468	37.574881
	10	0.000955	-5.000186	-0.339528	-2.498179	8.000267	47.581206	10.001670	8.000203	37.568319
27	1	10.001249	-7.999229	-0.111421	-9.999069	-7.999300	47.702086	2.501892	-7.999364	37.703226
	2	-9.999854	-7.999429	-0.214609	-2.498316	-7.999673	47.705969	10.002043	-7.999240	37.694166
	3	10.001284	-2.998944	-0.195311	-9.999023	-3.998653	47.675897	2.501003	-3.999146	37.675015
	4	-9.999855	-2.999144	-0.241497	-2.498281	-3.999520	47.684278	10.002075	-3.999089	37.667454
	5	10.001325	3.000959	-0.266280	-9.998977	0.000689	47.651207	2.501049	0.001551	37.648303
	6	-9.998494	3.000760	-0.303962	-2.498246	0.000324	47.663087	10.002108	0.000260	37.640242
	7	10.001359	8.000687	-0.339171	-9.998930	4.001355	47.623017	2.501094	4.000873	37.622092
	8	-10.000484	8.000989	-0.351351	-2.499133	4.000075	47.633896	10.001640	4.000511	37.614030
	9	0.001036	5.001450	-0.330809	-9.998884	8.000618	47.596327	2.501139	8.001472	37.596880
	10	0.001193	-4.998680	-0.213530	-2.498177	8.000680	47.601705	10.001673	8.000199	37.590317
28	1	10.001252	-7.999233	-0.309417	-9.999062	-7.999300	47.672088	2.501893	-7.999364	37.665729
	2	-9.999868	-7.999430	-0.223609	-2.498321	-7.999667	47.671471	10.002038	-7.999234	37.659169
	3	10.001285	-2.996983	-0.320809	-9.999016	-3.998658	47.648898	2.501011	-3.999146	37.640518
	4	-9.999916	-2.998644	-0.274997	-2.498285	-3.999946	47.653280	10.002070	-3.999510	37.629457
	5	10.001326	3.003024	-0.365779	-9.998971	0.000689	47.626708	2.501055	0.001545	37.617806
	6	-10.000429	3.001260	-0.307963	-2.498249	-0.000093	47.634088	10.002103	0.000266	37.607245
	7	10.001360	8.000686	-0.373170	-9.998925	4.001351	47.602018	2.501099	4.000873	37.595094
	8	-10.000504	8.000988	-0.359851	-2.498214	4.000079	47.607397	10.001636	4.000517	37.583032
	9	0.001037	4.999863	-0.373809	-9.998880	8.001530	47.578828	2.501144	8.001468	37.570382
	10	0.001043	-5.000097	-0.298029	-2.498180	8.000268	47.579206	10.001669	8.000204	37.561319
29	1	10.001251	-7.999231	-0.237919	-9.999058	-7.999301	47.654089	2.501894	-7.999364	37.658230
	2	-10.000067	-7.999432	-0.329607	-2.498323	-7.999665	47.658972	10.002037	-7.999232	37.653170
	3	10.001285	-2.998946	-0.270810	-9.999012	-3.998662	47.629900	2.501012	-3.999146	37.633518
	4	-10.000141	-2.999147	-0.378495	-2.498287	-3.999944	47.639781	10.002070	-3.999510	37.627957
	5	10.001325	3.000958	-0.311779	-9.998967	0.000688	47.606710	2.501057	0.001544	37.610306
	6	-9.998492	3.000757	-0.415960	-2.498251	0.000331	47.620089	10.002103	0.000266	37.603745
	7	10.001360	8.000687	-0.359670	-9.998921	4.001347	47.582020	2.501102	4.000873	37.585094
	8	-10.000749	8.000987	-0.447849	-2.499131	4.000081	47.594398	10.001636	4.000517	37.580532
	9	0.001037	5.001449	-0.404307	-9.998875	8.000617	47.557330	2.501146	8.001466	37.561882
	10	0.000999	-4.998682	-0.319528	-2.499091	8.000270	47.565207	10.001669	8.000205	37.558319
30	1	10.001251	-7.999232	-0.264918	-9.999067	-7.999300	47.694087	2.501892	-7.999364	37.693227
	2	-9.999937	-7.999430	-0.264108	-2.498318	-7.999671	47.697969	10.002042	-7.999238	37.685167
	3	10.001285	-2.998946	-0.278310	-9.999021	-3.998654	47.669897	2.501005	-3.999146	37.667516
	4	-9.999947	-2.999145	-0.291496	-2.498282	-3.999520	47.677278	10.002074	-3.999089	37.658955
	5	10.001325	3.000958	-0.313279	-9.998976	0.000689	47.647707	2.501050	0.001550	37.642804
	6	-9.998493	3.000759	-0.322462	-2.498246	0.000325	47.657587	10.002107	0.000261	37.635242
	7	10.002868	8.000687	-0.341671	-9.998929	4.001355	47.621517	2.501095	4.000873	37.618592
	8	-10.000528	8.000988	-0.370350	-2.498212	4.000076	47.630896	10.001640	4.000511	37.614030
	9	0.001036	5.001450	-0.363308	-9.998884	8.000618	47.597327	2.501139	8.001472	37.595380
	10	0.001056	-4.998681	-0.291528	-2.499093	8.000764	47.600705	10.001673	8.000200	37.588817

N° part	N° point	Plan 0 (CMM)			Plan 1 (CMM)			Plan 2 (CMM)		
		X	Y	Z	X	Y	Z	X	Y	Z
31	1	10.001250	-7.999230	-0.197919	-9.999082	-7.999298	47.755083	2.501890	-7.999364	37.753222
	2	-9.999840	-7.999429	-0.206610	-2.498309	-7.999681	47.756466	10.002051	-7.999750	37.743163
	3	10.001284	-2.998445	-0.248810	-9.999036	-3.998641	47.730893	2.500992	-3.999146	37.728511
	4	-9.999829	-2.999143	-0.225998	-2.498274	-3.999960	47.737774	10.002083	-3.999089	37.717950
	5	10.001325	3.000959	-0.277780	-9.998990	0.000691	47.709703	2.501036	0.000626	37.704799
	6	-10.000275	3.000761	-0.223464	-2.498238	0.000315	47.719583	10.002116	0.000250	37.694738
	7	10.002818	8.000688	-0.319172	-9.998944	4.001368	47.685014	2.501081	4.000873	37.681087
	8	-10.000250	8.000991	-0.233353	-2.498203	4.000499	47.694392	10.001649	4.000929	37.672025
	9	0.001035	5.001452	-0.255810	-9.998898	8.000620	47.661824	2.501126	8.001483	37.658875
	10	0.001153	-4.999985	-0.238030	-2.498169	8.000255	47.665701	10.001682	8.000189	37.650313
32	1	10.001250	-7.999231	-0.218919	-9.999063	-7.999300	47.677588	2.501893	-7.999364	37.681728
	2	-9.999932	-7.999430	-0.261608	-2.498320	-7.999238	47.682970	10.001611	-7.999736	37.671168
	3	10.001284	-2.998945	-0.220811	-9.999018	-3.998657	47.655898	2.501008	-3.999146	37.656016
	4	-9.999935	-2.999145	-0.285497	-2.498284	-3.999521	47.665279	10.002073	-3.999089	37.647956
	5	10.001324	3.000959	-0.241280	-9.998973	0.000689	47.635708	2.501052	0.000626	37.634304
	6	-9.998493	3.000759	-0.334462	-2.498248	0.000327	47.647587	10.002106	0.000762	37.626243
	7	10.002695	8.000689	-0.256672	-9.998927	4.001353	47.612018	2.501096	4.000873	37.612092
	8	-10.000453	8.000989	-0.338351	-2.498212	4.000497	47.623896	10.001639	4.000929	37.605030
	9	0.001035	5.001451	-0.300309	-9.998882	8.000618	47.590828	2.501140	8.001471	37.591380
	10	0.001074	-4.998681	-0.282029	-2.498177	7.999850	47.596705	10.002086	8.000200	37.585818
33	1	10.001251	-7.999232	-0.267418	-9.999073	-7.999299	47.718085	2.501892	-7.999364	37.705226
	2	-9.999773	-8.000613	-0.160611	-2.498315	-7.999238	47.712968	10.002042	-7.999739	37.688167
	3	10.001285	-2.998946	-0.298809	-9.999026	-3.998650	47.690896	2.501003	-3.999146	37.677515
	4	-9.999828	-2.999143	-0.224498	-2.498280	-3.999520	47.690277	10.002074	-3.999089	37.659955
	5	10.001326	3.000957	-0.367778	-9.998980	0.000690	47.665206	2.501048	0.000626	37.650803
	6	-9.998494	3.000760	-0.267963	-2.498245	0.000751	47.668086	10.002107	0.000261	37.633743
	7	10.001360	8.000686	-0.370670	-9.998933	4.001358	47.638016	2.501094	4.000873	37.623091
	8	-10.000463	8.000989	-0.342851	-2.498210	4.000497	47.638895	10.001639	4.000929	37.606530
	9	0.001037	5.001449	-0.371308	-9.998886	8.000619	47.608827	2.501139	8.001472	37.596880
	10	0.001138	-4.998681	-0.247029	-2.499093	7.999847	47.606705	10.001672	8.000201	37.582318
34	1	10.000130	-8.001202	-0.318505	-10.000164	-8.001272	47.592015	2.500361	-8.002246	37.593656
	2	-10.001338	-8.001402	-0.389693	-2.499867	-8.002124	47.593897	10.000906	-8.002601	37.583096
	3	10.000163	-3.001401	-0.335897	-10.000121	-4.001539	47.579824	2.499904	-4.001099	37.578443
	4	-10.001318	-3.001101	-0.397582	-2.499415	-4.002390	47.585705	10.000940	-4.002452	37.567382
	5	10.001784	2.998997	-0.368867	-10.000077	-0.001267	47.565633	2.499947	-0.001327	37.561730
	6	-10.001315	2.998798	-0.414048	-2.499378	-0.002616	47.575513	10.000975	-0.001770	37.553669
	7	10.001878	7.998727	-0.391759	-10.000034	3.998474	47.550943	2.499989	3.998915	37.548017
	8	-10.001370	7.999028	-0.445937	-2.499341	3.998536	47.559321	10.001009	3.998471	37.540455
	9	-0.000085	4.997292	-0.412896	-9.999990	7.998658	47.537252	2.500031	7.998599	37.536304
	10	0.000337	-5.000637	-0.382114	-2.499305	7.998219	47.539629	10.001044	7.998155	37.528741
35	1	10.001251	-7.999231	-0.227419	-9.998635	-7.999300	47.705086	2.501892	-7.999864	37.704226
	2	-9.999909	-7.999430	-0.248109	-2.498316	-7.999238	47.706969	10.002043	-7.999740	37.693666
	3	10.001285	-2.998946	-0.269810	-9.999023	-3.998652	47.678397	2.501003	-3.999146	37.675515
	4	-9.999933	-2.999145	-0.283997	-2.498281	-3.999520	47.684278	10.002075	-3.999517	37.665454
	5	10.001326	3.001457	-0.335279	-9.998977	0.000689	47.651707	2.501049	0.000626	37.647803
	6	-10.000463	3.000759	-0.323962	-2.498246	0.000751	47.662087	10.002107	0.000761	37.637242
	7	10.003009	8.000686	-0.398171	-9.998930	4.001355	47.622017	2.501094	4.000873	37.619592
	8	-10.000494	8.000988	-0.356351	-2.497790	4.000076	47.631896	10.001640	4.000929	37.611530
	9	0.001037	5.001449	-0.370308	-9.998883	8.000618	47.595327	2.501139	8.001472	37.593380
	10	0.001081	-4.998681	-0.278529	-2.498677	7.999849	47.598705	10.001673	8.000200	37.585817
36	1	10.001250	-7.999231	-0.207919	-9.999085	-7.999298	47.767582	2.501890	-7.999364	37.760222
	2	-9.999767	-7.999428	-0.156111	-2.498308	-7.999683	47.765465	10.002051	-7.999750	37.746662
	3	10.001284	-2.998945	-0.244810	-9.999038	-3.998639	47.739393	2.500991	-3.999146	37.730511
	4	-9.999785	-2.999143	-0.197998	-2.498273	-3.999960	47.741774	10.002083	-3.999526	37.716450
	5	10.002766	3.000958	-0.310280	-9.998991	0.000691	47.712703	2.501037	0.000626	37.702299
	6	-10.000356	3.000760	-0.269963	-2.498238	0.000753	47.719083	10.002115	0.000751	37.688738
	7	10.002936	8.000686	-0.370171	-9.998944	4.001367	47.683514	2.501083	4.000873	37.674088
	8	-10.000399	8.000989	-0.313351	-2.499135	4.000067	47.688393	10.001647	4.000503	37.661026
	9	0.001036	5.001451	-0.313309	-9.998897	8.000620	47.654324	2.501128	8.001481	37.646376
	10	0.001234	-4.998679	-0.187031	-2.498670	8.000256	47.655202	10.001680	8.000692	37.634814

N° part	N° point	Plan 0 (CMM)			Plan 1 (CMM)			Plan 2 (CMM)		
		X	Y	Z	X	Y	Z	X	Y	Z
	1	10.001252	-7.999233	-0.299417	-9.999061	-7.999300	47.669088	2.501893	-7.999864	37.665729
	2	-10.000038	-7.999432	-0.315607	-2.498321	-7.999667	47.670471	10.002037	-7.999307	37.656669
	3	10.001286	-2.998947	-0.334809	-9.999016	-3.998659	47.645899	2.501011	-3.999146	37.641518
	4	-10.000009	-2.999145	-0.321496	-2.498285	-3.999946	47.652280	10.002070	-3.999089	37.631457
37	5	10.001326	3.000957	-0.347279	-9.998971	0.000689	47.624709	2.501055	0.000626	37.618806
	6	-9.998493	3.000759	-0.341461	-2.498249	0.000751	47.634088	10.002103	0.000765	37.609244
	7	10.001360	8.000686	-0.381170	-9.998925	4.001350	47.600019	2.501099	4.000873	37.596093
	8	-10.000526	8.000988	-0.369350	-2.498714	4.000497	47.608897	10.001637	4.000516	37.587032
	9	0.001036	5.001450	-0.339809	-9.998880	8.000618	47.577828	2.501143	8.001468	37.573881
	10	0.001011	-4.998682	-0.313528	-2.498679	8.000267	47.580706	10.001670	8.000704	37.565319
	1	10.001251	-7.999231	-0.236419	-9.999066	-7.999300	47.690587	2.501893	-7.999364	37.688728
	2	-10.000024	-7.999431	-0.308607	-2.498318	-7.999238	47.691470	10.002041	-7.999737	37.679168
	3	10.001284	-2.998946	-0.259310	-9.999020	-3.998655	47.664397	2.501006	-3.999146	37.661016
	4	-10.000011	-2.998645	-0.322496	-2.498283	-3.999949	47.670279	10.002073	-3.999514	37.650956
38	5	10.001325	3.000958	-0.310279	-9.998974	0.000689	47.640708	2.501052	0.000626	37.634804
	6	-9.998493	3.000759	-0.353461	-2.498248	0.000751	47.648587	10.002106	0.000763	37.625243
	7	10.002878	8.000687	-0.346171	-9.998928	4.001353	47.612518	2.501097	4.000873	37.608593
	8	-10.000579	8.000988	-0.390350	-2.498213	4.000497	47.619896	10.001638	4.000514	37.599531
	9	0.001037	5.001449	-0.370308	-9.998881	8.000618	47.585828	2.501141	8.001470	37.582881
	10	0.001048	-4.998682	-0.295528	-2.498678	8.000266	47.588206	10.001671	8.000702	37.574818
	1	10.001251	-7.999232	-0.254918	-9.999065	-7.999300	47.683088	2.501893	-7.999364	37.685228
	2	-9.999898	-7.999430	-0.242109	-2.498319	-7.999670	47.687470	10.002041	-7.999737	37.678168
	3	10.001285	-2.998946	-0.274810	-9.999019	-3.998656	47.658898	2.501007	-3.999146	37.660516
	4	-9.999961	-3.000561	-0.298497	-2.498283	-3.999520	47.667779	10.002073	-3.999514	37.652955
39	5	10.001325	3.000958	-0.319279	-9.998973	0.000689	47.635708	2.501052	0.000626	37.635304
	6	-10.000516	3.000759	-0.347962	-2.498247	0.000751	47.652087	10.002106	0.000683	37.628243
	7	10.002885	8.000687	-0.349171	-9.998927	4.001352	47.610018	2.501096	4.000873	37.610592
	8	-10.000553	8.000988	-0.380350	-2.499132	4.000497	47.621396	10.001639	4.000513	37.604530
	9	0.002562	5.001450	-0.348809	-9.998881	8.000618	47.585828	2.501140	8.000557	37.588380
	10	0.001459	-4.998682	-0.303528	-2.499092	8.000680	47.590205	10.001672	8.000701	37.582318
	1	10.001252	-7.999233	-0.295917	-9.998634	-7.999300	47.683087	2.501893	-7.999364	37.682228
	2	-9.999982	-7.998931	-0.287608	-2.498319	-7.999669	47.685470	10.002040	-7.999736	37.674168
	3	10.001285	-2.998947	-0.308809	-9.999019	-3.999582	47.660398	2.501007	-3.999146	37.658516
	4	-9.999947	-3.000547	-0.291497	-2.498283	-3.999093	47.667779	10.002073	-3.999514	37.649956
40	5	10.002821	3.000957	-0.335279	-9.998974	0.000689	47.638708	2.501052	0.000626	37.635304
	6	-10.000439	3.000759	-0.312463	-2.498247	0.000751	47.649087	10.002106	0.000683	37.627243
	7	10.002937	8.000686	-0.370171	-9.998928	4.000435	47.614518	2.501096	4.000873	37.612592
	8	-10.000509	8.000988	-0.361851	-2.498712	4.000577	47.624396	10.001639	4.000512	37.607030
	9	0.002547	5.001450	-0.342309	-9.998883	8.000618	47.591828	2.501140	8.000557	37.590380
	10	0.001458	-5.000597	-0.298028	-2.498678	8.000680	47.595705	10.001173	8.000700	37.584817
	1	10.001252	-7.999233	-0.300417	-9.998633	-7.998872	47.671588	2.501893	-7.999864	37.671229
	2	-10.000002	-8.000848	-0.298608	-2.498321	-7.999667	47.673971	10.002038	-7.999734	37.661669
	3	10.002728	-2.998947	-0.311310	-9.998592	-3.998658	47.648398	2.501009	-3.999146	37.647017
	4	-9.999992	-3.000594	-0.313996	-2.498285	-3.999521	47.655779	10.002071	-3.999511	37.635957
41	5	10.002801	3.000958	-0.326280	-9.998971	0.000689	47.626708	2.501054	0.000626	37.623805
	6	-10.000467	3.000759	-0.325962	-2.498249	0.000751	47.637588	10.002104	0.000683	37.614744
	7	10.002878	8.000687	-0.346171	-9.998925	4.000851	47.603018	2.501098	4.000873	37.600093
	8	-10.000474	8.000989	-0.347851	-2.498714	4.000997	47.611897	10.001637	4.000515	37.592031
	9	0.002551	4.999435	-0.344310	-9.998880	8.000618	47.579828	2.501142	8.000557	37.578381
	10	0.001458	-4.998682	-0.299528	-2.499092	8.000680	47.585206	10.001671	8.001113	37.570818
	1	10.001251	-7.999232	-0.278418	-9.999066	-7.998868	47.688587	2.501893	-7.999864	37.679728
	2	-9.999816	-7.998929	-0.190610	-2.498319	-7.999669	47.685970	10.002039	-7.999735	37.667169
	3	10.002714	-2.998947	-0.304310	-9.999020	-3.999582	47.662898	2.501008	-3.999146	37.653017
	4	-9.999899	-2.999144	-0.266497	-2.498284	-3.999448	47.665779	10.002071	-3.999512	37.639956
42	5	10.002893	3.000957	-0.365779	-9.998974	0.000689	47.638708	2.501053	0.000626	37.627805
	6	-10.000439	3.000759	-0.312463	-2.498248	0.000751	47.644088	10.002104	0.000683	37.614744
	7	10.002974	8.000686	-0.385171	-9.998927	4.000435	47.612018	2.501098	4.000873	37.603093
	8	-10.000539	8.000988	-0.374850	-2.499132	4.000997	47.617397	10.001637	4.000515	37.590031
	9	0.002569	4.999417	-0.351809	-9.998882	8.000618	47.586828	2.501142	8.000557	37.578381
	10	0.001458	-5.000591	-0.295028	-2.498679	8.000680	47.586706	10.001670	8.000703	37.567319

N° part	N° point	Plan 0 (CMM)			Plan 1 (CMM)			Plan 2 (CMM)		
		X	Y	Z	X	Y	Z	X	Y	Z
43	1	10.001251	-7.999232	-0.266918	-9.998635	-7.999300	47.708086	2.501892	-7.999864	37.711226
	2	-9.999868	-7.999430	-0.223609	-2.498316	-7.999674	47.711968	10.002045	-7.999308	37.704166
	3	10.001285	-2.998946	-0.287310	-9.998594	-3.999582	47.683396	2.501001	-3.999146	37.685514
	4	-9.999922	-3.000522	-0.277997	-2.498280	-3.999452	47.693777	10.002077	-3.999519	37.679453
	5	10.001325	3.000958	-0.310279	-9.998979	0.000690	47.662706	2.501046	0.000626	37.662302
	6	-10.000410	3.001260	-0.297963	-2.498244	0.000323	47.674586	10.002110	0.000683	37.656241
	7	10.002853	8.000687	-0.334671	-9.998933	4.000436	47.638016	2.501090	4.000873	37.638090
	8	-10.000463	8.000989	-0.342351	-2.499134	4.000998	47.648895	10.001643	4.000508	37.632028
	9	0.002522	5.001450	-0.331309	-9.998888	8.000619	47.614826	2.501135	8.000557	37.615878
	10	0.001458	-4.998681	-0.272528	-2.498675	8.000262	47.619704	10.001676	8.000613	37.611815
44	1	10.000131	-7.999593	-0.381504	-9.999750	-8.002185	47.588515	2.500775	-8.001832	37.589156
	2	-10.001345	-7.999764	-0.392193	-2.499867	-8.001710	47.591897	10.000905	-8.002187	37.578596
	3	10.001723	-3.001402	-0.360897	-10.000120	-4.001539	47.577324	2.499904	-4.002010	37.574443
	4	-9.999664	-3.001601	-0.409581	-2.499415	-4.001977	47.583205	10.000940	-4.001542	37.564382
	5	10.000204	2.998997	-0.383366	-10.000077	-0.002177	47.565133	2.499947	-0.001827	37.560730
	6	-10.001293	2.998798	-0.407048	-2.499378	-0.001705	47.574513	10.000974	-0.001770	37.551669
	7	10.001816	7.998727	-0.368259	-10.000034	3.998474	47.551443	2.499989	3.998008	37.548017
	8	-10.001329	7.998528	-0.433437	-2.499342	3.998036	47.558821	10.001009	3.997566	37.538955
	9	0.001651	4.997252	-0.426396	-9.999990	7.997753	47.537252	2.500031	7.997694	37.535304
	10	-0.000163	-5.002746	-0.380614	-2.499305	7.998219	47.540629	10.001044	7.998155	37.527241
45	1	10.001250	-7.999230	-0.174920	-9.998640	-7.999297	47.809079	2.501889	-7.999864	37.796219
	2	-9.999705	-7.998927	-0.108612	-2.498302	-7.999689	47.803463	10.002057	-7.999757	37.781160
	3	10.002553	-2.998945	-0.215811	-9.999049	-3.999579	47.782890	2.500982	-3.999146	37.769508
	4	-9.999666	-3.000261	-0.111000	-2.498267	-3.999467	47.782272	10.002089	-3.999534	37.754948
	5	10.002632	3.000959	-0.239281	-9.999002	0.000692	47.758701	2.501028	0.000626	37.743796
	6	-10.000165	3.000763	-0.148466	-2.498232	-0.000192	47.761581	10.002121	0.000683	37.729235
	7	10.002749	8.000688	-0.285172	-9.998956	4.000438	47.732511	2.501073	4.000873	37.719084
	8	-10.000218	8.000991	-0.213353	-2.499138	4.001000	47.734390	10.001654	4.000495	37.703523
	9	0.002356	5.000952	-0.247311	-9.998909	8.000621	47.705321	2.501118	8.000557	37.693873
	10	0.001957	-4.999179	-0.178030	-2.499098	8.000248	47.704699	10.001686	8.000183	37.680311
46	1	10.001251	-7.997846	-0.283418	-9.998636	-7.999299	47.724585	2.501892	-7.999864	37.714725
	2	-9.999752	-7.999928	-0.145111	-2.498314	-7.999675	47.721468	10.002044	-7.999741	37.701666
	3	10.002665	-2.998946	-0.279810	-9.999028	-3.999581	47.698395	2.501001	-3.999146	37.687014
	4	-9.999755	-3.000352	-0.177999	-2.498279	-3.999453	47.699277	10.002076	-3.999518	37.672954
	5	10.001326	3.000957	-0.328779	-9.998982	0.000690	47.673706	2.501046	0.000626	37.661302
	6	-10.000259	2.999497	-0.213465	-2.498244	0.000322	47.677586	10.002109	0.000683	37.646742
	7	10.002876	8.000687	-0.345171	-9.998935	4.000436	47.646516	2.501091	4.000873	37.633591
	8	-10.000304	8.000990	-0.264852	-2.499134	4.000498	47.649395	10.001641	4.000510	37.621029
	9	0.001035	5.000951	-0.292809	-9.998889	8.000619	47.619326	2.501136	8.000557	37.608379
	10	0.001152	-5.000486	-0.238530	-2.498675	8.000262	47.617704	10.001674	8.001113	37.597316
47	1	10.001251	-7.999233	-0.289918	-9.998636	-7.999299	47.716085	2.501892	-7.999364	37.703226
	2	-9.998588	-7.998928	-0.130111	-2.498316	-7.999673	47.710469	10.002042	-7.999739	37.685667
	3	10.002717	-2.998947	-0.305810	-9.998594	-3.999582	47.688396	2.501004	-3.999146	37.673515
	4	-9.998546	-2.999143	-0.192498	-2.498281	-3.999451	47.686778	10.002074	-3.999515	37.655455
	5	10.001326	3.000957	-0.339279	-9.998979	0.000690	47.661206	2.501049	0.000626	37.645804
	6	-10.000308	3.000761	-0.242964	-2.498246	0.000324	47.663587	10.002106	0.000683	37.626743
	7	10.001360	8.000686	-0.367670	-9.998932	4.000435	47.632517	2.501095	4.000873	37.617092
	8	-10.000390	8.000989	-0.309351	-2.498211	3.999997	47.633896	10.001638	4.000929	37.599531
	9	0.001036	5.000950	-0.333809	-9.998886	8.000619	47.605327	2.501140	8.000557	37.589880
	10	0.001151	-4.999180	-0.239529	-2.498177	8.000265	47.599705	10.001671	8.000202	37.573818
48	1	10.001252	-7.999233	-0.293418	-9.998635	-7.999300	47.705586	2.501892	-7.999864	37.699727
	2	-9.999814	-7.998929	-0.189110	-2.498317	-7.999672	47.703469	10.002042	-7.999739	37.686667
	3	10.001285	-2.998946	-0.270810	-9.999024	-3.999582	47.679397	2.501004	-3.999146	37.672015
	4	-9.999868	-2.999144	-0.248997	-2.498281	-4.000381	47.682278	10.002074	-3.999516	37.660955
	5	10.001326	3.000957	-0.335279	-9.998977	0.000689	47.654707	2.501049	0.001126	37.646303
	6	-10.000389	2.999365	-0.287463	-2.498246	0.000325	47.661087	10.002107	-0.000161	37.633743
	7	10.001360	8.000686	-0.367670	-9.998931	4.000435	47.628017	2.501094	4.000873	37.620592
	8	-10.000512	8.000988	-0.363851	-2.498211	4.000497	47.633396	10.001640	4.000512	37.608530
	9	0.002539	4.999447	-0.338810	-9.998885	8.000619	47.601327	2.501139	8.000557	37.595380
	10	0.001458	-4.999181	-0.276528	-2.498177	8.000680	47.602705	10.001673	8.000613	37.584817

N° part	N° point	Plan 0 (CMM)			Plan 1 (CMM)			Plan 2 (CMM)		
		X	Y	Z	X	Y	Z	X	Y	Z
49	1	10.001250	-7.999230	-0.160920	-9.999066	-7.998869	47.687587	2.501893	-7.999864	37.687728
	2	-9.999915	-7.998930	-0.251109	-2.498318	-7.999238	47.690970	10.002041	-7.999737	37.680168
	3	10.001284	-2.998945	-0.234810	-9.999019	-3.999582	47.660398	2.501007	-3.999146	37.660016
	4	-9.999905	-2.999144	-0.268997	-2.498283	-3.999948	47.667779	10.002073	-3.999514	37.652955
	5	10.002699	3.000959	-0.276780	-9.998973	0.000689	47.636208	2.501052	0.000626	37.633804
	6	-10.000473	2.999280	-0.328463	-2.498248	0.000327	47.647088	10.002106	0.000262	37.626243
	7	10.002905	8.000687	-0.357671	-9.998927	4.000435	47.608518	2.501096	4.000873	37.611592
	8	-10.000536	8.000988	-0.373350	-2.498213	4.000997	47.619396	10.001639	4.000513	37.604530
	9	0.001036	5.000950	-0.349308	-9.998881	8.000618	47.582328	2.501141	8.000970	37.584880
	10	0.001143	-4.999180	-0.244029	-2.498179	8.000266	47.587206	10.001672	8.000613	37.578818
50	1	10.001249	-7.999229	-0.128921	-9.998635	-7.998866	47.698087	2.501892	-7.999864	37.697227
	2	-9.999885	-7.998930	-0.234109	-2.498317	-7.999238	47.698969	10.002042	-7.999739	37.687167
	3	10.002540	-2.998944	-0.207811	-9.999021	-3.999582	47.670897	2.501005	-3.999146	37.668515
	4	-9.999926	-2.999145	-0.280497	-2.498282	-3.999950	47.677778	10.002074	-3.999515	37.658955
	5	10.001325	3.002357	-0.289280	-9.998975	0.001113	47.645707	2.501050	0.000626	37.641304
	6	-10.000542	3.000258	-0.358962	-2.498247	0.000751	47.655587	10.002107	0.000683	37.632243
	7	10.001360	8.000686	-0.375170	-9.998929	4.000854	47.618018	2.501096	4.000873	37.614092
	8	-10.000565	8.000988	-0.384850	-2.498212	4.000497	47.626896	10.002056	4.000513	37.606030
	9	0.002612	5.000949	-0.369309	-9.998882	8.001032	47.589828	2.501141	8.000557	37.587380
	10	0.001457	-5.000469	-0.228530	-2.499092	8.000680	47.593705	10.002085	8.000201	37.581318

N° part	N° point	Cylinder (CMM)		
		X	Y	Z
	1	14.998087	0.001103	25.999971
	2	10.505575	-10.503522	26.000151
	3	0.001132	-14.773568	25.998916
	4	-10.468183	-10.469076	25.999455
	5	-14.931321	0.001331	25.999139
	6	-14.914488	0.001052	35.999534
1	7	-10.417132	-10.424189	35.998819
	8	0.002189	-14.696865	35.998889
	9	10.480335	-10.473892	35.998557
	10	15.015920	0.001285	35.998835
	11	-14.895339	0.001906	45.999370
	12	-10.374160	-10.372377	45.998998
	13	-0.698338	-14.599836	45.999369
	1	15.006588	0.001103	25.999971
	2	10.507002	-10.497596	25.999833
	3	0.001133	-14.768067	25.999725
	4	-10.464839	-10.464920	25.999955
	5	-14.929821	0.001331	25.999964
	6	-14.925989	0.001052	35.999535
2	7	-10.425998	-10.430808	35.998819
	8	0.002190	-14.703365	35.998889
	9	10.477177	-10.471549	35.998557
	10	15.013920	0.001285	35.998835
	11	-14.917340	0.001908	45.998870
	12	-10.395619	-10.391420	45.998999
	13	-0.698337	-14.619336	45.999370
	1	15.004087	0.001103	25.999971
	2	10.504342	-10.495755	26.000150
	3	0.001133	-14.767067	25.999725
	4	-10.467464	-10.464796	25.999955
	5	-14.931321	0.001331	25.999139
	6	-14.924989	0.001877	35.999211
3	7	-10.427498	-10.434829	35.999319
	8	0.002190	-14.703865	35.998889
	9	10.476490	-10.469237	35.999370
	10	15.009920	0.001285	35.998835
	11	-14.921340	0.001084	45.999370
	12	-10.397014	-10.396024	45.999000
	13	-0.698337	-14.620336	45.999370
	1	15.005088	0.001103	25.999638
	2	10.508844	-10.500253	25.999833
	3	0.001942	-14.767067	25.999725
	4	-10.469464	-10.466295	25.999955
	5	-14.930821	0.001331	25.999964
	6	-14.920489	0.001876	35.999210
4	7	-10.426669	-10.433151	35.999319
	8	0.002189	-14.699865	35.998889
	9	10.479148	-10.471079	35.998557
	10	15.012920	0.001285	35.998835
	11	-14.918340	0.001408	45.999370
	12	-10.396666	-10.394873	45.999000
	13	-0.698337	-14.620836	45.999370
	1	15.004588	0.001603	25.999638
	2	10.503525	-10.497072	26.000650
	3	0.001133	-14.770067	25.999725
	4	-10.468339	-10.468420	25.999955
	5	-14.930321	0.001331	25.999639
	6	-14.921489	0.001376	35.999210
5	7	-10.429277	-10.432518	35.999319
	8	0.002189	-14.702365	35.999389
	9	10.478460	-10.468767	35.999057
	10	15.012920	0.001619	35.998835
	11	-14.915340	0.001408	45.999370
	12	-10.396270	-10.391769	45.998999
	13	-0.698337	-14.619836	45.999370

N° part	N° point	Cylinder (CMM)		
		X	Y	Z
	1	15.003087	0.000770	25.999971
	2	10.503500	-10.494098	25.999834
	3	0.001133	-14.766067	25.999726
	4	-10.467964	-10.465296	25.999955
	5	-14.932821	0.001331	25.999639
	6	-14.923489	0.001377	35.999211
6	7	-10.427321	-10.432993	35.999319
	8	0.002189	-14.702365	35.999389
	9	10.475146	-10.467081	35.999057
	10	15.007920	0.001285	35.998835
	11	-14.920840	0.001084	45.999370
	12	-10.401378	-10.393662	45.999000
	13	-0.698337	-14.616836	45.999370
	1	15.007088	0.001103	25.999638
	2	10.507844	-10.499254	26.000650
	3	0.001133	-14.767067	25.999725
	4	-10.464930	-10.459830	25.999954
	5	-14.923821	0.001331	25.999639
	6	-14.916989	0.001376	35.999210
7	7	-10.423372	-10.428940	35.999319
	8	0.002189	-14.699865	35.999389
	9	10.479491	-10.472235	35.999057
	10	15.015420	0.001285	35.998835
	11	-14.914840	0.001084	45.999370
	12	-10.390267	-10.385272	45.998999
	13	-0.698337	-14.619836	45.999370
	1	15.004088	0.000770	25.999638
	2	10.506660	-10.496938	26.000650
	3	0.001133	-14.771567	25.999725
	4	-10.466307	-10.463952	25.999955
	5	-14.929821	0.001331	25.999639
	6	-14.918989	0.001376	35.999210
8	7	-10.425682	-10.432138	35.999319
	8	0.002190	-14.706865	35.999390
	9	10.478147	-10.470080	35.999057
	10	15.011420	0.001285	35.998835
	11	-14.911340	0.001084	45.999370
	12	-10.391164	-10.389374	45.998999
	13	-0.698337	-14.621336	45.999370
	1	15.000087	0.000770	25.999638
	2	10.508003	-10.499095	26.000650
	3	0.001133	-14.772567	25.999725
	4	-10.468496	-10.467764	25.999955
	5	-14.927821	0.001656	25.999639
	6	-14.922989	0.001052	35.999211
9	7	-10.428802	-10.434512	35.999319
	8	0.001255	-14.707865	35.999390
	9	10.479992	-10.472735	35.999057
	10	15.007420	0.001618	35.998835
	11	-14.918340	0.001408	45.999370
	12	-10.396817	-10.394222	45.999000
	13	-0.699132	-14.622836	45.999371
	1	15.002087	0.000770	25.999638
	2	10.506502	-10.497596	26.000650
	3	0.001133	-14.770067	25.999725
	4	-10.464495	-10.463764	25.999955
	5	-14.923821	0.001655	25.999639
	6	-14.916489	0.001052	35.999534
10	7	-10.425524	-10.432802	35.998819
	8	0.001690	-14.705365	35.999389
	9	10.478647	-10.470580	35.999057
	10	15.012420	0.001285	35.998835
	11	-14.913340	0.001084	45.999370
	12	-10.396119	-10.391920	45.998999
	13	-0.698337	-14.619836	45.999370

N° part	N° point	Cylinder (CMM)		
		X	Y	Z
	1	15.002587	0.000770	25.999971
	2	10.505685	-10.498413	26.000650
	3	0.001133	-14.770567	26.000534
	4	-10.467151	-10.466108	25.999955
	5	-14.930321	0.001656	25.999639
	6	-14.919989	0.001052	35.999210
11	7	-10.427144	-10.431156	35.999319
	8	0.001255	-14.705365	35.999389
	9	10.478178	-10.472549	35.999057
	10	15.013420	0.001619	35.998835
	11	-14.921340	0.001408	45.999370
	12	-10.395014	-10.394524	45.999000
	13	-0.698337	-14.621836	45.999370
	1	15.007588	0.000770	25.999638
	2	10.508527	-10.502070	26.000651
	3	0.001132	-14.774568	25.999725
	4	-10.468964	-10.465795	25.999955
	5	-14.926821	0.001656	25.999639
	6	-14.916989	0.001052	35.999210
12	7	-10.426986	-10.431821	35.999319
	8	0.001255	-14.705365	35.999389
	9	10.479835	-10.473892	35.999057
	10	15.016920	0.001285	35.998835
	11	-14.909339	0.001407	45.999370
	12	-10.393363	-10.393676	45.998999
	13	-0.699132	-14.620836	45.999370
	1	15.004087	0.001103	25.999638
	2	10.502499	-10.493098	26.000650
	3	0.001133	-14.769067	25.999725
	4	-10.464839	-10.464920	25.999955
	5	-14.930821	0.001331	25.999639
	6	-14.923989	0.001377	35.999211
13	7	-10.426808	-10.429985	35.999319
	8	0.001689	-14.701865	35.999389
	9	10.474177	-10.468550	35.999057
	10	15.013420	0.001619	35.998835
	11	-14.918840	0.001408	45.999370
	12	-10.394014	-10.393025	45.998999
	13	-0.699132	-14.618336	45.999370
	1	15.000587	0.000770	25.999638
	2	10.506185	-10.498912	26.000650
	3	0.001133	-14.765567	25.999726
	4	-10.465463	-10.462797	25.999955
	5	-14.930321	0.001656	25.999639
	6	-14.918989	0.001376	35.999210
14	7	-10.425663	-10.429637	35.999319
	8	0.001689	-14.702865	35.999389
	9	10.480305	-10.471922	35.999057
	10	15.011920	0.001619	35.998835
	11	-14.912340	0.001408	45.999370
	12	-10.393316	-10.391223	45.998999
	13	-0.699132	-14.617836	45.999370
	1	15.003087	0.001103	25.999638
	2	10.502525	-10.496072	26.000650
	3	0.001133	-14.769067	25.999725
	4	-10.466275	-10.461985	25.999955
	5	-14.928321	0.001656	25.999639
	6	-14.920489	0.000404	35.999211
15	7	-10.426017	-10.433809	35.999319
	8	0.001690	-14.705865	35.999389
	9	10.477177	-10.472049	35.999057
	10	15.013420	0.000785	35.998835
	11	-14.915840	0.001408	45.999370
	12	-10.395514	-10.395024	45.999000
	13	-0.699132	-14.622336	45.999370

N° part	N° point	Cylinder (CMM)		
		X	Y	Z
	1	15.003587	0.001103	25.999638
	2	10.504501	-10.496097	26.000650
	3	0.001133	-14.769067	25.999725
	4	-10.468776	-10.464484	25.999955
	5	-14.927821	0.001656	25.999639
	6	-14.919989	0.000728	35.999211
16	7	-10.428942	-10.431346	35.999319
	8	0.001255	-14.702865	35.999389
	9	10.475677	-10.470049	35.999057
	10	15.011920	0.000785	35.998835
	11	-14.912840	0.001408	45.999370
	12	-10.393467	-10.390572	45.998999
	13	-0.699132	-14.619836	45.999370
	1	15.002587	0.001103	25.999638
	2	10.502341	-10.494756	26.000650
	3	0.001133	-14.770567	25.999725
	4	-10.467776	-10.463484	25.999955
	5	-14.930321	0.000831	25.999639
	6	-14.920989	0.001052	35.999211
17	7	-10.427308	-10.429985	35.999319
	8	0.001255	-14.701365	35.999389
	9	10.472488	-10.465238	35.999057
	10	15.010420	0.001619	35.998835
	11	-14.916840	0.001408	45.999370
	12	-10.394060	-10.395978	45.999500
	13	-0.698837	-14.618836	45.999870
	1	15.003087	0.001103	25.999638
	2	10.508319	-10.497779	26.000650
	3	0.001132	-14.774568	25.999725
	4	-10.470089	-10.464172	25.999955
	5	-14.929821	0.001656	25.999639
	6	-14.921989	0.000728	35.999211
18	7	-10.427973	-10.433334	35.999319
	8	0.001255	-14.706865	35.999390
	9	10.482806	-10.474421	35.999057
	10	15.012420	0.001619	35.998835
	11	-14.913840	0.001408	45.999370
	12	-10.394514	-10.394025	45.998999
	13	-0.699132	-14.623836	45.999371
	1	15.007088	0.001103	25.999638
	2	10.508027	-10.501570	26.000650
	3	0.001132	-14.774067	25.999725
	4	-10.466275	-10.461985	25.999955
	5	-14.924821	0.000831	25.999639
	6	-14.916489	0.000728	35.999210
19	7	-10.423372	-10.428940	35.999319
	8	0.001691	-14.708865	35.999390
	9	10.479491	-10.472735	35.999057
	10	15.014920	0.000785	35.998835
	11	-14.908839	0.001407	45.999370
	12	-10.390966	-10.388073	45.998999
	13	-0.698837	-14.622336	45.999370
	1	14.999087	0.000270	25.999639
	2	10.502683	-10.495414	26.000650
	3	0.001133	-14.768567	25.999725
	4	-10.467964	-10.465296	25.999955
	5	-14.932321	0.001656	25.999639
	6	-14.925989	0.000727	35.999211
20	7	-10.429454	-10.434354	35.999319
	8	0.001689	-14.702865	35.999389
	9	10.474176	-10.469050	35.999057
	10	15.005419	0.001285	35.998835
	11	-14.922340	0.001409	45.999370
	12	-10.396619	-10.392920	45.999499
	13	-0.698837	-14.616336	45.999870

N° part	N° point	Cylinder (CMM)		
		X	Y	Z
	1	15.001087	0.000270	25.999638
	2	10.501024	-10.495072	26.000650
	3	0.001133	-14.770567	25.999725
	4	-10.469120	-10.465640	25.999955
	5	-14.931821	0.001656	25.999639
	6	-14.920989	0.000728	35.999211
21	7	-10.429277	-10.432518	35.999319
	8	0.001255	-14.704365	35.999389
	9	10.476334	-10.470393	35.999057
	10	15.009920	0.001618	35.998835
	11	-14.918840	0.001408	45.999370
	12	-10.397969	-10.395070	45.999000
	13	-0.699132	-14.619336	45.999370
	1	15.004588	0.000270	25.999638
	2	10.507344	-10.499754	26.000650
	3	0.001133	-14.773067	25.999725
	4	-10.465619	-10.462141	25.999955
	5	-14.924821	0.000831	25.999639
	6	-14.915488	0.001052	35.999210
22	7	-10.423872	-10.429440	35.999319
	8	0.001690	-14.706865	35.999390
	9	10.479335	-10.473392	35.999057
	10	15.013920	0.001619	35.998835
	11	-14.916340	0.001408	45.999370
	12	-10.392014	-10.391525	45.998999
	13	-0.698837	-14.620336	45.999370
	1	15.001587	0.000603	25.999638
	2	10.506185	-10.499412	26.000650
	3	0.001132	-14.773567	25.999725
	4	-10.472935	-10.467826	25.999955
	5	-14.931321	0.000831	25.999639
	6	-14.923989	0.001052	35.999211
23	7	-10.430923	-10.433366	35.999319
	8	0.001691	-14.711365	35.999390
	9	10.479491	-10.472735	35.999057
	10	15.010420	0.001619	35.998835
	11	-14.922340	0.001409	45.999370
	12	-10.398772	-10.394268	45.999000
	13	-0.698837	-14.622336	45.999370
	1	15.000587	0.000603	25.999638
	2	10.501867	-10.496730	26.000650
	3	0.001133	-14.771067	25.999725
	4	-10.470277	-10.465983	25.999955
	5	-14.932321	0.000831	25.999639
	6	-14.920989	0.001052	35.999211
24	7	-10.431892	-10.431378	35.999319
	8	0.001690	-14.704865	35.999389
	9	10.476646	-10.469081	35.999057
	10	15.011420	0.001619	35.998835
	11	-14.917340	0.001408	45.999370
	12	-10.396514	-10.396024	45.999000
	13	-0.698837	-14.618336	45.999370
	1	15.008088	0.000603	25.999637
	2	10.508186	-10.501411	26.000650
	3	0.001133	-14.770067	25.999725
	4	-10.465307	-10.463452	25.999955
	5	-14.926821	0.000831	25.999639
	6	-14.918989	0.001052	35.999210
25	7	-10.425827	-10.428966	35.999319
	8	0.001690	-14.704865	35.999389
	9	10.481305	-10.472922	35.999057
	10	15.016920	0.000785	35.998835
	11	-14.912340	0.001408	45.999370
	12	-10.391315	-10.389223	45.998999
	13	-0.698837	-14.619836	45.999370

N° part	N° point	Cylinder (CMM)		
		X	Y	Z
	1	15.006588	0.000603	25.999638
	2	10.511004	-10.502593	26.000651
	3	0.001132	-14.775068	25.999725
	4	-10.467620	-10.464140	25.999955
	5	-14.926821	0.000831	25.999639
	6	-14.918489	0.001052	35.999210
26	7	-10.425840	-10.431473	35.999319
	8	0.001691	-14.708865	35.999390
	9	10.483590	-10.471139	35.999057
	10	15.012920	0.000785	35.998835
	11	-14.910340	0.001407	45.999370
	12	-10.395270	-10.390769	45.998999
	13	-0.698836	-14.624836	45.999371
	1	15.001087	0.000270	25.999638
	2	10.505184	-10.498413	26.000650
	3	0.001133	-14.769567	25.999725
	4	-10.468932	-10.463828	25.999955
	5	-14.937821	0.000831	25.999639
	6	-14.920489	0.000228	35.999211
27	7	-10.425840	-10.431973	35.999319
	8	0.001689	-14.702865	35.999389
	9	10.474989	-10.468237	35.999057
	10	15.013420	0.000785	35.998835
	11	-14.917340	0.001408	45.999370
	12	-10.394816	-10.392722	45.998999
	13	-0.698837	-14.621836	45.999870
	1	15.002587	0.000603	25.999638
	2	10.511846	-10.504251	26.000651
	3	0.001133	-14.773067	25.999725
	4	-10.467652	-10.466608	25.999955
	5	-14.930821	0.000831	25.999639
	6	-14.918989	0.001052	35.999534
28	7	-10.426156	-10.430643	35.999319
	8	0.001255	-14.706365	35.999390
	9	10.478678	-10.473549	35.999057
	10	15.013420	0.000785	35.998835
	11	-14.914340	0.001408	45.999370
	12	-10.395468	-10.392571	45.999499
	13	-0.698836	-14.625836	45.999871
	1	14.996587	0.000603	25.999639
	2	10.509638	-10.497961	26.000650
	3	0.001132	-14.776068	25.999416
	4	-10.471309	-10.469451	25.999955
	5	-14.931821	0.000831	25.999639
	6	-14.920989	0.000552	35.999211
29	7	-10.430942	-10.435867	35.999319
	8	0.001255	-14.709365	35.999390
	9	10.479648	-10.472579	35.999057
	10	15.007420	0.000785	35.998835
	11	-14.919840	0.001408	45.999370
	12	-10.402122	-10.398418	45.999000
	13	-0.698836	-14.627836	45.999871
	1	15.008588	0.000603	25.999637
	2	10.510687	-10.503910	26.000651
	3	0.001133	-14.772568	25.999416
	4	-10.464618	-10.461141	25.999955
	5	-14.920321	0.000831	25.999639
	6	-14.908988	0.000552	35.999210
30	7	-10.422226	-10.428591	35.999319
	8	0.001255	-14.711365	35.999390
	9	10.485494	-10.478733	35.999057
	10	15.020920	0.000785	35.998835
	11	-14.902839	0.001407	45.999370
	12	-10.389362	-10.389676	45.999499
	13	-0.698836	-14.625336	45.999871

N° part	N° point	Cylinder (CMM)		
		X	Y	Z
31	1	15.000087	0.000603	25.999638
	2	10.502709	-10.498388	26.000650
	3	0.001133	-14.772067	25.999725
	4	-10.470652	-10.469607	25.999955
	5	-14.931321	0.000831	25.999639
	6	-14.921489	0.000228	35.999211
	7	-10.427663	-10.433658	35.999319
	8	0.001690	-14.705365	35.999389
	9	10.475051	-10.473675	35.999057
	10	15.011920	0.000785	35.998835
	11	-14.919840	0.001408	45.999370
	12	-10.396468	-10.393571	45.999499
	13	-0.698837	-14.622836	45.999870
32	1	15.000087	0.000603	25.999638
	2	10.518001	-10.491612	26.000650
	3	0.001133	-14.770067	25.999725
	4	-10.474555	-10.453218	25.999954
	5	-14.927821	0.000831	25.999639
	6	-14.918489	0.000552	35.999210
	7	-10.435394	-10.419762	35.999319
	8	0.001690	-14.705365	35.999389
	9	10.490731	-10.462514	35.999056
	10	15.014420	0.000785	35.998835
	11	-14.915340	0.001408	45.999370
	12	-10.392816	-10.390223	45.999499
	13	-0.698837	-14.621836	45.999870
33	1	15.004087	0.000603	25.999971
	2	10.514227	-10.494378	26.000650
	3	0.001132	-14.778568	25.999725
	4	-10.474276	-10.456992	25.999954
	5	-14.927321	0.000831	25.999639
	6	-14.917489	0.000552	35.999210
	7	-10.438025	-10.421627	35.999319
	8	0.001691	-14.709365	35.999390
	9	10.489446	-10.464794	35.999056
	10	15.014420	0.000785	35.998835
	11	-14.914340	0.001408	45.999370
	12	-10.406303	-10.385248	45.999499
	13	-0.698836	-14.626836	45.999871
34	1	14.959464	-0.001678	26.001560
	2	10.522592	-10.521351	26.001930
	3	0.000003	-14.845535	26.001560
	4	-10.527768	-10.525083	26.001738
	5	-14.968943	-0.001622	26.002559
	6	-14.958610	-0.002229	36.001459
	7	-10.502778	-10.505614	36.001245
	8	0.000587	-14.810333	36.001315
	9	10.508748	-10.507825	36.000978
	10	14.971797	-0.001997	36.001254
	11	-14.955960	-0.001043	46.001292
	12	-10.485464	-10.484669	46.001427
	13	-0.699447	-14.760306	46.001799
35	1	15.006588	0.000603	25.999638
	2	10.503367	-10.498230	26.000650
	3	0.001133	-14.772067	25.999725
	4	-10.467464	-10.465296	25.999955
	5	-14.927821	0.000831	25.999639
	6	-14.920989	0.000552	35.999211
	7	-10.429106	-10.430675	35.999319
	8	0.001690	-14.705365	35.999389
	9	10.473676	-10.469050	35.999057
	10	15.013920	0.000785	35.998835
	11	-14.912340	0.000908	45.999370
	12	-10.393514	-10.393025	45.998999
	13	-0.698837	-14.620836	45.999370

N° part	N° point	Cylinder (CMM)		
		X	Y	Z
36	1	15.003087	0.000603	25.999638
	2	10.507027	-10.501070	26.000650
	3	0.001133	-14.771067	25.999725
	4	-10.470308	-10.468951	25.999955
	5	-14.931821	0.000831	25.999639
	6	-14.922989	0.001052	35.999211
	7	-10.430430	-10.432859	35.999319
	8	0.001690	-14.704365	35.999389
	9	10.480962	-10.471766	35.999057
	10	15.010920	0.000785	35.998835
	11	-14.920340	0.000584	45.999370
	12	-10.401167	-10.399372	45.999000
	13	-0.698836	-14.626336	45.999871
37	1	15.003587	0.000603	25.999638
	2	10.506343	-10.498754	26.000650
	3	0.001132	-14.775568	25.999725
	4	-10.471965	-10.469294	25.999955
	5	-14.931321	0.000831	25.999639
	6	-14.923989	0.000228	35.999211
	7	-10.432417	-10.437894	35.999320
	8	0.001691	-14.709865	35.999390
	9	10.479148	-10.472079	35.999057
	10	15.010920	0.000785	35.998835
	11	-14.920340	0.001408	45.999370
	12	-10.402167	-10.401372	45.999500
	13	-0.698836	-14.630836	45.999871
38	1	15.003087	0.000270	25.999638
	2	10.508186	-10.501411	26.000650
	3	0.001133	-14.771067	25.999725
	4	-10.466931	-10.462329	25.999955
	5	-14.925821	0.001156	25.999639
	6	-14.914988	0.001052	35.999210
	7	-10.422904	-10.430427	35.999319
	8	0.001689	-14.702365	35.999389
	9	10.478334	-10.472892	35.999057
	10	15.013420	0.000785	35.998835
	11	-14.910339	0.000584	45.999370
	12	-10.394816	-10.392222	45.999499
	13	-0.698837	-14.620336	45.999870
39	1	15.005088	0.000270	25.999638
	2	10.506026	-10.500071	26.000650
	3	0.001133	-14.770567	25.999725
	4	-10.464963	-10.462797	25.999955
	5	-14.929821	0.000831	25.999639
	6	-14.919989	0.000228	35.999211
	7	-10.425195	-10.431624	35.999319
	8	0.001690	-14.707365	35.999390
	9	10.479021	-10.474705	35.999057
	10	15.014920	0.000785	35.998835
	11	-14.916340	0.000584	45.999370
	12	-10.394514	-10.393525	45.999499
	13	-0.698837	-14.621336	45.999870
40	1	15.003087	0.000270	25.999638
	2	10.506185	-10.499412	26.000650
	3	0.001133	-14.773068	25.999416
	4	-10.465651	-10.464608	25.999955
	5	-14.930321	0.000831	25.999639
	6	-14.919489	0.000228	35.999211
	7	-10.424030	-10.428775	35.999319
	8	0.001690	-14.708365	35.999390
	9	10.479865	-10.476861	35.999057
	10	15.011920	0.000785	35.998835
	11	-14.917340	0.000584	45.999370
	12	-10.391862	-10.391676	45.999499
	13	-0.698837	-14.621836	45.999870

N° part	N° point	Cylinder (CMM)		
		X	Y	Z
	1	15.002087	0.000270	25.999638
	2	10.507844	-10.500253	26.000650
	3	0.001133	-14.771567	25.999725
	4	-10.469432	-10.464828	25.999955
	5	-14.927821	0.000831	25.999639
	6	-14.916489	0.000229	35.999210
41	7	-10.427802	-10.430992	35.999319
	8	0.001690	-14.704865	35.999389
	9	10.479648	-10.472079	35.999057
	10	15.009920	0.000785	35.998835
	11	-14.915840	0.000584	45.999370
	12	-10.397514	-10.397024	45.999000
	13	-0.698836	-14.627336	45.999371
	1	15.003587	0.000603	25.999638
	2	10.507527	-10.501570	26.000650
	3	0.001132	-14.778068	25.999415
	4	-10.470933	-10.466327	25.999955
	5	-14.930821	0.000831	25.999639
	6	-14.919989	0.000228	36.000035
42	7	-10.429125	-10.433176	35.999319
	8	0.001691	-14.710865	35.999390
	9	10.480178	-10.475048	35.999057
	10	15.013920	0.000785	35.999335
	11	-14.915340	0.000584	45.999370
	12	-10.396817	-10.394222	45.999499
	13	-0.698836	-14.627836	45.999871
	1	15.004087	0.000603	25.999638
	2	10.510638	-10.499461	26.000650
	3	0.001132	-14.776068	25.999416
	4	-10.470745	-10.464016	25.999955
	5	-14.929821	0.000831	25.999639
	6	-14.922489	0.001052	36.000035
43	7	-10.431417	-10.433372	35.999319
	8	0.001691	-14.710865	35.999390
	9	10.487063	-10.472167	35.999057
	10	15.011920	0.000785	35.998835
	11	-14.918840	0.000584	45.999370
	12	-10.397468	-10.394571	45.999000
	13	-0.698837	-14.623336	45.999371
	1	14.961464	-0.002178	26.001560
	2	10.525432	-10.525510	26.002750
	3	0.000002	-14.850535	26.001559
	4	-10.528448	-10.527403	26.002558
	5	-14.972943	-0.001951	26.002229
	6	-14.962110	-0.001901	36.001459
44	7	-10.502951	-10.507455	36.001245
	8	0.000588	-14.813333	36.001315
	9	10.509907	-10.508166	36.000978
	10	14.972797	-0.001668	36.001254
	11	-14.956960	-0.001370	46.001293
	12	-10.486436	-10.482698	46.001427
	13	-0.699754	-14.763306	46.001799
	1	15.000087	0.000270	25.999638
	2	10.511882	-10.491724	26.000650
	3	0.001132	-14.774568	25.999725
	4	-10.477593	-10.458677	25.999955
	5	-14.928321	0.000831	25.999639
	6	-14.919989	0.001052	35.999210
45	7	-10.435789	-10.428427	35.999319
	8	0.001690	-14.707365	35.999390
	9	10.482809	-10.461428	35.999056
	10	15.007920	0.000785	35.998835
	11	-14.910339	0.000584	45.999370
	12	-10.401785	-10.388758	45.999499
	13	-0.698836	-14.629836	45.999871

N° part	N° point	Cylinder (CMM)		
		X	Y	Z
	1	15.002087	0.000270	25.999638
	2	10.516364	-10.493744	26.000650
	3	0.001132	-14.774068	25.999416
	4	-10.478439	-10.460330	25.999955
	5	-14.930821	0.000831	25.999639
	6	-14.919989	0.000228	35.999211
46	7	-10.436575	-10.425611	35.999319
	8	0.001690	-14.708865	35.999390
	9	10.488318	-10.466918	35.999057
	10	15.011920	0.000785	35.999669
	11	-14.916840	0.000584	45.999370
	12	-10.405896	-10.390150	45.998999
	13	-0.698837	-14.622836	45.999870
	1	14.999587	0.000270	25.999638
	2	10.514677	-10.489933	26.000650
	3	0.001133	-14.772568	25.999416
	4	-10.480157	-10.453124	25.999954
	5	-14.931821	0.000831	25.999639
	6	-14.920489	0.001052	35.999210
47	7	-10.436581	-10.425105	35.999319
	8	0.001690	-14.708365	35.999390
	9	10.487598	-10.462143	35.999056
	10	15.018420	0.000785	35.998835
	11	-14.918840	0.000584	45.999370
	12	-10.403939	-10.390105	45.999499
	13	-0.698836	-14.625336	45.999871
	1	15.002087	0.000270	25.999638
	2	10.517842	-10.492270	26.000650
	3	0.001132	-14.778068	25.999416
	4	-10.482695	-10.458082	25.999955
	5	-14.929821	0.000831	25.999639
	6	-14.920489	0.001052	35.999210
48	7	-10.440498	-10.424155	35.999319
	8	0.001691	-14.711865	35.999390
	9	10.491105	-10.465636	35.999056
	10	15.011420	0.000785	35.998835
	11	-14.915840	0.000584	45.999370
	12	-10.410112	-10.387441	45.999499
	13	-0.698836	-14.628336	45.999871
	1	15.004587	0.000270	25.999971
	2	10.510563	-10.492041	26.000650
	3	0.001133	-14.773568	25.999416
	4	-10.477437	-10.459332	25.999955
	5	-14.931821	0.000831	25.999639
	6	-14.922489	0.001052	36.000035
49	7	-10.439192	-10.424972	35.999319
	8	0.001690	-14.708865	35.999390
	9	10.485783	-10.462456	35.999056
	10	15.013920	0.000785	35.998835
	11	-14.921840	0.000584	45.999370
	12	-10.404894	-10.388652	45.999499
	13	-0.698837	-14.622336	45.999870
	1	15.004588	0.000270	25.999638
	2	10.510195	-10.488413	26.000650
	3	0.001133	-14.772568	25.999416
	4	-10.476278	-10.458490	25.999955
	5	-14.930321	0.000831	25.999639
	6	-14.919989	0.001052	36.000034
50	7	-10.436422	-10.425769	35.999319
	8	0.001690	-14.706365	35.999390
	9	10.483466	-10.461271	35.999056
	10	15.013920	0.000785	35.999335
	11	-14.917840	0.000584	45.999370
	12	-10.403043	-10.387003	45.999499
	13	-0.698837	-14.623336	45.999870

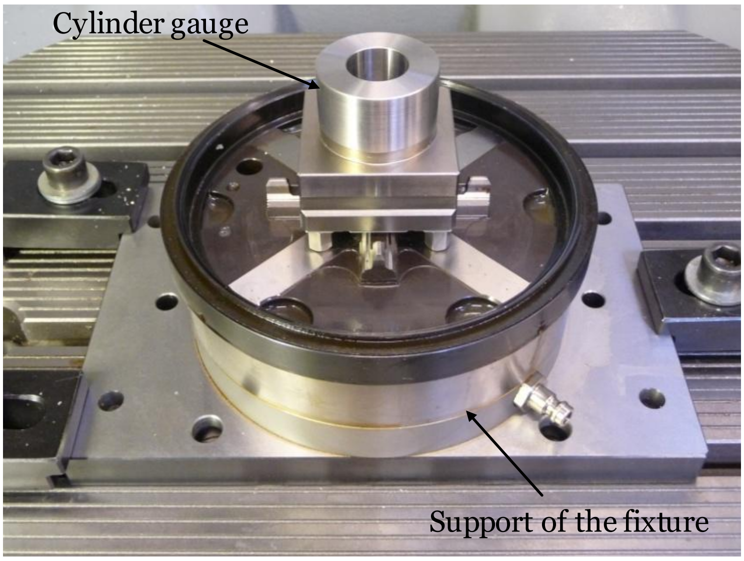
APPENDIX 4 – LOCATING THE THREE-SOFT-JAW CHUCK ON THE CNC MACHINE (DMG DEKEL MAHO-DMU50)

The three-soft-jaw chuck (fixture) is located on the rotary table of the CNC machine by the following steps:

First step

Firstly, a cylinder gauge is fixed in the center of the rotary table of the CNC machine. This cylinder gauge is used to adjust the origin (zero-offset) of the probe of the machine.

The fixture is then located on the support (EROVA) that fixed on the rotary table of the CNC machine. Therefore, the center of the support must be adjusted to coincide with the fourth axes of the machine. To do this, a cylinder gauge is located on the support for adjustment as following figure.

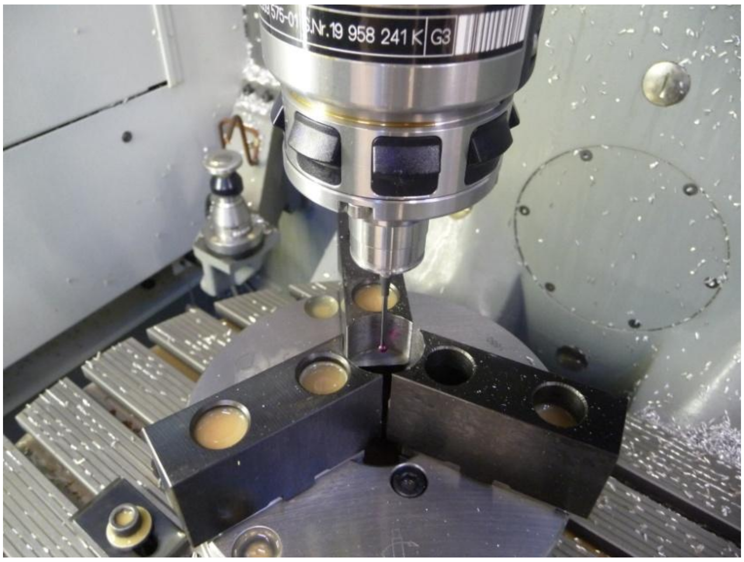


Second step

The three-soft-jaw chuck is fixed on the support. A cylinder 30 mm diameter and 20 mm high is then machined on these three soft jaws by an end mill of this machine (the following figure).

Third step

A machine's probe is used to measure the machined cylinder surfaces (cylinder surface and plane surface) for creating a machine coordinate system (MCS) on this fixture. This coordinate system has the Z-axis that coincides with the cylinder axis and the OXY plan is on the machined plane of the three soft jaws.



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Identification et simulation des incertitudes de fabrication

Résumé :

L'étude présente les méthodes pour identifier et simuler les défauts de fabrication tridimensionnels. Les méthodologies ont été élaborées sur la base des travaux antérieurs, tels que la méthode de simulation MMP (Model of Manufactured Part) présentée par F. Villeneuve et F. Vignat, associée à la méthode de la double mesure présentée par S. Tichadou.
Dans cette thèse, la première méthode proposée, basée sur la méthode des petits déplacements (TPD) est présentée et permet l'identification des défauts de fabrication. Cette méthode permet de distinguer les défauts d'usinage et les défauts de positionnement d'un lot de pièces au cours d'un processus de fabrication. Les résultats obtenus dans cette méthode représentent les dispersions géométriques des pièces usinées. En outre, une méthode d'analyse modale de défauts a été réalisée pour analyser les défauts de forme d'une pièce mesurée sur une MMT avec un nombre restreint de points de mesure (10 points sur chaque surface usinée). Les résultats montrent que les modes des défauts de forme sont obtenus correctement (bombé, ondulation, vrillage, etc.)

En raison de l'importance du rôle du défaut de positionnement dans la qualité d'un produit en cours de fabrication, ensuite deux indicateurs simples ont été proposés pour évaluer la qualité globale d'un montage de fixation de pièces.

Par ailleurs, un modèle permettant de simuler les défauts de positionnement d'une pièce fixée sur un mandrin à trois mors a été développé. Le modèle final de simulation est une combinaison de trois méthodes: plan d'expérience, simulation par éléments finis, et simulation de Monte Carlo. Pour la méthode des plans d'expérience, trois facteurs, qui sont supposés être les plus importants dans les défauts de positionnement, sont utilisés dans le modèle. Les résultats obtenus à partir des simulations sont exprimés sous forme de distributions et de paramètres statistiques caractéristiques. Ceux-ci sont ensuite utilisés pour effectuer les simulations en appliquant la méthode de Monte Carlo.

Enfin, un modèle global est proposé, pour simuler la gamme de fabrication d'une pièce fraisée. Ce modèle permet de vérifier la gamme choisie avec des tolérances fonctionnelles de la pièce imposée. De plus, cette méthode permet de vérifier une gamme de fabrication en garantissant les tolérances fonctionnelles imposées ou une utilisation inverse qui permet de déterminer les tolérances garantissant un nombre de pièces usinées hors des zones de tolérance.

Mots-clés :
Incertitudes de fabrication, identification des défauts de production, simulation de gamme, analyse de tolérance en fabrication, indicateurs de qualité

Identification and simulation of manufacturing uncertainties

Abstract:

The research presents methodologies to identify and simulate manufacturing defects in three-dimension. The methodologies have been developed based on the previous works, such as the MMP (Model of Manufactured Part) simulation method presented by F. Villeneuve and F. Vignat, and the double measurement method is presented by S. Tichadou.
In this thesis, the first proposed method based on the Small Displacement Torsor (SDT) concept is presented for identification of manufacturing defects. This method allows distinguishing the machining defects and positioning defects of a batch of parts during a process plan. The results obtained in this method represent geometric dimension errors of machined parts. In addition, we applied the parameterization method, which is usually used to analyze form defects of a part measured on a CMM with hundreds of measurement points, to complete the analysis of the form defects with a restricted number of measurement points (10 points on each machined surface). Even though this number appears to be low, the modes of the form defects are almost obtained (comber, undulation, twist, etc).

Because of the important role of the positioning defect in the quality of a product during manufacturing, we then propose two simple indicators for evaluating the global quality of a fixture.

Furthermore, we developed a model for simulating positioning defects of a workpiece fixed on a three-jaw chuck. The model is a combination of three methods: design of experiments, finite element simulation, and Monte Carlo simulation. Three factors, which are assumed to be the most important in positioning defects, are used in this model. Based on the simulated results, the influences of these factors are estimated. The results obtained from simulations can be expressed by form of distributions or statistical parameters. These allow using simulation of tolerance analysis based on Monte Carlo simulation.

Finally, a model is developed based on MMP for tolerance analysis. This model allows us to verify a given process plan with functional tolerances of the machined part by determination of a number of machined parts out of tolerance zones or determine functional tolerances of a batch of machined parts based on a given process plan (without functional tolerances) and a number of rejected parts per million.

Key words:
Manufacturing defects, identification of manufacturing defects, simulation a process plan, tolerance analysis, indicators of quality